Gravity or Dummies?
The Limits of Identification in Gravity Estimations

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Abstract
This paper argues that identification of trade policy effects with a gravity equation that includes country-time dummies to control for the theoretical Multilateral Trade Resistances (MTR) is severely limited. In most cases heterogeneous policy effects, i.e. more than one policy dummy, cannot be identified separately, because the policy dummies and the country-time dummies are perfectly collinear. Although a single policy dummy can be identified, the estimate may not be meaningful, because country-time dummies absorb too much of the useful variation of the data. Standard estimation techniques often do not reveal these problems. The paper demonstrates these arguments by taking four typical research questions on the effect of a trade policy, checking the identifiability of the corresponding policy effects and deriving the estimates. Empirical exercise on estimating the trade effects of EU enlargement complements the analytical findings.

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1 Introduction

Measuring the effects of trade policy changes on bilateral trade flows has always been a central issue in the empirical trade literature. These effects can be heterogeneous across groups of country pairs or asymmetric regarding the direction of trade. Rose (2004) examines differential effects of one-sided and joint WTO membership. Baldwin, Skudelný and Taglioni (2005) and Flam and Nordström (2006) find that the euro increased trade not only among members but, to a lesser extent, also between members and non-members. Learning about such heterogeneous effects is essential to know how trade policies work. In what way such effects can be identified in empirical research is not always clear, however. Since the seminal paper of Anderson and van Wincoop (2003) empirical applications of the gravity equation, the workhorse model of trade, has been challenged by the need to account for the so-called Multilateral Trade Resistances (henceforth, MTR). The MTRs of the exporter and the importer in the theoretical gravity equation are average measures of trade barriers the exporter faces in, and the importer imposes on, all the countries in the world, and are linked to all bilateral trade costs and to each other non-linearly.

This paper considers one particular empirical specification of the gravity equation, which aims at controlling for the MTRs with dummies, and examines its ability to identify (potentially heterogeneous) effects of trade policy changes. The gravity specification is what I call the “fixed-effects country-time dummies gravity specification” for panel data. It includes fixed effects for all country pairs and a full set of exporter-time and importer-time dummies, the latter controlling for the time-varying MTRs. The same specification was proposed by Baltagi, Egger and Pfaffermayr (2003) and Baldwin and Taglioni (2006) as the theory-consistent fixed effects specification of the gravity equation. In this paper, I argue that the fixed-effects country-time dummies gravity specification severely limits the set of trade policy effects that can be identified from the data. In many applications, identifying heterogeneous effects is not possible at all because of perfect collinearity among the trade policy dummies and the country-time dummies. Moreover, the problem may not be apparent for the first sight, when one uses standard estimation techniques.

Let me consider an example to illustrate the main argument. Take a set of countries that enter a customs union at the same time. A researcher may be interested in using this episode to measure the trade-creating effect of the customs union. She may believe that entering the union has a one-off effect on trade, which can be measured by comparing the growth of trade of the entrants between the pre-entry and the post-entry years, relative to some benchmark. When a country has long been a member (insider), there is no such union effect any more. This enables the researcher to use the growth of trade among insider countries as a comparison group (benchmark). There are three different groups of country pairs with possible trade-creating effect: trade among entrants, trade from entrants to insiders, and trade from insiders to entrants. Suppose that trade growth was 5 per cent in the first and 1-1 per cents in the second and third groups, while there was zero growth in trade among insiders.

1This specification has a cross section analogue with exporter and importer dummies. The findings of this paper also apply to the cross section case.
The researcher may believe that the entry affected only trade among entrants and she may decide to put trade between entrants and insiders also in the benchmark. This may be justified, if e.g. trade between the union and the entrants was governed by a free trade agreement (FTA) well before the time of entry. FTA and customs union differ only in the trade protection with third countries, which is automatically controlled for by the country-time dummies in the fixed-effects country-time dummies gravity specification. Alternatively, the researcher may think that all the three groups of pairs with at least one entrant are affected by the entry and wants to measure an average effect across the three, compared to trade among insiders. If she uses the fixed effects country-time dummies gravity specification, the estimated trade-creating effect she gets is 3 per cent in the first and -3 per cent in the second case.

I demonstrate in this paper that the astonishing result of getting estimates that are negatives of each other under two ultimately similar research questions is because the fixed-effects country-time dummies gravity specification leaves too little variation in the data. In the above example, there is in fact only one parameter the gravity specification is able to identify. In other words, estimates under different research questions degenerate to simple transformations (like the negative) of a single parameter. A direct consequence of this is that it is not possible to identify more than one effects simultaneously (heterogeneous effects). The above researcher may want to estimate separate effects for trade among entrants and trade between entrants and insiders by including two policy dummies in the estimating equation. I argue that it is not possible in the above setup, because one of the policy dummies will be perfectly collinear with the set of country-time dummies.

Perfect collinearity of the policy dummy with other dummies in the regression is a trivial case for unidentification. Yet, the problem may not be apparent for the first sight, if the researcher uses standard estimation techniques and software. I demonstrate this for Fixed Effects Least Squares Dummy Variables (FE-LSDV) and 'OLS on the demeaned' estimations, using STATA. In the case of FE-LSDV estimation, the software often drops one of the country-time dummies and reports policy effect estimates orderly. If the estimation involves hundreds of country-time dummies, it is likely that the researcher overlooks the problem. OLS on the demeaned reveals the problem and drops the perfectly collinear policy dummy, given that it is properly performed. The demeaning transformation that is used to demean the data before estimation requires a data set that also includes trade of a country with itself (domestic trade). However, if such data is not included, which is usually the case with foreign trade databases, OLS on the demeaned also reports false estimates for the perfectly collinear policy dummy.

A possible solution to the identification problem is to extend the database with countries that are, in none of the trading pairs they form, affected by the policy change. In the above example these may be countries that are outside of the customs union in the whole sample period (so-called third countries). I will show that at most four different policy effects can be identified simultaneously in such an extended database. It is crucial, however, that changes in trade barriers with third countries are not correlated with the policy or they are appropriately controlled for by hard data (e.g. data on tariff changes). Entering a customs union involves adopting the union’s common external trade policy, i.e. trade protection between entrants and third countries must change. If
the researcher does not account for this in the estimation, third countries’ trade is not a valid benchmark. If, lacking hard data on trade protection, she wants to control for the third country effects by including additional dummies, the previous identification problem can return.

This paper is a contribution to the literature on the proper econometric specification of the gravity equation. With the development of panel data econometrics, several authors emphasized the importance of country or country pair fixed effects in accounting for the unobserved (time-constant) heterogeneity in the gravity equation (Mátayás (1997), Glick and Rose (2001), Egger and Pfaffermayr (2003), Cheng and Wall (2005)). After the contribution of Anderson and van Wincoop (2003), however, trade economists realized that time-constant fixed effects are insufficient to capture the unobservable time-varying MTR. This lead to the proposition of the fixed-effects country-time dummies gravity specification by Baltagi, Egger and Pfaffermayr (2003) and Baldwin and Taglioni (2006). Estimating gravity with some sets of fixed effects or dummies has been popular among empirical trade researcher, because it can be performed easily and it offers a robust way to control for the unobserved. Alternative solutions to the MTR problem are all imperfect in one way or another. Structural estimation (Anderson and van Wincoop (2003), Bergstrand, Egger and Larch (2011)) is computationally burdensome and requires strict assumptions. Other methods, developed in the recent years, are more data-demanding and/or cannot treat asymmetric trade barriers (Head and Ries (2001), Combes, Lafourcade and Mayer (2005), Novy (2008), Baier and Bergstrand (2009)).

The contribution of this paper is, in general, to show an important drawback of relying on dummies extensively to control for the unobserved heterogeneity in the estimation. Dummies offer a robust control, but they can also absorb too much of the useful variation in the data. In particular, I discourage empirical trade researchers to use the fixed-effects country-time dummies gravity specification, except for some special cases. I demonstrate the above findings with an empirical example: the enlargement of the European Union (EU) in 2004 with eight Central and Eastern European countries and its trade consequences.

The findings of the paper are more general than the presented examples. They equally apply to cross section gravity estimations, when the estimating equation includes a full set of exporter and importer dummies. Moreover, they apply not only to trade policy dummies, but also to other dummy regressors in the gravity equation (e.g. common language). Finally, these lessons can be useful for empirical researchers in other fields of Economics as well. Multidimensional panel estimations and the tendency to use dummies to control for the unobserved heterogeneity is not confined to the empirical trade research.

The paper is structured as follows. Section 2 presents the fixed-effects country-time dummies gravity specification. Section 3 describes four research questions on the effect of a trade policy. Section 4 examines whether the policy effects under the four research questions are identified and, if they are, what is the estimated effect. Section 5 considers the extension of the database with third countries. Section 6 presents a summary and discussion.

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2 Application of this gravity specification, however, has not become widespread. One example is Eicher and Henn (2009) on the effect of WTO membership.
2 The fixed-effects country-time dummies gravity

The gravity equation derived from the model of Anderson and van Wincoop (2003) is

\[ x_{ij} = \frac{y_i y_j}{y_w} \left( \frac{\tau_{ij}}{\Pi_i P_j} \right)^{1-\sigma}, \]  

subject to the expressions for \( \Pi_i \) and \( P_j \),

\[ P_j^{1-\sigma} = \sum_i \frac{y_i}{y_w} \left( \frac{\tau_{ij}}{\Pi_i} \right)^{1-\sigma}, \]  

and

\[ \Pi_i^{1-\sigma} = \sum_j \frac{y_j}{y_w} \left( \frac{\tau_{ij}}{P_j} \right)^{1-\sigma}, \]

where \( x_{ij} \) is exports from country \( i \) to \( j \), \( y_i \) and \( y_j \) are nominal income of the exporter and the importer, respectively, \( y_w \) is world income, \( \tau_{ij} \) is the bilateral trade costs between the exporter and the importer, and \( \sigma \) is the elasticity of substitution between all goods. \( \Pi_i \) and \( P_j \) are the Multilateral Trade Resistances (MTR) for the exporter and the importer, respectively. More precisely, \( \Pi_i \) is a measure of trade barriers that country \( i \)'s exports face in the rest of the world and \( P_j \) is a measure of trade barriers that country \( j \) imposes on imports from the rest of the world.

Let us define the logarithm of income-adjusted exports, \( z_{ij} = \ln \left( \frac{x_{ij} y_w}{y_i y_j} \right) \), introduce the time dimension \( t \), and express (1) in logarithms,

\[ z_{ijt} = (1-\sigma) \ln \tau_{ijt} - (1-\sigma) \ln \Pi_{it} - (1-\sigma) \ln P_{jt}. \]  

Putting income-adjusted exports on the left-hand side implies unit income elasticity, consistent with the theory. Although this assumption is often relaxed in empirical applications, we keep it for the simplicity of the exposition.³

Notice that both bilateral trade costs and the MTR terms can vary with time. Let the bilateral trade cost function, \( \ln \tau_{ijt} \), be additively separable in its time-varying and time-constant cost components and assume, for simplicity, that the time-varying component can be captured by a policy dummy variable, \( T_{ijt} \), with some parameter and an additive error term, \( u_{ijt} \), uncorrelated with the policy dummy. Then, the fixed-effects country-time dummies specification for panel data, consistent with (4), can be expressed as

\[ z_{ijt} = \beta T_{ijt} + \zeta_{ij} + \delta_{it} + \theta_{jt} + u_{ijt}, \]

where \( \zeta_{ij} \) are (direction-specific) pair fixed effects and \( \delta_{it} \) and \( \theta_{jt} \) are a full set of exporter-time and importer-time dummies. The trade effect of the policy is captured by \( \beta \), which is the product of \( 1-\sigma \) and the parameter of the policy dummy in the bilateral trade cost function.

³Relaxing the assumption would not change the findings, because the country-time dummies net out all country-time-specific variables, including the income levels.
The presence of the exporter-time and importer-time dummies in (5) is required, because the MTRs in (4) are not observable and potentially vary with time. In contrast, the inclusion of the pair fixed effects ($\zeta_{ij}$) is not directly motivated by the theory. Nevertheless, in panel data applications, when the regressor of interest varies with time, it is customary to control for time-invariant unobservable bilateral trade costs via country pair fixed effects. Many elements of bilateral trade costs, like those related to culture or institutions, cannot be observed and hardly change with time. Researchers tend to choose to avoid omitted variable biases stemming from these unobserved bilateral costs by including pair fixed effects.\footnote{Egger and Pfaffermayr (2003) argue for pair fixed effects over separate exporter and importer fixed effects. Baltagi, Egger and Pfaffermayr (2003) and Baldwin and Taglioni (2006) also suggest a gravity specification with pair fixed effects.} Some research explicitly follows a difference-in-differences strategy to capture the effect of a policy change (Hornok, 2011). This strategy identifies from the time changes across different groups of country pairs and, hence, requires that time-invariant factors are netted out by pair fixed effects.

The panel gravity specification in (5) has a cross section equivalent under some conditions. Assume that the trade policy change occurs at one point in time, which defines a two-period panel with $t = 1$ pre-policy and $t = 2$ post-policy periods. Time-differencing (5) on the two-period panel yields

$$dz_{ij} = \beta dt_{ij} + \alpha_i + \eta_j + \epsilon_{ij},$$

(6)

where $d$ denotes the time change from $t=1$ to $t=2$, $\alpha_i$ and $\eta_j$ are exporter and importer fixed effects and the error is $\epsilon_{ij} = du_{ijt}$.

In what follows I assume a two-period panel with pre-policy and post-policy periods and derive the analytical results for (6). I exploit that the fixed effects and the first-difference panel estimation methods are identical in two-period panels. This can be done without loss of generality. The analytical findings directly extend both to traditional cross section gravity estimations with exporter and importer dummies and to multiple-period panel estimations of the form (5), given that the panel has well-defined pre-policy and a post-policy periods (i.e. no sequential policies).

### 3 Four research questions

To demonstrate the limits of the fixed-effects country-time dummies gravity specification I consider four research questions on the same data set and trade policy episode. The episode is the enlargement of the EU in 2004 with 8 Central and Eastern European countries.\footnote{I do not consider Cyprus and Malta, which also joined the EU in May 2004.} The EU is a customs union, which means tariff-free intra-EU trade and a common external trade protection. Although trade was free for most products due to bilateral FTAs between the pre-2004 EU and the entrants and among the entrants themselves years before the enlargement, evidence shows that the enlargement brought further trade-creation.\footnote{See Hornok (2010, 2011).}
Let the sample include two types of countries: entrants and insiders to the customs union. For the moment abstract from outsiders, the third country type. The two types of countries form four groups of country pairs, shown in Figure 1. Group $G_{11}$ includes pairs, where both the exporter and the importer are entrants, $G_{12}$ are pairs with an entrant exporter and an insider importer, and so on. The number of countries in each type can be arbitrary. If the number is only one, then the within-group trade is trade of a country with itself (domestic trade).

![Figure 1: Groups of pairs with entrants and insiders](image)

When a researcher wants to measure the effect of a trade policy, she needs to define two sets of country pairs: those who are “treated” by the policy (treated) and those who are not (benchmark). Which pairs are treated and which are the benchmark is ultimately an empirical issue. Depending on how this choice is made I define four different research questions:

1. $G_{11}$ is treated, the other three are the benchmark;
2. $G_{11}$, $G_{12}$ and $G_{21}$ are treated and a common effect is estimated for them, $G_{22}$ is the benchmark;
3. $G_{11}$, $G_{12}$ and $G_{21}$ are treated, a separate effect is estimated for $G_{11}$ and a common effect for the other two, $G_{22}$ is the benchmark;
4. $G_{12}$ and $G_{21}$ are the treated and a common effect is estimated for them, $G_{11}$ and $G_{22}$ are the benchmark.

The research question determines the exact formulation of the policy dummy, $T_{ijt}$, in (5). In the first case, it is 1 for country pairs in $G_{11}$ in $t=2$ and 0 otherwise. In the second case, it is 1 for pairs in $G_{11}$, $G_{12}$ and $G_{21}$ in $t=2$ and 0 otherwise. In the third case, there are two policy dummies. The first takes value 1 for pairs in $G_{11}$ in the post-policy period and 0 otherwise, the second is 1 for pairs in $G_{12}$ and $G_{21}$ in the post-policy period and 0 otherwise. In the last case, there is one policy dummy that takes 1 for pairs in $G_{12}$ and $G_{21}$ in $t=2$ and 0 otherwise.

In the first research question, the researcher wants to estimate a policy effect for trade among entrants ($G_{11}$), while she puts trade between entrants and insiders ($G_{12}$ and $G_{21}$), together with $G_{22}$, in the benchmark. She may believe that EU enlargement could bring no further trade creation in $G_{12}$ and $G_{21}$, because free trade of most goods was achieved by FTAs ("Europe Agreements")
between the pre-2004 EU and the entrants already in the first half of the 1990s.\footnote{Trade among entrants was also subject to FTAs (CEFTA, BAFTA). These were formed somewhat later, in the second half of the 1990s.} Indeed, trade growth after 2004 was much faster among entrants than between entrants and insiders.

The second research question puts trade between entrants and insiders ($G_{12}$ and $G_{21}$) also in the treated group. The researcher may believe that EU membership decreases some non-tariff trade barriers, which are not eliminated by an FTA, and the fall of these costs affects all trading pairs with at least one entrant equally. Such a non-tariff trade barrier can be e.g. the time cost of trade: trade is faster for country pairs within the EU, because there are no border controls and customs procedures (Hornok, 2011). In this case, the researcher wants to estimate a common effect for the above three groups of country pairs.

The third question is similar to the second, with one difference. It wants to estimate two separate policy effects simultaneously: one effect for $G_{11}$ and a separate (common) effect for $G_{12}$ and $G_{21}$. This research question assumes that a policy has a significantly different effect on a country pair, where both countries are subject to the policy, than on a country pair with only one country, who introduced the policy. It is similar to the approach in Rose (2004, 2005), who examines separate trade effects for joint and unilateral WTO membership. De Benedictis, De Santis, Vicarelli (2005) also take a similar approach on European data, when they compare the pre-2004 regional FTAs among entrants (CEFTA, BAFTA) with the FTAs between entrants and insiders.

The fourth research question asks how trade between entrants and outsiders changed with enlargement, relative to trade among entrants and trade among insiders. Namely, the researcher looks at trade across, relative to within, country types. In the EU enlargement context, this question is not particularly relevant. In other applications, however, it is common. The best example is the so-called 'border effect' literature, which was initiated by the paper of McCallum (1995). This literature looks at how much smaller trade is across nations (international trade), relative to trade within nations (intranational trade). Similarly, research that examines the trade effect of sharing the same language or currency, e.g., is often of this type.\footnote{See e.g. Rose and van Wincoop (2001) on the effects of currency unions.}

Notice that what the researcher thinks about the evolution of trade costs with third countries is irrelevant as long as the panel estimating equation controls for the MTRs with country-time dummies and the sample does not include trade with third countries. Later in this paper, when I consider samples with third countries, this argument will no longer hold and controlling for trade cost changes with third countries will be an issue.

### 4 What is identified and what is not?

I derive the policy effect estimates, $\hat{\beta}$, for the first-differenced panel estimating equation (6) under each research question, check whether the effects are identified and, if they are, how the estimates relate to each other. I assume that the sample includes observations for all the country pairs that
can be formed with \( n_1 \) entrant and \( n_2 \) insider countries (\( n_1 \) can be different from \( n_2 \)). The sample also includes domestic trade for all the \( N = n_1 + n_2 \) countries.\(^9\)

A simple way to solve for the policy effect estimate analytically is to demean \( dz_{ij} \) and \( dT_{ij} \) from the exporter and importer dummies in (6) and then run OLS regression on the demeaned variables. The \( ij \)-th element of the demeaned left-hand side variable, \( \bar{dz}_{ij} \) is

\[
\bar{dz}_{ij} = dz_{ij} - \frac{1}{N} \sum_{i=1}^{N} dz_{ij} - \frac{1}{N} \sum_{j=1}^{N} dz_{ij} + \frac{1}{N^2} \sum_{j=1}^{N} \sum_{i=1}^{N} dz_{ij},
\]

(7)

and similarly for the demeaned policy dummy, \( \bar{dT} \).\(^10\) Then, the estimate for \( \beta \) can be obtained via the OLS formula \( \hat{\beta} = (\bar{dT}' dT)^{-1} \bar{dT}' \bar{dz} \).

To express the demeaned variables in vector form, take the vector of the left-hand side variable as \( \bar{dz}' = \begin{bmatrix} \bar{dz}_{11} & \bar{dz}_{12} & \bar{dz}_{21} & \bar{dz}_{22} \end{bmatrix} \), where the elements are simple averages of the \( z_{ij} \)'s across the country pairs belonging to the same group. Hence, \( \bar{dz}_{11} \) is the simple average of the \( n_1^2 \) country pair observations belonging to \( G_{11} \), \( \bar{dz}_{12} \) is the simple average of the \( n_1 n_2 \) country pair observations belonging to \( G_{12} \), and so on. It is straightforward to show that the vector of the demeaned left-hand side variable is \( \bar{dz}' = \Delta N^{-2} \begin{bmatrix} n_2^2 & -n_1 n_2 & -n_1 n_2 & n_1^2 \end{bmatrix} \), where \( \Delta = \bar{dz}_{11} - \bar{dz}_{12} - \bar{dz}_{21} + \bar{dz}_{22} \).

The demeaned vectors of policy dummies can be similarly obtained and the policy effect coefficients calculated using the OLS formula. I present the demeaned policy dummies and the \( \beta \) estimates for each research question separately in Table 1.

<table>
<thead>
<tr>
<th>Research question</th>
<th>Demeaned policy dummy ((dT))</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( N^{-2} ) ( n_2^2 ) (-n_1 n_2) (-n_1 n_2) ( n_1^2 )</td>
<td>( \Delta )</td>
</tr>
<tr>
<td>2</td>
<td>(-N^{-2} ) ( n_2^2 ) (-n_1 n_2) (-n_1 n_2) ( n_1^2 )</td>
<td>(-\Delta )</td>
</tr>
<tr>
<td>3.1</td>
<td>( N^{-2} ) ( n_2^2 ) (-n_1 n_2) (-n_1 n_2) ( n_1^2 )</td>
<td>not identified</td>
</tr>
<tr>
<td>3.2</td>
<td>(-2N^{-2} ) ( n_2^2 ) (-n_1 n_2) (-n_1 n_2) ( n_1^2 )</td>
<td>separately</td>
</tr>
<tr>
<td>4</td>
<td>(-2N^{-2} ) ( n_2^2 ) (-n_1 n_2) (-n_1 n_2) ( n_1^2 )</td>
<td>(-\frac{\Delta}{2} )</td>
</tr>
</tbody>
</table>

Notes: \( N = n_1 + n_2 \), where \( n_1 \) is the number of entrants, \( n_2 \) the number of insiders in the sample. \( \Delta = \bar{dz}_{11} - \bar{dz}_{12} - \bar{dz}_{21} + \bar{dz}_{22} \), where the \( \bar{dz}'s \) are averages of observations of the LHS variable in eq. (6) across country pair groups in Figure 1.

The answer to the first research question “How much more trade among entrants grew as a result of the policy, relative to the trade of other pairs?” is given by \( \hat{\beta} = \Delta = \bar{dz}_{11} - \bar{dz}_{12} - \bar{dz}_{21} + \bar{dz}_{22} \), where \( \bar{dz} \) is the change in the level of (income-adjusted) trade from the pre-policy to the post-policy period in either of the country pair groups. If, like in the Introduction, I assume that (income-adjusted) trade grew by 5% among entrants and by 1% between entrants and insiders, while it

\(^9\) Unlike the coefficient estimate, identifiability does not depend on whether domestic trade is included or not.

\(^10\) This formula, also called within transformation formula, is present in several Econometrics textbook like e.g. Baltagi (2001). The demeaning formula for the panel equation (5) is more complicated. I provide a derivation of it in Appendix A.
did not change among insiders, the estimated policy effect becomes 3%. If I modify the research question and ask “How much more trade of country pairs with at least one entrant grew as a result of the policy, relative to trade among insiders?” (second research question), the estimated coefficient changes sign and becomes $\beta = -\Delta$, i.e. -3%. Since the two research questions are not mirror images to each other, such a change in the estimated policy effect does not look reasonable.

The estimate under the fourth research question “How much different trade growth was between entrants and insiders, relative to trade growth among entrants or insiders?” is $\beta = -\frac{\Delta}{2}$, yet again a simple transformation of the same parameter, $\Delta$. This suggests that, for the fixed-effects country-time dummies gravity specification, the range of coefficient estimates under different research questions are severely restricted. In fact, for samples with only two types of countries (here entrant and insider), there is only one parameter that can be identified and the coefficient estimates under different research questions are simple transformations of this one parameter.

The two policy effects under the third research question cannot be identified separately. This is another consequence of the fact that the fixed-effects country-time dummies gravity specification on a sample of entrants and insiders cannot identify more than one policy effects. Notice that the two demeaned policy dummies in Table 1 are clearly perfectly collinear (3.1 stands for the effect on trade among entrants, 3.2 for the effect on trade between entrants and insiders). Another way to see that more than one policy dummies cannot be identified is to write out the matrix of regressors in (6) for the third research question,

$$\begin{bmatrix} \alpha & \eta & dT \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad (8)$$

where the elements of the matrix are vectors of ones or zeros of dimensions $n_1^2$ in the first, $n_1n_2$ in the second and third and $n_2^2$ in the fourth rows of the matrix. The first two columns of the matrix are the exporter dummies, the third column includes the importer dummies for entrants (importer dummies for insiders omitted) and the last two columns are the two policy dummies. Since the number of linearly independent columns should always be equal to the number of linearly independent rows, the five column vectors of this matrix cannot be linearly independent. The exporter and importer dummies already take three out of the maximum four linearly independent column vectors. Hence, there is room left for only one linearly independent policy dummy.\(^{11}\)

I demonstrate the above findings on the example of EU enlargement by estimating policy effects under all the four research questions on a panel of country pairs formed by 8 entrants and 12

\(^{11}\)Of course, having only one policy dummy is a necessary but not sufficient condition for linear independence and, hence, identification. Even if there is only a single policy dummy, identification is not possible if the regressor matrix is of deficient rank. This means that the policy dummy is constructed so that it is perfectly collinear with one or more of the country dummies. This would be the case if the researcher wanted to estimate e.g. the effect on $G_{11}$ and $G_{12}$, relative to $G_{21}$ and $G_{22}$. In this case the policy dummy is, by construction, perfectly collinear with the exporter dummies for the entrants (first column of the regressor matrix).
I use annual data and not a two-period panel to show that the analytical findings extend to multiple period panels. I estimate (5) with income-adjusted bilateral exports on the left-hand side, where income is captured by nominal GDP levels. I use two estimation methods, FE-LSDV with pair fixed effects and exporter-year and importer-year dummies and OLS on the demeaned variables, both performed in STATA. For the latter method it is important that the database also includes observations for trade of a country with itself (domestic trade). Since such data is not available, I construct it as gross output of all non-services sectors minus total exports of goods. For OLS on the demeaned I demean the left-hand side variable and the policy dummy using the formula derived in Appendix A.

The estimation results are shown in Table 2, FE-LSDV in the first four columns, OLS on the demeaned in the last column. The \( \hat{\beta} \) estimates reinforce the analytical findings. The value of the parameter \( \triangle \) is -0.007, given that the elements of the \( dz' \) vector in this particular sample are, in order, 0.041, 0.072, -0.181, and -0.157. The \( \hat{\beta} \) estimates under the different research questions relate to each other as expected. The estimate for the second question is the negative of the estimate for the first question, and the estimate for the fourth question is half of the estimate for the second question. None of them is statistically different from zero.

The policy effects under the third research question cannot be separately identified. Yet, quite misleadingly, the FE-LSDV estimation method reports sizeable and strongly significant estimates. If one checks the number of exporter-year and importer-year dummies that are dropped, it turns out that FE-LSDV drops one of these dummies, instead of the policy dummy, due to the perfect collinearity. In contrast, OLS on the demeaned drops the perfectly collinear policy dummy. It

Table 2: Estimates for EU with entrants and insiders

<table>
<thead>
<tr>
<th>Research question</th>
<th>FE-LSDV</th>
<th>OLS on demeaned</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \hat{\beta} )</td>
<td>Cluster s.e.</td>
</tr>
<tr>
<td>1</td>
<td>-0.007</td>
<td>0.059</td>
</tr>
<tr>
<td>2</td>
<td>0.007</td>
<td>0.059</td>
</tr>
<tr>
<td>3.1</td>
<td>0.450(a)</td>
<td>0.157</td>
</tr>
<tr>
<td>3.2</td>
<td>0.229(b)</td>
<td>0.077</td>
</tr>
<tr>
<td>4</td>
<td>0.003</td>
<td>0.029</td>
</tr>
</tbody>
</table>

Notes: Eq. (5) is estimated with FE-LSDV and OLS on demeaned. No of obs: 2400. No of groups: 400. The sample includes country pairs of 12 of the EU-15 countries and 8 of the countries that joined the EU in 2004. Dependent variable is log bilateral exports normalized by GDPs. Time dimension is years between 2001 and 2006. Pair fixed effects, exporter-year and importer-year dummies included. \(a\) Number of extra country-year dummies dropped in bracket. \(b\) significant at 1%, \(c\) at 5%. Last column shows coefficient estimates from OLS on demeaned, where significance is not reported.

12 Entrants: Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia, Slovenia. Insiders: Austria, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Portugal, Spain, Sweden, United Kingdom.
13 The post-policy period starts with 2004, although the date of enlargement was precisely 1 May 2004.
14 All data is from Eurostat and OECD.
15 Domestic trade is similarly constructed, among others, in Wei (1996), Noy (2008), Jacks, Meissner and Noy (2011) and Hornok (2011).
is important to emphasize, however, that OLS on the demeaned reports perfect collinearity only with a database that also includes domestic trade observations. If domestic trade is not part of the database, OLS on the demeaned also reports “false” estimates.\footnote{It is because the demeaning (within transformation) formula is derived for a full trade matrix.} In this case, the reported estimates are different from the FE-LSDV estimates.

5 Third countries: a solution?

I extend the sample with countries that are outside the EU’s customs union. I call these countries outsiders or third countries interchangeably. In the extended sample the number of country pair groups increases to nine (Figure 2). Outsiders export to all the three types of countries ($G_3$ in last row) and the three types of countries export to outsiders ($G_{3.3}$ in last column).

Figure 2: Groups of pairs with entrants, insiders and outsiders

\begin{center}
\begin{tabular}{ccc}
  \hline
  \textbf{i \backslash j} & entrant & insider & outsider \\
  \hline
  entrant & $G_{11}$ & $G_{12}$ & $G_{13}$ \\
  insider & $G_{21}$ & $G_{22}$ & $G_{23}$ \\
  outsider & $G_{31}$ & $G_{32}$ & $G_{33}$ \\
  \hline
\end{tabular}
\end{center}

I consider the four research questions as before with unchanged treated pair groups. This implies that the benchmark, relative to which the policy effect is identified, automatically extends with the country pairs of outsiders ($G_{3.3}$, $G_{3.3}$). It is by no means an innocuous modification. The choice of the benchmark observations, which is ultimately the researcher’s responsibility, is crucial to get a reliable estimate for the policy effect. Outsider country pairs are valid benchmark only if their trade is not affected by the policy change or, if it is affected, the policy-induced change in their trade costs is appropriately controlled for in the estimation.

5.1 All are benchmark

Let us assume for the moment that country pair groups with outsiders are valid benchmark and check the identifiability of the policy effects under the four research questions. Let the number of outsider countries in the sample be $n_3$. It is straightforward to see that all the four research questions are identifiable on the extended database.\footnote{With third countries in the sample the $\beta$ estimates cannot be expressed as simply as before. An alternative way to check identifiability is to write out the regressor matrix ($X$) and check whether the determinant of $X'X$ is zero (singular matrix) or approximately zero (near singular matrix). A singular or near singular matrix indicates perfect collinearity.} In particular, the matrix of regressors in (6)
for the third research question is now

\[
\begin{bmatrix}
\alpha & \eta & dT
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

where the order of observations is \(G_{11}, G_{12}, G_{13}, G_{21}, G_{22}, G_{23}, G_{31}, G_{32}, G_{33}\) and the elements of the matrix are vectors of ones or zeros of the following dimensions: \(n_1^2\) in the first, \(n_1n_2\) in the second and fourth, \(n_1n_3\) in the third and seventh, \(n_2^2\) in the fifth, \(n_2n_3\) in the sixth and eighth, and \(n_3^2\) in the ninth rows of the matrix. The first three columns of the matrix are the exporter dummies, the fourth and fifth columns are the importer dummies (importer dummies for outsiders omitted) and the last two columns are the two policy dummies of the third research question. The extension of the database with outsider countries increases the number of rows of the regressor matrix to nine, which also increases the maximum possible number of linearly independent column vectors to nine. Since five columns are reserved for the country dummies, the researcher is able to identify at most four policy effects separately.

Does the inclusion of outsider countries also lead to a less restrictive range of estimated effects? The answer is yes. One can solve for the \(\beta\) estimates by following the same steps as in the previous section. Again, I assume that the sample includes observations of all the country pairs formed by \(n_1\) entrants, \(n_2\) insiders and \(n_3\) outsiders, also including domestic trade observations. The vector of the left-hand side variable in (6) is

\[
dz' = \begin{bmatrix}
dz_{11} & dz_{12} & dz_{13} & dz_{21} & dz_{22} & dz_{23} & dz_{31} & dz_{32} & dz_{33}
\end{bmatrix},
\]

where the elements are simple averages of the observations across country pairs belonging to the same group. The estimated \(\beta\) coefficients for each research question can be expressed as linear combinations of the elements of \(dz\) with some parameter vector. The elements of the parameter vectors are functions of \(n_1\), \(n_2\) and \(n_3\). Details of the analytical solution are shown in Appendix B.

I present the parameter vectors for the four research questions under the simplifying assumption that the number of countries by type is equal, i.e. \(n_1 = n_2 = n_3\). The elements of the parameter vectors are in the rows of Table 3. Linear combinations of the elements of \(dz\) with these give the \(\beta\) estimates for each research question. For instance, the estimated effect under the first research question can be expressed as \(\hat{\beta} = dz_{11} - 0.5 \cdot (dz_{12} + dz_{13} + dz_{21} + dz_{31}) + 0.25 \cdot (dz_{22} + dz_{23} + dz_{32} + dz_{33})\).

Estimation results for the EU enlargement episode confirm that the inclusion of third countries enables identification under all the four research questions. The EU database is augmented with 8
Table 3: \( \beta \) estimates in panels with outsiders \((n_1 = n_2 = n_3)\)

| Research question | Elements of vector of LHS variable | \( \hat{\beta} \)  
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( d_{z11} ) ( d_{z12} ) ( d_{z13} ) ( d_{z21} ) ( d_{z22} ) ( d_{z31} ) ( d_{z32} ) ( d_{z33} )</td>
<td>1.041 0.085 0.283 -0.171 -0.146 -0.071 0.272 0.014 -0.014</td>
</tr>
<tr>
<td>2</td>
<td>( d_{z11} ) ( d_{z12} ) ( d_{z13} ) ( d_{z21} ) ( d_{z22} ) ( d_{z31} ) ( d_{z32} ) ( d_{z33} )</td>
<td>0.248</td>
</tr>
<tr>
<td>3.1</td>
<td>( d_{z11} ) ( d_{z12} ) ( d_{z13} ) ( d_{z21} ) ( d_{z22} ) ( d_{z31} ) ( d_{z32} ) ( d_{z33} )</td>
<td>-0.5</td>
</tr>
<tr>
<td>3.2</td>
<td>( d_{z11} ) ( d_{z12} ) ( d_{z13} ) ( d_{z21} ) ( d_{z22} ) ( d_{z31} ) ( d_{z32} ) ( d_{z33} )</td>
<td>-0.5</td>
</tr>
<tr>
<td>4</td>
<td>( d_{z11} ) ( d_{z12} ) ( d_{z13} ) ( d_{z21} ) ( d_{z22} ) ( d_{z31} ) ( d_{z32} ) ( d_{z33} )</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

Notes: \( \beta \) estimates are linear combinations of the elements of \( dz \) with the parameter values in the rows.

Table 4: Estimates for EU with entrants, insiders and outsiders

<table>
<thead>
<tr>
<th>Research question</th>
<th>FE-LSDV</th>
<th>OLS on demeaned</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\beta} )</td>
<td>Cluster s.e.</td>
<td>Within R²</td>
</tr>
<tr>
<td>1</td>
<td>-0.248a</td>
<td>0.074</td>
</tr>
<tr>
<td>2</td>
<td>-0.254a</td>
<td>0.060</td>
</tr>
<tr>
<td>3.1</td>
<td>-0.502a</td>
<td>0.115</td>
</tr>
<tr>
<td>3.2</td>
<td>-0.254a</td>
<td>0.059</td>
</tr>
<tr>
<td>4</td>
<td>-0.053</td>
<td>0.034</td>
</tr>
</tbody>
</table>

Notes: Eq. (5) is estimated with FE-LSDV and OLS on demeaned. No of obs: 3456. No of groups: 576. The sample includes country pairs of 8 of the EU-15 countries, 8 of the countries that joined the EU in 2004 and 8 non-EU countries. Dependent variable is log bilateral exports normalized by GDPs. Time dimension is years in 2001-2006. Pair fixed effects, exporter-year and importer-year dummies included. a significant at 1%, b sign. at 5%. Last column shows coefficient estimates from OLS on demeaned, where significance is not reported.

One can check whether the analytical solutions for the \( \beta \) estimates in Table 3 are correct. In this particular database the elements of the \( dz \) vector take the following values:

\[
dz' = \begin{bmatrix} 0.041 & 0.085 & 0.283 & -0.171 & -0.146 & -0.071 & 0.272 & 0.014 & -0.014 \end{bmatrix}.
\]

The \( \beta \) estimate for the first research question, e.g., can be calculated as linear combination of the elements of this vector with the corresponding parameter values in the first row of Table 3, i.e. \( \hat{\beta} = 0.041 - 0.5 \cdot (0.085 + 0.283 - 0.171 + 0.272) + 0.25 \cdot (-0.146 - 0.071 + 0.014 - 0.014) = -0.248. \)

18 The choice of outsiders is determined by data availability. They are Switzerland, Israel, Iceland, Japan, South Korea, Mexico, Norway, United States.

19 Denmark, Greece, Ireland and Portugal are dropped from the original 12 insiders. The choice is arbitrary.
The $\beta$ estimates in Table 4 are strikingly different from the estimates in Table 2; they are all negative, mostly large in absolute value and statistically significant. The big difference between the two sets of estimates is due to the change in the benchmark observations, which now include all country pairs with outsiders. Income-adjusted trade with outsiders, and especially between entrants and outsiders (3rd and 7th elements of the $dz'$ vector), increased faster than elsewhere, which causes the $\beta$ estimates to be significantly negative.

5.2 Dummies for third country effects

Are country pairs with outsiders valid benchmark for the estimation of a policy effect? It is ultimately an empirical question. If there are good reasons to believe that changes in trade barriers with outsiders are uncorrelated with the policy, the answer is positive. If they are correlated with the policy, but appropriately controlled for in the estimation (e.g. with hard data on trade costs), the answer is still positive. If however such “third-country effects” are not accounted for properly, country pairs with outsiders are not valid benchmark. If, e.g., the trade policy change involves a decrease in trade costs between entrants and outsiders, which increases their bilateral trade, leaving entrant-outsider country pairs in the benchmark without controlling for this change results in the underestimation of the policy effect.

Getting back to the example of EU enlargement, entering the EU involves entering a customs union and adopting its external trade policy. Available data suggests that tariffs of entrants with outsiders had to change considerably with EU entry (Table 5). Before enlargement most entrants faced higher tariffs as exporters in, and imposed higher tariffs as importers on, the eight outsiders in the sample, relative to the level of tariffs faced and imposed by the EU member Germany. The difference from the EU’s external protection was especially large for import tariffs of some entrants (Poland, Slovenia, Hungary). In contrast, outsiders were only marginally more protective towards the entrants than towards the pre-2004 EU.

Table 5: Difference in tariffs with 8 outsiders relative to Germany in 2001

<table>
<thead>
<tr>
<th>Faced by exporter</th>
<th>diff in tariff (%point)</th>
<th>Imposed by importer</th>
<th>diff in tariff (%point)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Czech Republic</td>
<td>0.4</td>
<td>Czech Republic</td>
<td>4.4</td>
</tr>
<tr>
<td>Estonia</td>
<td>0.4</td>
<td>Estonia</td>
<td>-2.0</td>
</tr>
<tr>
<td>Hungary</td>
<td>0.8</td>
<td>Hungary</td>
<td>7.6</td>
</tr>
<tr>
<td>Lithuania</td>
<td>0.5</td>
<td>Lithuania</td>
<td>1.2</td>
</tr>
<tr>
<td>Latvia</td>
<td>0.1</td>
<td>Latvia</td>
<td>0.9</td>
</tr>
<tr>
<td>Poland</td>
<td>-0.2</td>
<td>Poland</td>
<td>14.7</td>
</tr>
<tr>
<td>Slovakia</td>
<td>1.2</td>
<td>Slovakia</td>
<td>4.4</td>
</tr>
<tr>
<td>Slovenia</td>
<td>-0.1</td>
<td>Slovenia</td>
<td>8.8</td>
</tr>
</tbody>
</table>

Notes: Manufacturing tariffs, average of 3-digit ISIC industries. Sources: CEPII. Outsiders: Switzerland, Israel, Iceland, Japan, South Korea, Norway and the United States.

Controlling for changes in trade barriers with hard data is often problematic. Available data on bilateral tariffs is deficient and often not good quality, let alone data on non-tariff trade barriers. The
empirical researcher is tempted to simply include additional dummies to account for the changes in third-country trade costs between the pre- and post-policy periods. Because the decline in third-country tariffs of entrants at EU enlargement was apparently asymmetric, I consider the inclusion of two separate dummies, one for the (smaller) entrant-outsider and another for the (larger) outsider-entrant effects. I demonstrate that with such a modification to the estimating equation the identification problems discussed in Section 4 can return.

The panel fixed-effects country-time dummies estimating equation (5), augmented with the entrant-outsider and outsider-entrant dummies, is

$$z_{ijt} = \beta T_{ijt} + \gamma_1 D_{13,t} + \gamma_2 D_{31,t} + \zeta_{ij} + \delta_{it} + \theta_{jt} + u_{ijt},$$  

(10)

where $D_{13,t}$ is a dummy variable taking value 1 for country pairs in $G_{13}$ in $t = 2$ and 0 otherwise, $D_{31,t}$ is a dummy taking value 1 for country pairs in $G_{31}$ in $t = 2$ and 0 otherwise, and $\gamma_1$ and $\gamma_2$ are parameters to estimate. Estimation results for the four research questions in the EU enlargement example are in Table 6.

<table>
<thead>
<tr>
<th>Research question</th>
<th>Coefficient</th>
<th>FE-LSDV Estimate</th>
<th>Cluster s.e.</th>
<th>Within $R^2$</th>
<th>Identified?</th>
<th>OLS on demeaned Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\beta$</td>
<td>0.006</td>
<td>0.068</td>
<td>0.22</td>
<td>Yes</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>$\gamma_1$</td>
<td>0.174$^b$</td>
<td>0.080</td>
<td></td>
<td></td>
<td>0.174</td>
</tr>
<tr>
<td></td>
<td>$\gamma_2$</td>
<td>0.334$^a$</td>
<td>0.089</td>
<td></td>
<td></td>
<td>0.334</td>
</tr>
<tr>
<td>2</td>
<td>$\beta$</td>
<td>-0.006</td>
<td>0.068</td>
<td>0.22</td>
<td>Yes</td>
<td>-0.006</td>
</tr>
<tr>
<td></td>
<td>$\gamma_1$</td>
<td>0.167</td>
<td>0.086</td>
<td></td>
<td></td>
<td>0.167</td>
</tr>
<tr>
<td></td>
<td>$\gamma_2$</td>
<td>0.328$^a$</td>
<td>0.103</td>
<td></td>
<td></td>
<td>0.328</td>
</tr>
<tr>
<td>3.1</td>
<td>$\beta$ ($G_{11}$)</td>
<td>0.695</td>
<td>0.379</td>
<td>0.22</td>
<td>No (1)</td>
<td>dropped</td>
</tr>
<tr>
<td>3.2</td>
<td>$\beta$ ($G_{12}, G_{21}$)</td>
<td>0.341</td>
<td>0.190</td>
<td></td>
<td></td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>$\gamma_1$</td>
<td>0.518$^a$</td>
<td>0.192</td>
<td></td>
<td></td>
<td>0.171</td>
</tr>
<tr>
<td></td>
<td>$\gamma_2$</td>
<td>0.679$^a$</td>
<td>0.210</td>
<td></td>
<td></td>
<td>0.331</td>
</tr>
<tr>
<td>4</td>
<td>$\beta$</td>
<td>-0.003</td>
<td>0.034</td>
<td>0.22</td>
<td>Yes</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>$\gamma_1$</td>
<td>0.171$^b$</td>
<td>0.076</td>
<td></td>
<td></td>
<td>0.171</td>
</tr>
<tr>
<td></td>
<td>$\gamma_2$</td>
<td>0.331$^a$</td>
<td>0.000</td>
<td></td>
<td></td>
<td>0.331</td>
</tr>
</tbody>
</table>

Notes: Eq. (10) is estimated with FE-LSDV and OLS on demeaned. No. obs: 3456. No. groups: 576. The sample includes country pairs of 8 of the EU-15 countries, 8 of the countries that joined the EU in 2004 and 8 non-EU countries. Dependent variable is log bilateral exports normalized by GDPs. Time dimension dimension is years between 2001 and 2006. Pair fixed effects, exporter-year and importer-year dummies included. $^a$ Number of extra country-year dummies dropped in bracket. $^b$ Significant at 1%. $^c$ Significant at 5%. Last column shows coefficient estimates from OLS on demeaned, where significance is not reported.

Both estimated third-country effects are positive, and $\tilde{\gamma}_2$ is larger and more strongly significant than $\tilde{\gamma}_1$. That entrants adopted the EU’s external trade policy, which is less restrictive than their pre-enlargement trade protection was, seems to have promoted trade between entrants and outsiders in both directions. In contrast, with the inclusion of the two additional dummies, the $\beta$ coefficient estimates become small and not different from zero statistically. Recall that the benchmark now includes outsider country pairs of $G_{23}$, $G_{32}$ and $G_{33}$, but not country pairs of $G_{13}$ and $G_{31}$. Despite the fact that outsider countries are also in the sample, the relationship among the $\beta$ estimates is like
in Section 4. The $\beta$ estimate of the second research question is the negative of the $\beta$ estimate of the first one, and the estimate of the fourth research question is half of the second's. Yet again similar to Section 4, the two policy effects of the third research question cannot be identified separately.

The identification problem under the third research question is due to a deficient rank regressor matrix. The number of columns in the regressor matrix (nine) equals the number of rows, which would allow identification. However, there is perfect collinearity among the columns. The regressor matrix (9), extended with the two third-country dummies becomes

$$
\begin{bmatrix}
\alpha & \eta & dT & D_{13} & D_{31} \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
$$

where $D_{13}$ and $D_{31}$ are the last two columns. Perfect collinearity arises from the linear relationship among the exporter and importer dummies for entrants, the two policy dummies and $D_{31}$ of the form $2v_6 + v_7 + v_8 + v_9 - v_1 - v_4 = 0$, where the $v$s are the column vectors in order.

All in all, the advantages of adding outsider country observations to the sample are completely lost, when one has to control for (direction-specific) third-country effects via additional dummies. Of course, depending on the empirical application, additional third country dummies may take different forms. In some applications, a common entrant-outsider dummy (i.e. not direction-specific dummies) may be sufficient. In others, insider-outsider effects, or both entrant-outsider and insider-outsider effect, should be controlled for.

Table 7: Identifiability with additional third country dummies

<table>
<thead>
<tr>
<th>Research question</th>
<th>$G_{13}, G_{31}$ common</th>
<th>$G_{31}$ separate</th>
<th>$G_{23}, G_{32}$ common</th>
<th>$G_{23}, G_{32}$ separate</th>
<th>$G_{13}, G_{31}, G_{23}, G_{32}$ common</th>
<th>$G_{13}, G_{31}, G_{23}, G_{32}$ separate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>2</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>3</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>4</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>

Notes: "yes" and "no" refer to identifiability of the policy effect under research questions 1-4, when additional dummies for country pair groups with outsiders in $t = 2$ are also included. "Common" stands for a common dummy, "separate" for separate dummies by country pair group.

Table 7 shows how the different sets of third-country dummies determine the identifiability of the policy effects under the four research questions. There is no identification problem with only insider-outsider dummies. When entrant-outsider dummies are included (common or separate), the
policy dummies in the third research question cannot be identified. Finally, none of the policy effects can be identified, when separate dummies are included for both insider-outsider and entrant-outsider groups.

6 Summary and Discussion

The findings of this paper point to the fact that the country-time dummies in the fixed-effects country-time dummies gravity specification absorb too much of the variation in the data. In most cases the variation left is so narrow that heterogeneous policy effects cannot be identified, because the country-time dummies and the policy dummies are perfectly collinear. Being aware of this limitation is important, because standard estimation techniques do not report the problem clearly. Little variation left is problematic even if the policy effect of interest is identified, because the estimated coefficients may not be meaningful estimates. I demonstrate this problem, when I compare the policy estimates under the first and the second research questions in Table 2. The estimates are negatives of each other, while the research questions are not mirror images.

The message of this paper is not limited to the presented examples. Cross section gravity estimations are equally subject to the above limitations, given that the estimating equation includes a full set of exporter and importer dummies. The same findings apply to all regressors that are captured by dummies and not only to policy dummies. For instance, looking at the trade effect of a common language (captured by a dummy) in a cross section estimation is similar to the fourth research question of this paper. Finally, these findings can also serve useful in other fields of empirical research, where dummies are extensively used as control variables.

Researchers, who want to estimate a theory-consistent gravity equation, are advised to find other methods to control for the MTRs in a theoretically consistent way. Alternative methods are numerous, though none is perfect. The researcher should choose among them, based on what assumptions are reasonable to make and what data is available. Anderson and van Wincoop (2003) use structural estimation, assuming symmetric trade costs. In their structural estimation, Bergstrand, Egger and Larch (2011) relax the trade cost symmetry assumption. Other authors regress the gravity equation on some ratio of international to intranational trade (Head and Ries (2001), Novy (2008)) to net out the MTRs. Baier and Bergstrand (2009) develop a linear reduced-form gravity equation with first-order log-linear Taylor series approximation of the MTRs.

Needless to say that all the above methods are more data-demanding than the fixed-effects country-time dummies specification. The method of Baier and Bergstrand (2009), e.g., requires comprehensive bilateral trade cost data for all country pairs in the world. A well-designed empirical strategy like a quasi-experimental framework or matching country pairs can help reduce the data requirement. Nevertheless, the need for an improvement in the availability and quality of data on trade barriers remains a central issue in the empirical trade research.
References


A The demeaning formula for the panel specification

I derive the demeaning (within transformation) formula for the error structure of the fixed effects panel estimation (5). The derivation is based on the general solution in Davis (2002).

The fixed effects panel specification for international trade data can be represented with the error structure

$$u_{ijt} = \zeta_{ij} + \delta_{it} + \theta_{jt} + \nu_{ijt}, \quad (12)$$

where $i = 1, \ldots, N$ denote exporters, $j = 1, \ldots, M$ importers and $t = 1, \ldots, T$ time, $\zeta_{ij}$, $\delta_{it}$ and $\theta_{jt}$ are the unobservable pair-specific, exporter-year and importer-year effects, respectively. In vector form,

$$u = Z_\zeta \zeta + Z_\delta \delta + Z_\theta \theta + \nu, \quad (13)$$

where $\zeta$, $\delta$ and $\theta$ are vectors of parameters to estimate of dimension $NMT \times NM$, $NMT \times NT$ and $NMT \times MT$, respectively, and $Z_\zeta = I_{NM} \otimes \iota_T$, $Z_\delta = I_N \otimes \iota_M \otimes I_T$ and $Z_\theta = \iota_N \otimes I_{MT}$. $I$ is the identity matrix and $\iota$ is the vector of ones of given dimension and $\otimes$ denotes the Kronecker product.\(^{21}\)

The projection matrix, which projects onto the range of $Z = (Z_\zeta; Z_\delta; Z_\theta)$, is $P_{[Z]} = Z (Z' Z)^{-1} Z'$. The orthogonal projection matrix is $Q_{[Z]} = I - P_{[Z]}$. $P$ and $Q$ are symmetric and idempotent. Note that $P_{[Z_{\zeta}]} = I_{NM} \otimes \bar{J}_T$ averages the data over $t$, where $\bar{J}_T = \frac{1}{T} J_T$ with $J_T$ being the matrix of ones of dimension $T$. Similarly, $P_{[Z_{\delta}]} = I_N \otimes \bar{J}_M \otimes I_T$ averages the data over $j$ and $P_{[Z_{\theta}]} = \bar{J}_N \otimes I_{MT}$ averages the data over $i$. For example, in the last case, $(\bar{J}_N \otimes I_{MT}) u$ has a typical element $\bar{u}_{ijt} = \frac{1}{N} \sum_{i=1}^{N} u_{ijt}$.

The general solution for the within transformation matrix according to Davis (2002) is

$$Q_{[Z]} = Q_{[A]} - P_{[B]} - P_{[C]}, \quad (14)$$

where $A = Z_\theta$, $B = Q_{[A]} Z_\delta = Q_{[Z_\theta]} Z_\delta$ and $C = Q_{[B]} Q_{[A]} Z_\mu = Q_{[Z_{\theta}]} Z_\delta Q_{[Z_\theta]} Z_{\zeta}$.

It is straightforward to show that

$$Q_{[A]} = (I_N - \bar{J}_N) \otimes I_{MT}.$$ 

The second term can be expressed as

$$P_{[B]} = (I_N - \bar{J}_N) \otimes \bar{J}_M \otimes I_T,$$

where I used that $Q_{[A]} Z_\delta = (I_N - \bar{J}_N) \otimes \iota_M \otimes I_T$. The third term is

$$P_{[C]} = (I_N - \bar{J}_N) \otimes (I_M - \bar{J}_M) \otimes \bar{J}_T,$$

where I used that $Q_{[A]} Q_{[B]} Z_{\zeta} = (I_N - \bar{J}_N) \otimes (I_M - \bar{J}_M) \otimes \iota_T$.

\(^{20}\)The formula for the cross section equation (6) is widely known and can be found in Econometrics textbooks like Baltagi (2001, p. 32). The textbook formula is derived for individual and time dimensions, which should be replaced by the exporter and importer dimensions.

\(^{21}\)A useful property of the Kronecker product (mixed-product property) is that $(A \otimes B) \cdot (C \otimes D) = AC \otimes BD$, given that the dimensions of the matrices are such that taking their product is possible.
Collecting terms,

\[
Q_{[Z]} = (I_N - \bar{J}_N) \otimes (I_M - \bar{J}_M) \otimes (I_T - \bar{J}_T)
\]

\[
= I_{NMT} - \bar{J}_N \otimes I_{MT} - I_N \otimes \bar{J}_M \otimes I_T - I_{NM} \otimes \bar{J}_T + I_N \otimes \bar{J}_{MT} + \bar{J}_N \otimes I_{MT} + \bar{J}_{NM} \otimes I_T - \bar{J}_{NMT}
\]

with a typical element

\[
\bar{u} = Q_{[Z]} u = u_{ijt} - \bar{u}_{.,jt} - \bar{u}_{i.,t} - \bar{u}_{.,i} + \bar{u}_{i..} + \bar{u}_{.j.} + \bar{u}_{..t} - \bar{u}_{...},
\]

where \( \bar{u}_{.,jt} = N^{-1} \sum_i u_{ijt} \), \( \bar{u}_{i.,t} = M^{-1} \sum_j u_{ijt} \), \( \bar{u}_{ij.} = T^{-1} \sum_t u_{ijt} \), \( \bar{u}_{i..} = (MT)^{-1} \sum_t \sum_j u_{ijt} \), \( \bar{u}_{.,j.} = (NT)^{-1} \sum_t \sum_i u_{ijt} \), \( \bar{u}_{.,t} = (NM)^{-1} \sum_j \sum_i u_{ijt} \) and \( \bar{u}_{...} = (NMT)^{-1} \sum_t \sum_j \sum_i u_{ijt} \).

The estimation method ‘OLS on the demeaned’ is done by demeaning the variables as in (15) and estimating the regression equation with the demeaned variables. It is important to add that this formula is derived for a “full” trade matrix. This means that, if some countries are both exporters and importers in the database (which is almost always the case), data on trade of these countries with themselves (domestic trade) should also be included.
B Deriving the $\hat{\beta}$'s without assuming $n_1 = n_2 = n_3$

If I do not assume $n_1 = n_2 = n_3$, the demeared left-hand side variable can be expressed as

$$\bar{d}z = a_1 d\bar{z}_{11} + a_2 d\bar{z}_{12} + a_3 d\bar{z}_{13} + a_4 d\bar{z}_{21} + a_5 d\bar{z}_{22} + a_6 d\bar{z}_{23} + a_7 d\bar{z}_{31} + a_8 d\bar{z}_{32} + a_9 d\bar{z}_{33},$$

where the $a$'s are vectors, whose elements are functions of $n_1$, $n_2$ and $n_3$. Expressing the $a$'s in terms of the number of countries we get

$$\begin{pmatrix}
    d\bar{z}_{11} \\
    d\bar{z}_{12} \\
    d\bar{z}_{13} \\
    d\bar{z}_{21} \\
    d\bar{z}_{22} \\
    d\bar{z}_{23} \\
    d\bar{z}_{31} \\
    d\bar{z}_{32} \\
    d\bar{z}_{33}
\end{pmatrix} = \frac{1}{N^2}\begin{pmatrix}
    (n_2 + n_3)^2 \\
    -n_1 (n_2 + n_3) \\
    -n_2 (n_2 + n_3) \\
    -n_1 (n_2 + n_3) \\
    n_1^2 \\
    n_2^2 \\
    -n_1 (n_2 + n_3) \\
    n_1^2 \\
    n_2^2
\end{pmatrix} d\bar{z}_{11} + \frac{1}{N^2}
\begin{pmatrix}
    -n_2 (n_2 + n_3) \\
    n_1 n_2 \\
    n_1 n_2 \\
    n_1 n_2 \\
    n_1 n_3 \\
    n_1 n_3 \\
    n_1 n_3 \\
    n_1 n_3 \\
    n_1 n_3
\end{pmatrix}
\begin{pmatrix}
    d\bar{z}_{12} \\
    d\bar{z}_{13} \\
    d\bar{z}_{21} \\
    d\bar{z}_{22} \\
    d\bar{z}_{23} \\
    d\bar{z}_{31} \\
    d\bar{z}_{32} \\
    d\bar{z}_{33}
\end{pmatrix} + \frac{1}{N^2}\begin{pmatrix}
    -n_4 (n_2 + n_3) \\
    -n_4 (n_2 + n_3) \\
    (n_4 + n_2) (n_2 + n_3) \\
    n_1 n_3 \\
    n_1 n_3 \\
    n_1 n_3 \\
    n_1 n_3 \\
    n_1 n_4
\end{pmatrix} d\bar{z}_{13} + \frac{1}{N^2}\begin{pmatrix}
    -n_2 (n_1 + n_3) \\
    n_2 n_3 \\
    n_2 n_3 \\
    n_2 n_3 \\
    n_2 n_3 \\
    n_2 n_3 \\
    n_2 n_3 \\
    n_2 (n_1 + n_2)
\end{pmatrix} d\bar{z}_{23} + \frac{1}{N^2}\begin{pmatrix}
    -n_4 (n_2 + n_3) \\
    -n_4 (n_2 + n_3) \\
    (n_4 + n_2) (n_2 + n_3) \\
    n_1 n_3 \\
    n_1 n_3 \\
    n_1 n_3 \\
    n_1 n_3 \\
    n_1 n_4
\end{pmatrix} d\bar{z}_{31} + \frac{1}{N^2}\begin{pmatrix}
    -n_2 (n_1 + n_3) \\
    n_2 n_3 \\
    n_2 n_3 \\
    n_2 n_3 \\
    n_2 n_3 \\
    n_2 n_3 \\
    n_2 n_3 \\
    n_2 (n_1 + n_2)
\end{pmatrix} d\bar{z}_{32} + \frac{1}{N^2}\begin{pmatrix}
    -n_4 (n_2 + n_3) \\
    -n_4 (n_2 + n_3) \\
    (n_4 + n_2) (n_2 + n_3) \\
    n_1 n_3 \\
    n_1 n_3 \\
    n_1 n_3 \\
    n_1 n_3 \\
    n_1 n_4
\end{pmatrix} d\bar{z}_{33},$$

where $N = n_1 + n_2 + n_3$ is the total number of countries in the sample.

The demeaned policy dummy is $\bar{d}T = a_1$ for the first research question, $\bar{d}T = a_1 + a_2 + a_3$ for the second research question and $\bar{d}T = a_2 + a_3$ for the fourth research question. The matrix of the two demeaned policy dummies under the third research question is $\bar{d}T = \begin{pmatrix} a_1 & a_2 + a_3 \end{pmatrix}$, where $a_1$ and $a_2 + a_3$ are column vectors of the matrix.

To express the policy effect estimates as functions of the $n$'s and the $\bar{d}z$'s, one needs to solve for the OLS formula $\hat{\beta} = \left(\bar{d}T' \bar{d}T\right)^{-1} \bar{d}T' \bar{d}z$ for each research question separately.

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