Richer countries supply and demand higher-quality products. We study this in an extended Heckscher-Ohlin framework on the supply side with a continuum of industries, each with differentiated products produced under monopolistic competition. In addition to choosing price, each firm in each country also simultaneously chooses quality. The cost of producing goods of a given quality depends on factor prices. We employ the AIDS demand system to model consumer demand with “quality” and “quantity” multiplying each other in the utility function. We estimate this system using detailed bilateral trade data and occupational wage data for over 100 countries for 1984-2002. Our system identifies quality-adjusted prices from which we will construct price indexes for imports and exports for each country. These price indexes have important applications. They directly enable the calculation of the terms of trade, and therefore they enable the calculation of output-side measures of real GDP (which measure the production possibilities of an economy) in addition to existing expenditure-side measures which are real expenditures adjusted for the trade balance. The difference between these two measures is essentially the terms of trade – and these differences can be substantial for small open economies.
1. Introduction

The idea is to develop a model of trade with quality choice by firms. From recent empirical work by Peter Schott and Juan-Carlos Hallak, we expect that richer countries will supply higher-quality products, and possibly also demand higher-quality products. We propose to study this in an extended HO framework, much like Romalis (2004) on the supply side. That is, there will be a continuum of industries with Cobb-Douglas demand over these. Each industry has a number $N$ of differentiated products, produced under monopolistic competition. In addition to choosing price, each firm in each country will also choose quality, denoted by the vector $z_i$ (we might change this to a scalar for convenience). In the notes below we do not distinguish the “industries” at all, but deal with only a single industry.

On the demand side, consumers in each industry demand the differentiated products, possibly with a non-homothetic utility function. (We presume that two-stage budgeting is still valid, in view of the Cobb-Douglas assumption across industries). In section 2, we deal with a general functional form for utility (also used in Feenstra, 2004, chapter 5), but which has the special conditions that:

(i) “quality” and “quantity” multiply each other in the utility function;

(ii) firms choose quality and prices simultaneously.

Under these two conditions, we show that there is a strong separability result: firm’s choice of product quality is independent of the “iceberg” transport costs, but will depend on the marginal cost of providing characteristics. We can model this as depending on factor price differences across countries, so country factor prices/endowments will determine quality.

In section 3 and 4, we begin to explore the special case of an AIDS demand system. This is a specific way to model non-homothetic demand that might be useful. The translog function –
which is a special case of the AIDS when demand is homothetic – has been used in monopolistic competition and trade model by Bergin and Feenstra (2000, 2001), and extended to varying numbers of products by Feenstra (2003) (see the Appendix here). That functional form allows markups to vary, which can be desirable in theory or for empirical work. These notes are open-ended and just indicate some progress to date.

2. Demand Model with Quality

Suppose that there are \(i=1,\ldots,N\) varieties of a differentiated product. We use the index \(i\) for products and also for the country of origin for each product. Country \(i\) need not send the same quality to each country \(j\), however: the origin country \(i\) will choose the quality characteristics \(z_i^j\) to send to country \(j\). We will suppose that the demand for the products in country \(j\) arises from an aggregate utility function, given by:

\[
U[f(z^i)c_i^j, \ldots, f(z^N)c_N^j],
\]

where \(c_i^j\) denotes the consumption of each variety, \(i=1,\ldots,N\), and the function \(f(\cdot)\) converts the vector of characteristics \(z_i^j\) into a scalar “quality” \(f(z_i^j)\), which then multiplies consumption. The functions \(f\) and \(U\) are common across destination countries \(j\).

We suppose there are a general form of “iceberg” transportation costs between the countries, so that \(T_i^j = T(z_i^j, d_i^j) \geq 1\) units of the good \(i\) must be exported in order for one unit to arrive in country \(j\), where \(d_i^j\) denotes the distance between the countries. Notice that we are allowing the transportation costs to depend on the quality choice (you might think of insurance costs as an example). We let \(p_i^j\) denote the f.o.b. price received by the firm in country \(i\), and then inclusive the transportation costs, the c.i.f. price abroad is \(T_i^j p_i^j\).
Consumers in country $j$ are presented with a set of $i=1,\ldots,N$ varieties, with characteristics $z_i^j$ and c.i.f. prices $T_i^j p_i^j$, and then choose the optimal quantity of each variety. It will be convenient to work with the “quality-adjusted” c.i.f. prices, which are defined by
\[
\bar{p}_i^j \equiv T_i^j p_i^j / f(z_i^j).
\]
That is, the higher is overall product quality $f(z_i^j)$, the lower are the quality-adjusted prices $\bar{p}_i^j$. The aggregate consumer maximizes utility in (1), subject to the budget constraint $\sum_{i=1}^N T_i^j p_i^j c_i^j \leq Y^j$. The Lagrangian for country $j$ is,
\[
L = U[f(z_1^j)c_1^j,\ldots,f(z_N^j)c_N^j] + \lambda (Y^j - \sum_{i=1}^N T_i^j p_i^j c_i^j)
\]
\[
= U(\bar{c}_1^j,\ldots,\bar{c}_N^j) + \lambda (Y^j - \sum_{i=1}^N \bar{p}_i^j \bar{c}_i^j),
\]
where the second line of (2) follows by defining $\bar{c}_i^j \equiv f(z_i^j)c_i^j$ as the effective “quality-adjusted” demand, and also using the quality-adjusted prices $\bar{p}_i^j \equiv T_i^j p_i^j / f(z_i^j)$. This re-writing of the Lagrangian makes it clear that instead of choosing $c_i^j$ given prices $T_i^j p_i^j$ and characteristics $z_i^j$, we can instead think of the aggregate consumer as choosing $\bar{c}_i^j$ given quality-adjusted prices $\bar{p}_i^j$, $i=1,\ldots,N$. Let us denote the solution to problem (2) by $\bar{c}_i^j(\bar{p}_i^j, Y^j)$, $i = 1,\ldots,N$, where $\bar{p}_i^j$ is the vector of quality-adjusted prices (the demand functions are common across countries).

Producing one unit of product $i$ with characteristics $z_i^j$ requires marginal costs of $g(z_i^j, w_1)$, where $w_i$ are factor prices in country $i$. Firms simultaneously choose prices f.o.b. prices $p_i^j$ and characteristics $z_i^j$ for each destination market. Revenue received from producing $y_i^j$ in country $i$ and exporting to country $j$ is $T_i^j p_i^j c_i^j = p_i^j y_i^j$, where output is related to consumption by $y_i^j = c_i^j T_i^j$. Then profits from exporting to all destination countries are:
The first equality in (3) converts from f.o.b. to c.i.f. prices, and the second equality converts to quality-adjusted prices \( \tilde{p}_i^j \) and demands \( \tilde{z}_i^j \). The latter transformation relies on our assumption that \( \text{prices and characteristics are chosen simultaneously} \), as well as our special functional form in (1), whereby quality multiplies quantity.

It is immediate that to maximize profits in (3), the firms must choose \( z_i^j \) to minimize

\[
T(z_i^j, d_i^j)g(z_i^j, w_i) / f(z_i^j),
\]

which is interpreted as the \textit{c.i.f. costs per unit of quality} for the good that country \( i \) send to \( j \). Then taking logs and minimizing over the choice of characteristics \( z_i^j \) leads to the first-order conditions,

\[
\frac{f_z(z_i^j)}{f(z_i^j)} = \frac{g_z(z_i^j, w_i)}{g(z_i^j, w_i)} + \frac{T_z(z_i^j, d_i^j)}{T(z_i^j, d_i^j)}, \quad i, j = 1, \ldots, N. \tag{4}
\]

Thus, we obtain equality between the relative marginal utility from each characteristic and its relative marginal cost, inclusive of transport costs, similar to Rosen (1974). Define the solution to minimizing \( T(z_i^j, d_i^j)g(z_i^j, w_i) / f(z_i^j) \) as:

\[
h(w_i, d_i^j) \equiv \min_{z_i^j} T(z_i^j, d_i^j)g(z_i^j, w_i) / f(z_i^j). \tag{5}
\]

This is the “envelope” of c.i.f. costs per unit of quality, depending on factor prices \( w_i \) and the distances between countries. We can substitute this expression back into (3). Then to determine the optimal quality-adjusted price \( \tilde{p}_i^j \), differentiate (3) to obtain:
\[ \tilde{c}_i^j (\tilde{p}_i^j, Y^j) + \left[ \tilde{p}_i^j - h(w_i, d_i^j) \right] \frac{\partial \tilde{c}_i^j}{\partial \tilde{p}_i^j} = 0. \quad i=1,\ldots,N. \] (6)

We let \( \eta_i^j (\tilde{p}_i^j, Y^j) \equiv -\partial \ln \tilde{c}_i (\tilde{p}_i^j, Y^j) / \partial \ln \tilde{p}_i^j \) denote the elasticity of demand from country \( i \) variety sold in country \( j \). Then (6) can be re-written as the familiar condition:

\[ \tilde{p}_i^j = \left[ \frac{\eta_i^j (\tilde{p}_i^j, Y^j)}{\eta_i'(\tilde{p}_i^j, Y^j) - 1} \right] h_i^j (w_i, d_i^j), \quad i=1,\ldots,N. \] (6')

Recalling that \( \tilde{p}_i^j = T_i^j p_i^j / f(z_i^j) \), We can therefore solve for the c.i.f. prices as a log-linear function:

\[ \ln(T_i^j p_i^j) = \ln f(z_i^j) + \ln h_i^j (w_i, d_i^j) + \ln \left[ \frac{\eta_i'(\tilde{p}_i^j, Y^j) - 1}{\eta_i'(\tilde{p}_i^j, Y^j) - 1} \right]. \] (7)

In addition, notice that the transport costs are:

\[ \ln(T_i^j p_i^j) - \ln(p_i^j) = \ln T(z_i^j, d_i^j). \] (8)

3. **Empirical Strategy**

Run (8) using the difference between c.i.f. importer prices and f.o.b. exporter prices as the dependent variable. On the RHS we include distances and exporter wages, since the latter indirectly affects the choice of characteristics. From this first stage regression we can get estimated transports cost \( \ln \hat{T}_i^j \).

Write the quality-adjusted c.i.f. costs \( h(w_i, d_i^j) \) as a Cobb-Douglas function over factor prices \( w_{ik} \), for factors \( k=1,\ldots,K \). We allow the factors in country \( i \) to each have their own efficiency level \( \pi_{ik} \), as in Trefler (1993), so that the effective factors prices are \( w_{ik}/\pi_{ik} \). Using
estimated transports cost $\ln \hat{T}^j_{it}$ from this first stage regression, and adding a year subscript $t$, the costs can be written as:

$$h(w_i, d^j_{it}) \equiv \sum_{k=1}^{K} \theta_{ik} [\ln w_{ikt} - \ln \pi_{ik}] + \ln \hat{T}^j_{it}.$$ (9)

We model quality as depending on exporter fixed effects, a time trend, and the distance to the importers:

$$\ln f(z^j_{it}) \equiv \delta_{1i} + \delta_{2i}t + \delta_{3} \ln d^j_{it}.$$ (10)

Substituting these in the c.i.f. pricing equation, we get:

$$\ln (T_{it}^j, p^j_{it}) = \delta_{1i} + \delta_{2i}t + \delta_{3} \ln d^j_{it} + \sum_{k=1}^{K} \theta_{ik} [\ln w_{ikt} - \ln \pi_{ik}] + \ln \hat{T}^j_{it} + \ln [\eta^j_{it} / (\eta^j_{it} - 1)].$$ (11)

In order to identify the factor efficiencies, we use the assumption:

$$\sum_{k=1}^{K} \theta_{ik} \pi_{ik} = \rho_0 + \rho_1 \ln f(z^j_{it}) - \varepsilon^j_{it}.$$ (12)

In other words, the average efficiency of factor is correlated with the quality of goods: goods of higher quality and produced by more productive factors. Substituting (12) into (11), the pricing equation becomes:

$$\ln p^j_{it} = -\rho_0 + (\bar{\delta}_{1i} + \bar{\delta}_{2i}t + \bar{\delta}_{3} \ln d^j_{it}) + \sum_{k=1}^{K} \theta_{ik} \ln w_{ikt} + \ln \hat{T}^j_{it} + \ln [\eta^j_{it} / (\eta^j_{it} - 1)] + \varepsilon^j_{it},$$ (15)

where $\bar{\delta}_{1i} \equiv (1 - \rho_1)\delta_{1i}$ and likewise for $\bar{\delta}_{2i}$, and $\bar{\delta}_{3}$. Initially (15) is estimated without the markup term on the RHS. To include that, we add a demand side. The demand side gives us additional power in identifying quality from the non-homotheticity of demand. In the next section we explore the AIDS system, to see whether the
demand side of this equation can help identify the quality-adjusted prices, and solve for the  
elasticities of demand that appear on the right of (8).

4. Demand System

The expenditure needed in country \( k \) to obtain a utility level of \( U_j \) under the AIDS is:

\[
\ln E(q^k, U^k) = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln q_i^k + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \ln q_i^k \ln q_j^k + u\beta_0 \prod_{j=1}^{N} (q_j^k)^{\beta_j}, \tag{16}
\]

where without loss of generality we set \( \gamma_{ij} = \gamma_{ji} \). The restrictions \( \sum_{i=1}^{N} \alpha_i = 1 \) and \( \sum_{j=1}^{N} \gamma_{ij} = \sum_{k=1}^{N} \beta_k = 0 \) ensure that \( E(q^k, U^k) \) is homogeneous of degree one in \( q^k \).

Differentiating (16) wrt. \( \ln q_j^k \) we obtain the expenditure shares:\(^1\)

\[
s_i^k = \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln q_j^k + u\beta_0 \prod_{j=1}^{N} (q_j^k)^{\beta_j}, \quad i = 1, \ldots, N,
\]

\[
= \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln q_j^k + \beta_i \ln (Y^k / P^k), \tag{17}
\]

where the second line follows from (16) by denoting expenditure as \( Y^k = E(q^k, U^k) \), and defining \( P^k \) as an aggregate of the prices:

\[
\ln P^k = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln q_i^k + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \ln q_i^k \ln q_j^k. \tag{18}
\]

Notice that good \( j \) is a luxury (necessity) as \( \beta_j >(<) 0 \), with an expenditure share rising (falling) in  
income, so the underlying utility function is non-homothetic. If \( \beta_j = 0 \) for \( j = 1, \ldots, N \), then (16)  

\(^1\) Notice that the expenditure shares defined in terms of quality-adjusted prices and quantities,  
or unadjusted prices and quantities, are identical.
reduces to the translog function. In the Appendix we summarize a few properties of the translog system as the number of goods varies, drawing on Feenstra (2003).

Working with (17) and (18), the elasticity of demand is:

\[ \eta_i^k = 1 - \left( \frac{d \ln s_i^k}{d \ln q_i^k} \right) = 1 - \frac{\gamma_{ii}}{s_i^k} + \beta_i \left( \frac{s_i^k - \beta_i \ln(Y^k / P^k)}{s_i^k} \right) \]

\[ = 1 + \beta_i - \left( \frac{\gamma_{ii} + \beta_i^2 \ln(Y^k / P^k)}{s_i^k} \right). \]  

(19)

Thus, once we have an estimate of the share equation in (17), then the elasticity of demand can be computed as in (19), and substituted back into the pricing equation (15).

5. Estimating Equations with Demand

Re-write (17) while including the time subscripts as:

\[ s_{it}^k = \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln(T_{ij}^k p_{jt}^k) - \ln f(z_{jt}^k) + \beta_i \ln(Y_{it}^k / P_{it}^k) \]

\[ = \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln(T_{ij}^k p_{jt}^k) - \frac{1}{(1-\rho_{it})} \sum_{j=1}^{N} \gamma_{ij} (\tilde{\delta}_{jt}^k) + \beta_i \ln(Y_{it}^k / P_{it}^k), \]  

(20)

where \( \tilde{\delta}_{jt}^k \equiv (\tilde{\delta}_{1jt} + \tilde{\delta}_{2jt} + \tilde{\delta}_{3jt} d_{jt}^k) \) is the estimated quality from (15). Thus, we estimate the share equation using observed prices on the right, as well as the estimated quality obtained from (15). Then we adjust the quality terms by the coefficient 1/(1-\( \rho_{it} \)) obtained from (20), to obtain the “true” quality \( \delta_{jt}^k = \tilde{\delta}_{jt}^k / (1-\rho_{it}) \). These are subtracted from the observed prices to obtain the quality-adjusted prices.

The empirical strategy for estimating (15) and (16) will be to estimate them iteratively. That is, we could begin with the share equations run on observed prices; then use those estimates
to obtain the elasticities of demand and run the pricing equations; then return to the share equations using those estimates of product quality, etc.

To simplify this system impose symmetry of the coefficients as follows, from the Appendix:

\[
\alpha_i = 1/N, \quad \gamma_{ii} = -\gamma(N - 1)/N, \quad \text{and} \quad \gamma_{ij} = \gamma/N \quad \text{for} \quad i \neq j, \quad \text{with} \quad i, j = 1, \ldots, N. \tag{21}
\]

Then the share equation becomes:

\[
s_{it}^k = \frac{1}{N} - \gamma \left[ \ln(T_{it}^k p_{it}^k) - \ln(T_{it}^k p_{it}^k) \right] + \gamma \left[ \tilde{\delta}_{jt} - \tilde{\delta}_{it} \right] + \beta_1 \ln(Y_t^k / P_t^k). \tag{22}
\]

6. Data

*International Trade Data*

Bilateral trade values and quantities at the 4-digit SITC Revision 2 level are from NBER-United Nations Trade Data, 1962-2000.

*Tariffs*

Bilateral tariff data is from Jon Haveman’s TRAINS extracts; the raw TRAINS files obtained from the World Bank’s WITS site; and tariff schedules scanned from the International Customs Journal. Tariffs are converted from the tariff-line level to 4-digit SITC Revision 2 level using simple averages.

*Wages*

Industry wage data at the 3-digit ISIC Revision 2 level are from UNIDO industry wage data and the ILO yearbook, Tables 5A and 5B.

*World Development Indicators*

Data on population and GDP are from the World Bank World Development Indicators.

7. Results
Observed unit values are first decomposed into quality and a quality-adjusted price components using the supply Equation (15). The quality and observed unit prices are then used in the demand Equation (22) - in a sense the decomposition has to prove itself in the demand equation. The first key parameter estimated in the demand Equation (22) is $\gamma$ - which gives how import share responds to observed price movements, conditional on quality. A separate $\gamma$ is estimated for each combination of importing country and 4-digit SITC Revision 2 product. The mean estimate for $\gamma$ is -.014 with the median at -0.013. The distribution of $\gamma$ estimates is given in Figure 1 - over 90% of estimates are negative. These estimates will be used to calculate demand elasticities below.

**Figure 1: How Import Shares Respond to Prices: Distribution of Estimates of $\gamma$**

The coefficients $\gamma$ can be converted into elasticities of demand $\eta$ using Equation (19). The mean demand elasticity is estimated as 6.8, while the median is 1.5. The distribution of demand elasticity estimates is given in Figure 2.
The second key coefficient estimated in the demand Equation (22) is on the “quality” variable in the demand equation (the relationship between the quality variable and true quality is explained above at Equation (15)). The mean and median estimate for how import share responds to “quality” is .011, and the distribution is depicted in Figure 3. Almost 90% of coefficient estimates are positive - import share responds positively to estimated quality.
The difference between the coefficients on the observed prices and on the “quality” variable allows us to estimate the coefficient $1/(1-\rho_1)$ using Equation (20), allowing us to observe “true” quality. These quality estimates can then be subtracted from the observed prices to obtain quality-adjusted prices. The parameter $\rho_1$ is also of direct interest - our identification assumption (12) says that factor-efficiency is correlated with the quality of goods. Figure 4 shows the distribution of the parameter - two-thirds of estimates suggest that factor efficiency is positively correlated with the quality of the goods produced. It will be possible to use the model to back out factor-efficiency estimates using Equation (14).

**Figure 4: Association Between Quality and Factor Efficiency**
8. Quality-Adjusted Prices and Terms of Trade

[Quality-adjusted prices will be used to construct import and export price indexes and terms of trade measures. These measures can be used to correct the PWT so that real GDP can also be measured as the growth in real output rather than just the growth in real expenditure - see Feenstra et al. (2006).]

9. Conclusion

We develop a supply and demand framework that enables us to decompose observed unit import and export prices into a quality component and a quality-adjusted price component. We estimate this system using detailed bilateral trade data and occupational wage data for over 100 countries for 1984-2002, and plan to extend this back to 1962. Our system identifies quality-adjusted prices from which we will construct price indexes for imports and exports for each country. These price indexes have important applications. They directly enable the calculation of the terms of trade, and therefore they enable the calculation of output-side measures of real GDP.
(which measure the production possibilities of an economy) in addition to existing expenditure-side measures which are real expenditures adjusted for the trade balance. The difference between these two measures is essentially the terms of trade – and these differences can be substantial for small open economies.
Appendix

Feenstra (2003) considers the translog function as the number of goods N varies. In principal, we should keep track of reservation prices for goods not available, i.e. prices at which demand is identically zero. But it turns out that keeping track of reservation prices can be simplified by using a symmetric translog function, by which we mean that the parameters $\alpha_i$ are equal across goods, and the parameters $\gamma_{ij}$, for $i \neq j$, are equal across goods. In that symmetric case, we can solve for the reservation prices, and substitute them back into the expenditure function to obtain (omitting the country superscript):

$$\ln E(q, U) = a_0 + \sum_{i=1}^{N} \alpha_i \ln q_i + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \ln q_i \ln q_j + \ln U, \quad (A1)$$

where there is a parameters $\gamma > 0$ with:

$$\alpha_i = 1/N, \; \gamma_{ii} = -\gamma(N-1)/N, \; \text{and} \; \gamma_{ij} = \gamma/N \; \text{for} \; i \neq j, \; \text{with} \; i, j = 1, \ldots, N. \quad (A2)$$

In other words, we can use the translog function shown in (A1)-(A2) without having to explicitly keep track of reservation prices: they are solved for in the background. This might be useful if we extend the theory to the free-entry case. We will later check that similar results holds for the AIDS system.
References


