Productivity Dispersion, Import Competition, and Specialization in Multi-product Plants

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Abstract

Import competition can increase plant level efficiency, as competitive pressure forces plants to reallocate more of their resources towards their highest performing products. There has been little empirical work directly documenting these gains, since within plant differences in quality and productivity, and the allocation of inputs across production lines, are rarely observed. This paper develops a flexible methodology that uses profit maximization conditions to solve for unobserved input allocations, TFP, and quality across product lines in multi-product plants. I apply this methodology to a panel of plants manufacturing machinery in India from 2000-2007. I emphasize three results. First, quality and quantity based TFP (TFPQ) are negatively correlated. Second, increases in Chinese import competition led plants to increase revenue based TFP (TFPR), which captures both TFPQ and quality upgrading. However, these increases in plant-level TFPR are fully explained by increases in mean TFPR within plants, rather than within-plant reallocations. Third, multi-product plants respond to Chinese import competition reallocating their inputs towards higher quality products and away from their high TFPQ products, thereby generating within plant quality improvements, rather than productivity gains.

JEL codes: D24, F12, F14.

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1 Introduction

Quantitative estimates of the gains from trade are of first-order importance for developing trade policy. Empirical work examining the effect of international trade on aggregate productivity has been very successful on this front, finding that trade liberalization can lead to sizeable efficiency gains that are straightforward to quantify.¹ These efficiency gains are generally found to come from two key sources: across firm reallocations, where high productivity firms gain market share at the expense of low productivity firms, and within firm efficiency gains, where existing firms increase their productivity.²

Recent studies, including Eckel and Neary (2010), Bernard et al. (2011), Mayer et al. (2014) and Mayer et al. (2016), have emphasized that trade induced efficiency gains may also be generated by within firm reallocations. Specifically, import competition may force firms to reallocate a larger proportion of their inputs towards their highest performing products, improving firm level efficiency. However, these within firm reallocations can only generate significant efficiency gains if there are sizeable differences in productivity and quality across production lines within a firm. Unfortunately, there has been very little empirical work measuring the magnitude of within firm heterogeneity.³ This is largely because data on multi-product firms will typically only record input use at the firm level, rather than the firm-product level, thereby making standard approaches to productivity estimation infeasible.

In this paper, I develop a flexible methodology that identifies unobserved input allocations, TFP, and quality across product lines in multi-product firms. This approach exploits the fact that firm-product level prices incorporate information on within firm input use. More formally, I show that profit maximization restrictions, combined with estimates of a firm-product level demand function, provide enough information for prices and quantities to reveal input use across production lines. One can then use these restrictions to estimate a firm-product level production function for multi-product firms, generating estimates of firm-product level TFP. Since the general approach requires estimation of a product-level demand function, I use differences in the demand residual across firm-products to estimate product specific quality, as in Khandelwal (2010) and Amiti and Khandelwal (2013).⁴ Altogether, the approach generates estimates of firm-product input use, and two dimensions of within firm heterogeneity, TFP and quality, allowing the researcher to answer questions related to within-firm reallocations and efficiency.

I then apply my methodology to a panel of plants manufacturing machinery in India, using data

¹See Melitz and Trefler (2012) and De Loecker and Goldberg (2014) for a review of this literature.

²For the theoretical underpinnings of these across firm reallocations, see Melitz (2003). See Pavcnik (2002), Bernard and Jensen (2004) and Trefler (2004), for empirical work documenting the existence of both across and within-firm productivity gains.

³Some notable exceptions include Hottman et al. (2016), who examine within firm marginal cost and quality dispersion in US retail, and Garcia-Marin and Voigtländer (2013) and Garcia-Marin and Voigtländer (2017), who estimate within-plant TFP dispersion in Chile using a novel dataset containing information on cost shares by product line within a plant.

⁴Since products with large demand residuals generate a higher level of purchases than can fully be explained by price, by revealed preference these products must include product characteristics that consumers find more appealing. I will refer to differences in these unobserved characteristics within a given product-code as quality variation. Note, however, that these product characteristics can also be immaterial characteristics, such as the quantity of advertising, that nevertheless increase quantity sold conditional on price.

from the 2000-2007 Annual Survey of Industries (ASI). One of the core findings of the study is that the vast majority of productivity upgrading in response to trade liberalization is due to increases in the average value of revenue-based TFP within plant, not within-plant reallocations. Note, however, that my identification strategy, which leverages plausibly exogeneous increases in Chinese import competition, also provides evidence that there are sizable within-plant reallocations in response to increased Chinese import competition. However, these reallocations are primarily towards a plant's highest quality products, but away from their products with the highest quantity based TFP, or TFPQ, varieties. These reallocations turn out to be revenue based TFP (TFPR) neutral, in large part because I find that TFPQ and quality are negatively correlated. Put differently, my empirical estimates imply that quality is costly to produce, in the sense that high quality goods often requires more inputs per unit of output. As a result, while I find evidence for within-plant reallocations in response to increased import competition, these reallocations generally do not increase plant-level productivity, but rather imply large gross changes in the composition of high versus low quality goods within the plant.

This paper generates some of the first estimates of the impact of within-plant reallocations on plant level efficiency, in large part because there is no standard approach for estimating productivity in datasets containing multiple outputs produced by the same firm or plant.⁵ To highlight the issues generated by multi-product firms in productivity analysis, consider a firm producing two outputs using only labour. Suppose that a researcher observes the level of output across the two production lines, as well as the total quantity of labour purchased by the firm. Further suppose that the researcher is confident that both production lines are characterized by constant returns to scale. Without further information, differences in output across production lines can either be due to differences in labour input, or differences in unit labour requirements. As a result, productivity and differences in input use are not separately identified.

I show that if product level prices are observed, then profit maximization restrictions can be used to separately identify input use from productivity dispersion. For example, if the markets for each output are characterized by perfect competition, the price of each good will equal its marginal cost. Given constant returns to scale, this implies that the price of each good will be equal to the wage over the unit labour requirement for that product. As a result, the ratio of output prices within the plant will be proportional to ratio of unit labour requirements, which means that price variation reveals productivity dispersion. One can then combine this information with the structure of the production technologies, as well as an aggregate resource constraint, to generate four equations that uniquely pin down the four unknowns; the two unobserved unit labour requirements, and the two unobserved labour allocations.⁶

While the above example imposes a number of strong assumptions (single input technologies, perfect competition, constant returns to scale), in this paper I show that one can use profit maximization to

⁵I survey some of the other approaches used in the literature in more detail in Section 2.

⁶More formally, since labour is the only input and production is characterized by constant returns to scale, we can write $Y^j = A^j L^j$, where j = 1, 2 indexes products within the plant, A^j is the unit labour requirement for output j, and L^j is the quantity of labour used to produce j. This provides two equations. As long as labour is completely attributable to each production line, this provides a third restriction, $L^1 + L^2 = L$, where L is the total quantity of observed labour. Profit maximization and perfect competition imply that $P^j = \frac{w}{A^j}$ for j = 1, 2, which implies that $\frac{P^1}{P^2} = \frac{A^2}{A^1}$. This is the fourth condition necessary to uniquely determine (A^1, A^2, L^1, L^2) using information on (Y^1, Y^2, P^1, P^2, L) .

separately identify input allocations from within-firm productivity heterogeneity for a much wider class of pricing models and production technologies. These include models of oligopolistic competition and collusion, as well as more general multiple input production technologies with increasing or decreasing returns to scale. These are new identification results that should prove useful to other researchers who wish to estimate the productivity of multi-product firms in datasets that only contain information on aggregate inputs.⁷

The identification results developed in this paper shows that one cannot ignore the demand side of an industry when engaging in productivity analysis, if one wishes to separately identify within-firm input use from within-firm productivity dispersion. In particular, I show that allowing firms to have pricing power usually requires that a product-level demand function be estimated, before one can identify within-firm input allocations.⁸ This is because separately identifying input allocations from productivity will require information on demand elasticities, since one must be able to distinguish between variation in prices due to productivity, versus variation in prices due to differences in market power.

In my empirical application, I estimate a product-level discrete/continuous choice demand system based on Björnerstedt and Verboven (2013), under the assumption of Bertrand-Nash price competition. This approach allows for within-plant cannibalization effects, where individual plants account for the fact that decreasing the price of one of their product lines will tend to decrease demand for their other products. To estimate the parameters governing this demand system, I make use of a novel set of instrumental variables, which harness exogenous variation in input prices due to demand and supply shocks in other output markets that use similar inputs. This new identification strategy takes advantage of the detailed input price variation observed in my data, while avoiding the endogeneity problems generated by the potential correlation between individual input prices and output quality.⁹

I further show that as long as the production function satisfies some relatively standard restrictions consistent with much of the empirical work on productivity analysis, then one can uniquely determine input allocations from price, quantity, and demand elasticity data, without prior knowledge of the production function parameters. This is a useful identification result, since one can estimate within plant input allocations before production function parameters are estimated. As a result, one can use these estimates of plant-product level input use to directly estimate a plant-product level production function, using standard tools from the production function estimation literature. This property allows me to use all of the plants in my sample to estimate the production function, rather than using a selected sample of single product plants as in De Loecker et al. (2016).

⁷Examples of this type of data include the U.S. Census of Manufactures used in Foster et al. (2008), the Belgian PRODCOM data used in Dhyne et al. (2017), the Canadian manufacturing data used in Baldwin and Gu (2009), the Indian Prowess data used in Goldberg et al. (2010b) and De Loecker et al. (2016), as well as the Indian ASI data used in this paper.

⁸I also show that the restrictions on the demand system necessary for identification of within-firm input allocations are simply a variant of those required by Berry and Haile (2014) for the nonparametric identification of demand systems in differentiated product markets. As noted in their paper, these restrictions are satisfied by almost all the demand systems used in applied work.

⁹See Kugler and Verhoogen (2012) for evidence on the correlation between plant level input prices and output quality. ¹⁰See Ackerberg et al. (2007) for a review of this literature.

¹¹Note that De Loecker et al. (2016) account for sample selection using a variant of the control function approach described in Olley and Pakes (1996), which partly restricts the nature of selection into multi-product production. Since

Having estimated the demand and production function parameters for the Indian machinery industry, I then examine whether import shocks lead to within-plant reallocations towards higher performing products. Using a series of simple OLS and IV regressions, where I leverage plausibly exogenous increases in Chinese exports, I find that increased import competition is associated with reallocations towards high quality goods, but away from high TFPQ products. This potentially counter-intuitive result is largely due to the fact that high TFPQ products are found to be lower quality, a finding that has also been uncovered in recent papers by Jaumandreu and Yin (2016) and Forlani et al. (2016), who estimate production and demand functions for single product firms. Intuitively, this means that quality is costly to produce (i.e. it requires more inputs), and therefore reallocations towards quality also generate increased costs. Altogether, these results imply that within-plant reallocations in response to Chinese import competition do not do much to change plant-level productivity- rather, I find that the productivity enhancing effects of trade liberalization, previously documented in a number of studies including Pavcnik (2002), Amiti and Konings (2007), Fernandes (2007), De Loecker (2011), Topalova and Khandelwal (2011), and Bloom et al. (2016), are almost entirely explained by mean TFPR upgrading.

These findings add to the empirical literature focused on trade's impact on multi-product producers, including Baldwin and Gu (2009), Liu (2010), Goldberg et al. (2010a), Goldberg et al. (2010b), Bernard et al. (2011), Mayer et al. (2014), and Eckel et al. (2015), and Medina (2017). In particular, I show that within-firm reallocations may partly explain the positive impact of import competition on quality upgrading, a feature of the data recently explored in Amiti and Khandelwal (2013) and Medina (2017). Moreover, these results highlight that the within-firm productivity gains from trade, emphasized by theoretical models developed in Eckel and Neary (2010), Bernard et al. (2011), Mayer et al. (2014) and Mayer et al. (2016), primarily operate through a quality improvement margin in this industry, rather than productivity upgrading per se.

This paper also contributes to a growing literature on separately identifying the different sources of firm heterogeneity, including Foster et al. (2008), Forlani et al. (2016), Jaumandreu and Yin (2016), who focus on separately identifying demand versus supply-side heterogeneity, De Loecker and Warzynski (2012) and De Loecker et al. (2016) who focus on separately identifying TFP and markups, Goldberg et al. (2010a) and Goldberg et al. (2010b), who focus on identifying within-firm improvements driven by product adding and dropping, Valmari (2016) and Garcia-Marin and Voigtländer (2017) who focus on identifying within-firm TFP heterogeneity, and Hottman et al. (2016) who provides a framework for separately identifying markups, quality, and marginal costs across and within firms. The approach described in this paper provides a unified framework that identifies all of these margins of firm heterogeneity, including across and within-firm variation in TFP (both TFPQ and TFPR), quality, markups, and marginal costs, while previous approaches have only developed techniques which can uncover a subset of these different margins. One exception to this is recent work by Blum et al. (2018), who also provide a framework to examine the various margins of firm heterogeneity, although with a slightly different notion of demand heterogeneity than that used in this paper. In particular, their approach uses estimated markups to provide a local linear approximation of the slope and intercept of a firm's

my approach uses information on multi-product plants directly, I do not need these restrictions.

demand function around equilibrium price and quantity, while I estimate a global demand function, and recover the estimates of the demand shifters (quality) from there.

The paper is structured as follows. In Section 2, I outline the problems generated by multi-product firms for productivity analysis, and outline how my approach to solving these problems differs from other methods that have previously been proposed in the literature. In Section 3, I present a general model of industry demand and supply, and show how the structure of this model can be used to obtain within-firm input allocations as a function of observable demand side data. I then discuss my data in Section 4, and describe the estimation strategy I use to estimate the demand and production function in Section 5. Section 6 presents the empirical results, and I conclude in Section 7.

2 The General Productivity Estimation Problem

2.1 The "Standard" Problem: Single Product Firms

The standard approach to measuring firm-level productivity involves estimating a firm-level production function. For example, a typical study examining firm productivity will estimate a model of the form:

$$y_{it} = f(\vec{X}_{it}) + \omega_{it} \tag{1}$$

where y_{it} is log output of firm i at time t, \vec{X}_{it} is an $1 \times P$ vector of P different input quantities, f(.) is the log production function and ω_{it} is firm-level log TFP.¹² Note I will denote the natural log of any variable X, by its corresponding lower case letter x, e.g. $y_{it} \equiv \ln(Y_{it})$, where Y_{it} is the level of output produced by firm i.

Typically, it is assumed that the researcher has data on output and all of the relevant inputs at the firm level, while the exact form f(.) takes in (1) is unknown. To obtain TFP estimates, the researcher must determine a way to estimate the shape of the production function.

The key challenge to estimating f(.) is the endogeneity of input use, or, as described by Griliches and Mairesse (1995), the "transmission bias" problem. Since firms know their own productivity, input usage will tend to be correlated with ω_{it} , which means that one cannot simply estimate (1) using OLS. As a result, much of the literature on production function estimation is concerned with solutions to the transmission bias problem, with some popular approaches being the proxy-variable estimation methods described by Olley and Pakes (1996), Levinsohn and Petrin (2003), and Ackerberg et al. (2015), as well as the dynamic panel approaches described by Arellano and Bond (1991), Blundell and Bond (1998), Blundell and Bond (2000) and Bond and Söderbom (2005).

 $^{^{12}}$ If y_{it} is measured in revenue units, ω_{it} would correspond to log TFPR, while if y_{it} is measured in quantity units, ω_{it} measures log TFPQ. Note that this distinction is not important for the general identification results discussed in this and the following section, as long as a well defined TFPR or TFPQ generating production function exists. As a result, I will simply refer to ω_{it} as TFP when discussing the general approach to identification.

2.2 Complications Generated by Multi-product Production

While the transmission bias problem inherent in estimating the production function f(.) has been widely studied, with many estimation routines available to applied researchers who wish to estimate productivity, firm-level datasets containing multi-product firms have generated new problems that the researcher must deal with to obtain estimates of TFP, even if f(.) were known. Specifically, most firm-level datasets only contain information on input use at the firm-level, even if they provide output information at the firm-product level.¹³ This generates significant difficulties, since if one wishes to estimate firm-product varying TFP, one must estimate some variant of the following model

$$y_{it}^j = f^j(\vec{X}_{it}^j) + \omega_{it}^j, \tag{2}$$

where y_{it}^j is output of product j produced by firm i at time t, \vec{X}_{it}^j is the $1 \times P$ vector of inputs used in the production of good j, $f^j(.)$ is the production function, which may vary with j, and ω_{it}^j is the overall productivity of good j. Unfortunately, while y_{it}^j is generally observed in most firm-level datasets, the researcher generally only has access to data on aggregate inputs, \vec{X}_{it} . As a result, all of the variables on the right-hand side of (2) are unobserved, making standard production function estimation infeasible.

There have been, roughly speaking, two general classes of solutions to the problems posed by multiproduct firms. The first solution makes observing \vec{X}_{it}^j unnecessary for productivity analysis, by redefining the object of interest. For example, one can instead focus on simply recovering estimates of firm-level TFP, either by creating a firm-level output index to deflate firm-level revenues, as in Eslava et al. (2004) and Smeets and Warzynski (2013), or by making use of an explicit within-firm aggregation model that allows one to estimate the transformation rates of outputs across uses within a firm, as in Balat et al. (2016). While these approaches are useful for answering questions related to across-firm variation in productivity, they by construction remain silent on within-firm productivity dispersion and allocative efficiency.

Another approach in this vein, recently pursued by Dhyne et al. (2017), examines firm as well as firm-product level productivity by estimating a transformation function, which is a generalization of the production function for multi-product firms which only requires information on aggregate input use. This approach allows for joint-production of outputs within a firm, which may allow for more careful analysis of the sources of economies of scope. On the other hand, the approach is not well suited for examining within-firm allocative gains, largely because the input allocation problem has already implicitly been solved in the formulation of the transformation function, and therefore cannot be used to consider changes in within-plant specialization.¹⁴

¹³Although there are some exceptions, including the data on vertically integrated Chinese steel producers used in Brandt et al. (2017), and the Chilean data used Garcia-Marin and Voigtländer (2013, 2017).

¹⁴Note that results from Hall (1973) imply that transformation functions that are specified to be additively separable in inputs and ouputs, as in Dhyne et al. (2017), almost always describe *non-joint* production technologies, i.e. technologies that *cannot* be described by a series of product line specific production functions. It is extremely difficult to capture the notion of specialization in non-joint environments, since this framework rules out a standard input allocation problem. See also Appendix H, which considers the relationship between transformation functions and the approach to recovering input allocations described in this paper.

A second solution involves using information at the firm-level to construct estimates of the withinfirm input allocations. While many authors simply use revenue shares to allocate inputs, including
Foster et al. (2008), Atalay (2014) and Collard-Wexler and De Loecker (2015), a number of recent
papers, including De Loecker et al. (2016) and Valmari (2016), have shown how one may use restrictions
on the nature of the technology and market structure to recover equilibrium input allocations. To
see why restrictions are necessary, note that if both \vec{X}_{it}^j and ω_{it}^j are unobserved in (2), then researchers
face a significant identification problem, since increased output across production lines can be due
to both increases in productivity, which are unobserved, or increases in the quantity of inputs going
into a particular production line j, which are also unobserved. More formally, even if $f^j(.)$ is known
ex-ante, then for each multi-product firm producing J_{it} products, there are $J_{it}(P+1)$ unobservables
that the researcher must identify: J_{it} unknown ω_{it}^j terms, and $J_{it} \times P$ unknown input allocations.
Unfortunately, (2) only provides, at most, J_{it} restrictions, leaving productivity and input allocations
severely underidentified.

Hence, if the researcher wishes to separately identify input allocations from productivity, the researcher must impose some restrictions on the nature of production. De Loecker et al. (2016) (henceforth DGKP), have recently generated some important insights to this problem, by showing that one can separately estimate TFP and input allocations under the following key restrictions:¹⁶

- (DGKP 1): Aggregate inputs, \vec{X}_{it} , are completely attributable to each production line, i.e. $X_{it}^{j} = S_{it}^{jX} X_{it}$, where $S_{it}^{jX} \in [0,1]$ and $\sum_{j=1}^{J_{it}} S_{it}^{jX} = 1$, for any input X.
- (DGKP 2): Input shares do not vary by input, i.e. $S_{it}^{jX} = S_{it}^{j}$ for any input X.
- (DGKP 3): There is no within-firm TFP dispersion, i.e. $\omega_{it}^j = \omega_{it}$ for all j.
- (DGKP 4): $f^{j}(.)$ is known.

These restrictions imply that there are only $J_{it} + 1$ unobservables that need to be identified by the researcher: the J_{it} input shares, S_{it}^j , and the single unknown TFP term, ω_{it} . More importantly, there are now enough equations to pin down the remaining unknowns, since (2) provides the J_{it} equations necessary to determine the input shares, while restrictions (DGKP 1) and (DGKP 2) generate the extra equation, $\sum_{j=1}^{J_{it}} S_{it}^j = 1$, necessary to pin down firm-level TFP. While this requires that $f^j(.)$ be known, DGKP accomplish by estimating $f^j(.)$ using a sample of single product firms.¹⁷ They then use $\sum_{j=1}^{J_{it}} S_{it}^j = 1$ and (2) to solve for the unknown input share and firm-level TFP terms numerically.¹⁸

 $^{^{15}}$ De Loecker (2011) and Collard-Wexler and De Loecker (2015) also describe sufficient conditions for input shares to be given by equal weights, or revenue shares, respectively, although this is not their primary focus.

¹⁶My presentation of their assumptions differs slightly from their own, since their goal is to estimate within-firm input expenditure shares, rather than quantity shares, as expenditure shares (combined with an estimate of within-firm input prices) are the only information needed to estimate product-level markups. Note, however, that their algorithm for backing out the expenditure allocations can also be applied to back out the quantity allocations.

¹⁷The estimation procedure used in DGKP introduces a selection correction term that, under some further restrictions, will consistently estimate the production function parameters.

¹⁸Note that having J + 1 equations to pin down J + 1 unknowns is only a necessary condition for identification. For this to be sufficient, one would need to impose further restrictions on the production technology to guarantee that a

The key advantage of using these restrictions to identify input shares and TFP, is that they do not require any restrictions on demand or market structure. This allows researchers to investigate questions related to competition and market power, without imposing some of the (potentially strong) restrictions on demand and market structure, that are commonly used in empirical industrial organization studies.¹⁹ For example, DGKP combine this insight with the fact that markups can be calculated using only production side data, as in De Loecker and Warzynski (2012), to show that trade liberalization in India resulted in an *increase* in markups, due to incomplete pass-through of the costs savings generated by input-market trade liberalization.²⁰

Note, however, that the restriction (DGKP 3) simply rules out variation in within-firm productivity by assumption, which is precisely the sort of within-firm heterogeneity emphasized by Eckel and Neary (2010), Bernard et al. (2011), Mayer et al. (2014) and Mayer et al. (2016) that can lead to within-firm allocative efficiency gains. In this paper, I develop an alternative set of restrictions that can be used to separately identify TFP from input allocations, even in the presence of within-firm TFP dispersion. The approach is based on a fully specified demand and supply model of production, where I show that conditional cost-minimization for the $J_{it} \times P$, inputs combined with J_{it} pricing first-order conditions, generate enough restrictions to separately identify the $J_{it} \times P$ input allocations from the J_{it} TFP terms.²¹

Valmari (2016) also takes advantage of pricing information to estimate within-firm input use, although only under the special case of Cobb-Douglas production technologies and monopolistic pricing. I provide a general approach to this problem, providing a series of sufficient conditions for observable prices and quantities to reveal the unobservable equilibrium input allocations chosen by a firm. The general approach allows for relatively general production technologies, as well as models of multi-product firms incorporating upward pressure on prices due to cross-product cannibalization effects, as well as some models of collusive pricing. In the subsequent section, I outline a general model of industry demand and supply that includes each of these particular models as special cases, and show how to use this class of models to identify within firm input allocations.

3 Model

This section describes a simple model of industry-level demand and supply, the structure of which can be used to determine within-firm input allocations as a function of commonly observed firm-level data (output quantities, prices, and aggregate inputs), and parameters to be estimated. First I describe the basic structure of the model, including the key assumptions necessary to generate a mapping between

unique solution to this system of equations exists. However, log-linear production functions, such as the Cobb-Douglas and translog production technologies, which are used in the vast majority of applied empirical work, will satisfy these restrictions.

¹⁹See Ackerberg et al. (2007) for a review of some of this literature.

²⁰In particular, while trade liberalization led to price decreases, marginal costs fell *more* than prices, generating an increase in markups.

²¹Another way to state this is that $J_{it} \times P$ first-order conditions for inputs, combined with the J_{it} equations provided by (2), generate all of the necessary equations to pin down the $J_{it}(P+1)$ unknowns. However, since input use will generally depend on output, and hence price, it is generally easier to work with the $J_{it} \times P$ first order conditions for cost minimization conditional on output levels, and the J_t pricing first-order conditions conditional on the cost function.

input allocations and observables, that does not depend on production function parameters. I then derive this mapping, and discuss some simple examples and extensions.

3.1 Basic Environment

During each period t, a set of differentiated products, Ω_t , are sold on the market by N_t firms. Each product is produced by a particular firm i, with $\mathbb{Y}_{it} \subset \Omega_t$ denoting the set of products produced by firm $i = 1, 2, ..., N_t$. Each product (or variety) $j \in \mathbb{Y}_{it}$ is produced using the following production technology:

$$Y_{it}^j = \exp(\omega_{it}^j) F(\vec{X}_{it}^j), \tag{3}$$

where Y_{it}^j is total output of variety $j \in \mathbb{Y}_{it}$, \vec{X}_{it}^j is the vector of inputs used in the production of j, and ω_{it}^j the log TFP, or productivity, of variety j.

To obtain an input allocation rule that *only* depends on demand-side data (prices, quantities, and demand elasticities), I require that the production function satisfy the following restrictions:

Assumption 1. F(.) is continuous and differentiable, equal to zero if any of its arguments are equal to zero, strictly increasing in all arguments, quasi-concave, and homogeneous of degree $\phi > 0$.

Assumption 2. The production technology differs across product lines within a firm only due to differences in ω_{it}^j , i.e. F(.) does not depend on $j \in \mathbb{Y}_{it}$.

While Assumption 1 mostly imposes standard regularity conditions on the production technology, Assumption 2 requires some discussion. While restricting the shape of the production technology to not differ across production lines within a firm may appear quite restrictive, note that in practice most production data sets only have enough observations to feasibly estimate production function parameters at the *industry* level, rather than the *industry-product level*. For example, Levinsohn and Petrin (2003) only allow production function parameters to differ at the 3-digit ISIC code, while De Loecker et al. (2016) only allow production function parameters to differ at the 2-digit NIC level. Hence, in practice, Assumption 2 simply requires that one focus on firms that only sell products within the same *industry*.

Moreover, this assumption, along with a little more structure, will imply restriction (DGKP 2), i.e. input shares do not depend on the identity of the input. This will allow me to determine an input allocation rule that *only* depends on demand side data. This shall prove useful for the purpose of estimation, since it will allow the researcher to estimate within-firm productivity dispersion in three sequential steps. *First*, estimate the demand side of the model. In the *second* step, recover the equilibrium input allocations. Finally, in the *third* step, use the estimated input allocations to estimate a firm-product level production function.

However, as I show in Section 3.3, Assumptions 1 and 2 can both be relaxed, but at the cost of generating an input allocation rule that depends on production function parameters. If production technologies were already known, this would generate no additional difficulties. However, for the purpose of estimation, this can generate significant problems, which I discuss in Section 3.3. In either case, the

researcher should regard Assumption 2 simply as a sufficient condition for the input allocation rule to only depend on demand side data.

While the production technology can use an arbitrary number of inputs, I will generally assume that production may require at least one *dynamic input*, such as capital, which can only be obtained through a dynamic investment process, and requires at least one *static input*, such as materials, that can be purchased from the market each time period and has no dynamic implications. More formally:

Assumption 3. F(.) takes as an input at least one element from the set \mathbb{M} , where \mathbb{M} is set of static inputs, which can be purchased for one-period use from the market according to some known price schedule $W_{it}^M = \mathbf{W}^M \left(\sum_{j \in \mathbb{Y}_{it}} M_{it}^j, A_{it}^M \right)$ for each $M \in \mathbb{M}$, where A_{it}^M is a vector of input price shifters.

Note that Assumption 3 allows for, but does not require, input buyers having market power, as in Morlacco (2018), in that they recognize that buying more units of an input may affect the market price of that input. If producers have no market power, then $\mathbf{W}^M\left(\sum_{j\in\mathbb{Y}_{it}}M_{it}^j,A_{it}^M\right)$ is simply a constant for all levels of input purchases.

Similarly, I let \mathbb{K} denote the set of dynamic inputs, which face some form of adjustment costs at the firm-level.²² Each dynamic input $K \in \mathbb{K}$, evolves over time according to the law of motion $K_{i,t+1} = l^K \left(K_{it}, I_{it}^K, I_{i,t+1}^K \right)$, where K_{it} is the stock of dynamic input K in firm i at time t, and I_{it}^K is firm i's current investment in dynamic input K.²³ Investment, as well as upkeep of the current stock of dynamic input K, costs the firm $d^K(K_{i,t-1}, K_{i,t}, I_{it}^K)$, where the stock of the dynamic input K in the previous period is included in the dynamic cost function to allow for adjustment costs. This formulation of the cost function allows dynamic inputs to have adjustment costs at the firm-level. To obtain an allocation rule that only depends on demand side data, I require that there not be any adjustment costs within a firm. In particular, I assume that:

Assumption 4. All inputs can be costlessly transferred across production lines within a firm.

Note that Assumption 4 is immediately satisfied for static inputs. For dynamic inputs, Assumption 4 simply means that given a stock K_{it} of dynamic input $K \in \mathbb{K}$, each firm i can costless allocate this resource across their production lines, with K_{it}^{j} denoting the quantity of dynamic input $K \in \mathbb{K}$ going into production line $j \in \mathbb{Y}_{it}$.

Given these assumptions, it will be useful to distinguish between *input allocations*, which describe the allocation of inputs across uses within a firm *conditional* on the aggregate resources firm i commands, and aggregate input vectors, which correspond to total quantity of inputs used by the firm i in period t. Formally, I will denote input allocations by the input matrix X_{it} , with typical element (j, X), X_{it}^{j} ,

²²Labour could either be a static variable, or a dynamic variable, depending on the structure of the labour market. For example, if hiring and costs are significant, then it may be more appropriate to model labour as a dynamic variable, as I do in my empirical application.

²³Note that I allow investment at time t+1 to affect the total stock of dynamic input K at time t+1, but do not require that current investment affect the total stock of the dynamic input. In particular, the case of predetermined dynamic inputs simply corresponds to the law of motion where $l^K\left(K_{it}, I_{it}^K, I_{i,t+1}^K\right) = l^K\left(K_{it}, I_{it}^K, \widetilde{I}_{i,t+1}^K\right) \quad \forall I_{i,t+1}^K, \widetilde{I}_{i,t+1}^K$. Note that this restriction would be satisfied by the standard law of motion $K_{i,t+1} = I_{i,t} + \delta K_{it}$, that is often used in the literature. Note, however, that the identification results in this section do not require that the law of motion for capital take this linear form, nor do they require that K_{it} be predetermined.

denoting the total quantity of input $X \in (\mathbb{K}, \mathbb{M})$ allocated to production line $j \in \mathbb{Y}_{it}$. On the other hand, I will denote aggregate input vectors by $\vec{X}_{it} \equiv (\vec{K}_{it}, \vec{M}_{it})$, where \vec{K}_{it} is the vector dynamic input stocks owned by firm i at time t, and \vec{M}_{it} is the vector of total static inputs purchased by firm i.

To ensure that input allocations are well defined, I also make the following assumption:

Assumption 5. Aggregate inputs, \vec{X}_{it} , are completely attributable to each production line, i.e. $X_{it}^j = S_{it}^{jX} X_{it}$, where $S_{it}^{jX} \in [0,1]$ and $\sum_{j=1}^{J_{it}} S_{it}^{jX} = 1$, for any input X.

Note that Assumption 5 rules out public inputs, such as a machine that can be used in more than one production processes simultaneously. This may rule out some forms of economies of scope. While this is a standard assumption in the literature on estimating production functions with multi-product firms, in Section 3.3 I relax this assumption. Under some restrictions on the form of public inputs, I show that the *share* of *effective inputs* across production lines can still be identified, although the quantity of public versus private inputs cannot. As a result, TFP estimates from multi-product firms may be "scaled-up" due to public inputs, which, under the restrictions discussed in Section 3.3, will be observationally equivalent to a common TFP shifter for multi-product firms. This echoes the discussion in De Loecker et al. (2016) that Assumption 5 is compatible with some economies of scope, as long as they are embodied in differences in TFP across firms.

I also make the following restriction on relationship between inputs and outputs:

Assumption 6. The input sets (\mathbb{K}, \mathbb{M}) , do not contain any products produced within the same firm, i.e. $\mathbb{Y}_{it} \notin (\mathbb{K}, \mathbb{M})$

Assumption 6 rules out vertical integration of the production process. While this may be an interesting channel for thinking about productivity in some industries (e.g. Steel), the methods outlined in this paper do not easily generalize to this case.

The next set of assumptions describe the industry structure. I assume that each product sold on the market faces a downward sloping demand function, $Q_{it}^j(\vec{P_t})$. Note that the demand function depends on the entire *vector* of prices charged on the market, $\vec{P_t}$, allowing for fairly general patterns of cross-product substitutability. Letting $\vec{Q}_{it}(\vec{P_t})$ denote the vector of demand functions, I also require that the overall demand system satisfy the following restriction, described in more detail in Berry et al. (2013) and Berry and Haile (2014):

Assumption 7. The demand system $\vec{Q}_{it}(\vec{P}_t)$ exhibits connected substitutes in prices.

Assumption 7 is a fairly weak restriction that is satisfied by most of the demand systems used in applied work. Roughly speaking, this restriction requires that all goods be weak substitutes for each other (demand for each good j is non-decreasing in the price of all goods $k \neq j$), and some goods are strict substitutes for one another (demand for good j is strictly increasing in the price of some subset goods $m \neq j$).

Each firm then chooses their aggregate input vectors \vec{X}_{it} , i.e. total labour and capital, input allocations X_{it} , i.e., the quantity of labour and capital in each production line, prices \vec{P}_{it} , and their investment

levels, \vec{I}_{it} to maximize the present discounted value of their profits, taking into account the constraint that sales cannot be greater than quantities produced, as well as the laws of motion for the dynamic inputs. The Bellman equation associated with this problem is given by:²⁴

$$V_{t}(\chi_{it}) = \underset{\vec{I}_{it}, \mathbb{X}_{it}, \vec{P}_{it}}{\operatorname{Max}} \sum_{j \in \mathbb{Y}_{it}} P_{it}^{j} Q_{it}^{j} \left(\vec{P}_{t} \right) - \sum_{M \in \mathbb{M}} \sum_{j \in \mathbb{Y}_{it}} \mathbf{W}^{M} \left(\sum_{j \in \mathbb{Y}_{it}} M_{it}^{j}, A_{it}^{M} \right) M_{it}^{j}$$

$$- \sum_{K \in \mathbb{K}} d^{K} (K_{i,t-1}, K_{it}, I_{it}^{K}) + \beta \mathbb{E} \{ V_{t+1} \left(\chi_{i,t+1} \right) | \chi_{it} \}$$
subject to:
$$\exp(\omega_{it}^{j}) F(\vec{X}_{it}^{j}) \geq Y_{it}^{j} = Q_{it}^{j} \left(\vec{P}_{t} \right) \quad \forall j \in \mathbb{Y}_{it}$$

$$\sum_{j \in \mathbb{Y}_{it}} X_{it}^{j} = X_{it} \quad \forall X \in \mathbb{K}$$

$$K_{i,t+1} = l^{K} (K_{it}, I_{it}^{K}, I_{i,t+1}^{K}) \quad \forall K \in \mathbb{K},$$

$$(4)$$

where χ_{it} is the vector state variables, which include the set of products produced by the firm \mathbb{Y}_{it} , the set of products produced by *other* firms $\mathbb{Y}_{-i,t} \equiv \Omega_t \setminus \mathbb{Y}_{it}$, the vector of firm-level TFP terms, $\vec{\omega}_{it}$, the vector of firm-level input price shifters \vec{A}_{it} , the vector of prices charged by all other firm $P_{-i,t}$, and the vector of lagged stocks of dynamic inputs $\vec{K}_{i,t-1}$.²⁵

3.2 Using First-Order Conditions to Recover Input Allocations

I now show that Assumptions 1 though 7 imply a mapping from observable prices and quantities, to the unobserved input allocations chosen by a set of firms each independently solving (4).

First, note that since dynamic inputs are costlessly transferable across uses within the firm (Assumption 3), and F(.) is strictly increasing in all of its inputs (Assumption 1), each firm will choose an input allocation that minimizes total static input costs conditional on its stock of firm-level dynamic inputs, \vec{K}_{it} , and some desired set of output levels \vec{Y}_{it} . More formally, any solution to (4) will involve an input allocation X_{it} that minimizes static input costs, subject to some desired output levels Y_{it}^{j} for each $j \in Y_{it}$, and a given stock of dynamic inputs.²⁷ The Lagrangian for this conditional cost minimization

 $^{^{24}}$ I slightly abuse notation here, letting Y_{it}^{j} denote the *desired* output quantity a firm wishes to produce. In equilibrium this will always equal actual quantity produced through (3).

²⁵In the special case of a *pre-determined* dynamic inputs, where $l^K\left(K_{it}, I_{it}^K, I_{i,t+1}^K\right) = l^K\left(K_{it}, I_{it}^K, \widetilde{I}_{i,t+1}^K\right) \quad \forall I_{i,t+1}^K, \widetilde{I}_{i,t+1}^K$, then K_{it} will also be a state variable.

²⁶Note given $(\mathbb{X}_{it}, \vec{X}_{it})$, prices are implicitly pinned down in the above formulation of the firm's problem by the constraints $\exp(\omega_{it}^j)F(\vec{X}_{it}^j) \geq Q_{it}^j(\vec{P}_t)$, which hold with equality in equilibrium. Therefore, one could also write the problem as firms choosing prices and all static inputs except for one static input $M \in \mathbb{M}$, although this would be notationally cumbersome.

²⁷Note that reallocating dynamic inputs across production lines, conditional on any level of firm-level dynamic inputs and desired output levels Y_{it}^j , will not affect a firm's current costs or future profits, except by allowing the firm to potentially purchase less static inputs by reorganizing the dynamic inputs more efficiently. Hence, one can "concentrate out" the optimal levels of static inputs in this problem, conditional on some level of dynamic inputs, by solving the conditional cost minimization sub-problem for any given value of (\vec{K}_{it} , Y_{it}^j).

sub-problem is given by²⁸

$$L = -\sum_{M \in \mathbb{M}} \sum_{j \in \mathbb{Y}_{it}} \mathbf{W}^{M} \left(\sum_{j \in \mathbb{Y}_{it}} M_{it}^{j}, A_{it}^{M} \right) M_{it}^{j} + \sum_{j \in \mathbb{Y}_{it}} \lambda_{it}^{j} \left(\exp\left(\omega_{it}^{j}\right) F(\vec{X}_{it}^{j}) - Y_{it}^{j} \right) + \sum_{K \in \mathbb{K}} \mu_{it}^{K} \left(K_{it} - \sum_{j \in \mathbb{Y}_{it}} K_{it}^{j} \right),$$

$$(5)$$

where λ_{it}^j is the Lagrangian multiplier for the production constraint, and μ_{it}^K is the Lagrangian multiplier for resource constraint for dynamic input $K \in \mathbb{K}$.

Letting $\nu_{it}^X = W_{it}^X + \frac{\partial \mathbf{W}^{\bar{X}}(\sum_{j \in \mathbb{Y}_{it}} X_{it}^j, A_{it}^X)}{\partial(\sum_{j \in \mathbb{Y}_{it}} X_{it}^j)} \sum_{j \in \mathbb{Y}_{it}} X_{it}^j$ if $X \in \mathbb{M}$, and $\nu_{it}^X = \mu_{it}^X$ if $X \in \mathbb{K}$, the first-order necessary condition for any input X_{it}^j can be written as:

$$-\nu_{it}^{X} + \lambda_{it}^{j} \exp\left(\omega_{it}^{j}\right) \frac{\partial F(\vec{X}_{it}^{j})}{\partial X} = 0.$$
 (6)

To obtain a simple input allocation formula that only depends on demand side information, I make use of the following Lemma, which shows that property (DGKP 2) from De Loecker et al. (2016) follows from the previously stated assumptions:

Lemma 1. If Assumptions 1 through 6 hold, then there exists a solution to the firm's conditional cost minimization problem satisfying $X_{it}^j = S_{it}^j X_{it} \quad \forall X \in (\mathbb{K}, \mathbb{M}), \text{ where } S_{it}^j \in [0, 1] \text{ and } \sum_{j \in \mathbb{Y}_{it}} S_{it}^j = 1$

Proof. See Appendix A.
$$\Box$$

While a formal proof of the above is provided in the Appendix for completeness sake, the intuition for this result is a relatively straightforward implication of homogeneous production technologies. Since homogeneous production technologies generate isoquants that have constant slopes along any ray from the origin, this means that a cost minimizing firm will choose constant input ratios across production lines within the firm, i.e. $\frac{X_{it}^j}{Z_{it}^j} = \frac{X_{it}^k}{Z_{it}^k}$ for each $X, Z \in (\mathbb{K}, \mathbb{M})$ and each $j, k \in \mathbb{Y}_{it}$. This immediately implies that the input shares within a particular production line j do not depend on the identity of the input, and therefore $\frac{X_{it}^j}{X_{it}} = S_{it}^j \ \forall X \in (\mathbb{K}, \mathbb{M})$.

As a result, Assumptions 1 through 6 guarantee that the vector of inputs allocated to each production line can be written as $\vec{X}_{it}^j = S_{it}^j \vec{X}_{it}$. Substituting this into (6), and then using the fact that all of the partial derivatives of a homogeneous of degree $\phi > 0$ function are homogeneous of degree $\phi - 1$, one obtains:

$$\nu_{it}^{X} = \lambda_{it}^{j} \exp\left(\omega_{it}^{j}\right) \left(S_{it}^{j}\right)^{\phi-1} \frac{\partial F(\vec{X}_{it})}{\partial X}.$$
 (7)

To get rid of the unknown TFP terms in (7), divide this expression by $Y_{it}^j = \exp(\omega_{it}^j) F(S_{it}^j \vec{X}_{it}) = \exp(\omega_{it}^j) (S_{it}^j)^{\phi} F(\vec{X}_{it})$, which yields, after some minor manipulations:

²⁸Formal statement of the conditional cost minimization problem is stated in Appendix A.

$$S_{it}^{j} = \frac{\partial F(\vec{X}_{it})}{\partial X} \frac{\lambda_{it}^{j} Y_{it}^{j}}{F(\vec{X}_{it}) \nu_{it}^{X}}.$$
 (8)

One can then sum (8) over all $j \in \mathbb{Y}_{it}$, yielding $\frac{\partial F(\vec{X}_{it})}{\partial X} \frac{\sum_{j \in \mathbb{Y}_{it}} \lambda_{it}^{j} Y_{it}^{j}}{F(\vec{X}_{it}) \nu_{it}^{X}}$. Dividing (8) by this expression yields:

$$S_{it}^{j} = \frac{\lambda_{it}^{j} Y_{it}^{j}}{\sum_{k \in \mathbb{N}_{it}} \lambda_{it}^{k} Y_{it}^{k}} = \frac{M C_{it}^{j} Y_{it}^{j}}{\sum_{k \in \mathbb{N}_{it}} M C_{it}^{k} Y_{it}^{k}},\tag{9}$$

where the second equality in (9) follows from the envelope theorem $\lambda_{it}^j = \frac{\partial C(\vec{K}_{it}, \vec{Y}_{it}, \vec{\omega}_{it}, \vec{A}_{it})}{\partial Y_{it}^j} \equiv MC_{it}^j$, where $C(\vec{K}_{it}, \vec{Y}_{it}, \vec{\omega}_{it}, \vec{A}_{it})$ is the cost function for static inputs, conditional on the level of dynamic inputs \vec{K}_{it} and a desired level of output \vec{Y}_{it} .²⁹

While (9) may not at first glance appear all that useful, as marginal costs are almost always unobservable in any firm-level data set, the key insight that I exploit is that many models of inter-firm competition imply a direct mapping from observable demand-side variables to unobservable product level conditional marginal costs. This is an old insight, originally formalized by Rosse (1970) for the case of a monopolist, who noted that one can obtain marginal costs without supply-side data using pricing first-order conditions, since a monopolist will choose prices such that marginal revenues equal marginal costs. Hence, one can use demand-side price and quantity data to estimate demand elasticities, which can then be used to back out a firm's equilibrium marginal costs from marginal revenue.

The general insight that observed demand elasticities can be used to recover marginal costs applies to a much wider class of imperfect competition models beyond that of simple monopolists - see Berry and Haile (2014). To see this in the context of the model described in this paper, note that once the input allocation problem has been solved for any potential level of dynamic inputs \vec{K}_{it} , and desired output levels \vec{Y}_{it} , one can determine the static conditional cost function and substitute this into (4), yielding the simplified firm's problem:

$$V_{t}(\chi_{it}) = \underset{\vec{P}_{it}, \vec{I}_{it}, \vec{K}_{it}}{\text{Max}} \sum_{j \in \mathbb{Y}_{it}} P_{it}^{j} Q_{it}^{j} \left(\vec{P}_{t} \right) - C \left(\vec{K}_{it}, \vec{Q}_{it} \left(\vec{P}_{t} \right), \vec{\omega}_{it}, \vec{A}_{it} \right)$$
$$- \sum_{K \in \mathbb{K}} d^{K}(K_{i,t-1}, K_{it}, I_{it}^{K}) + \beta \mathbb{E} \{ V_{t+1} \left(\chi_{i,t+1} \right) | \chi_{it} \}$$
(10)

subject to:

$$K_{i,t+1} = l^K(K_{it}, I_{it}^K, I_{i,t+1}^K) \ \forall K \in \mathbb{K},$$

where I have used the fact that quantity produced will equal quantity sold in equilibrium, i.e. $Y_{it}^{j} = Q_{it}^{j} \left(\vec{P}_{t} \right)$.

Taking the first-order condition for any P_{it}^{j} in (10) yields:

²⁹More formally, $C\left(\vec{K}_{it}, \vec{Y}_{it}, \vec{\omega}_{it}, \vec{A}_{it}\right)$ is the objective function associated associated with solution to (CM) in Appendix A.

$$Q_{it}^j + \sum_{k \in \mathbb{Y}_{it}} \frac{\partial Q_{it}^k}{\partial P_{it}^j} \left(P_{it}^k - M C_{it}^k \right) = 0.$$
 (11)

One can then stack the $J_t \equiv |\Omega_t|$ first order conditions defined by (11), which, in matrix notation, defines the following system of equations:

$$\vec{Q_t} + \Delta_t \left(\vec{P_t} - \vec{MC_t} \right) = 0, \tag{12}$$

where $\Delta_t = \mathbb{O}_t \circ \partial_t$, with ∂_t corresponding to an $J_t \times J_t$ matrix of demand derivatives, with typical element (j,k) equal to $\frac{\partial Q_{it}^j}{\partial P_{it}^k}$, and \mathbb{O}_t being the *ownership matrix*, with element (j,k) equal to 1 if product j and k are both produced by the same firm, and equal to zero otherwise.

Note that one can use (12) to solve for the equilibrium marginal costs as a function of quantities produced, prices, and demand derivatives, by pre multiplying by Δ_t^{-1} , yielding:

$$\vec{MC_t} = g\left(\vec{Q_t}, \vec{P_t}, \partial_t, \mathbb{O}_t\right) = \Delta_t^{-1} \vec{Q_t} + \vec{P_t}.$$
 (13)

For the marginal cost inversion described by (13) to be valid, Δ_t needs to be invertible. Berry and Haile (2014) have recently shown that Δ_t will be invertible under fairly general conditions. In particular, as long as Assumption 7 holds, a relatively weak restriction that is satisfied by many of the demand systems used in applied work, then Δ_t will be invertible.³⁰

An important point worth emphasizing is that as long as Assumption 7 holds, Δ_t is invertible for any ownership matrix \mathbb{O}_t . This means that the this marginal cost inversion is also possible under imperfect competition with some forms of collusion. In particular, as noted by Nevo (1998), collusion between firms can be thought of as joint-profit maximization, i.e. legally separate firms choosing their prices as if they were one multi-product firm.³¹ As long as the set of colluding firms fully internalizes their pricing decisions across products, then this simply corresponds to an alternative ownership matrix, \mathbb{O}'_t , than that observed in the data. For example, full market collusion simply corresponds to the case where \mathbb{O}'_t is a $J_t \times J_t$ matrix of ones, implying that all prices are chosen to internalize all possible cannibalization effects.

Combining (13) and (9) yields the following Theorem:

Theorem 1. As long as the cost minimizing input allocation is unique, then Assumptions 1 though 7 imply that if each firm chooses prices, input quantities, and investment to solve (4), the share any input $X \in (\mathbb{K}, \mathbb{M})$ going into production line $j \in \mathbb{Y}_{it}$ satisfies $S_{it}^j = \frac{g_{it}^j(\vec{Q}_t, \vec{P}_t, \partial_t, \mathbb{Q}_t)Y_{it}^j}{\sum_{j \in \mathbb{Y}_{it}} g_{it}^k(\vec{Q}_t, \vec{P}_t, \partial_t, \mathbb{Q}_t)Y_{it}^k}$, where each $g_{it}^j(.)$ is a known function of prices, quantities, demand derivatives, and the ownership matrix.

Proof. Follows from the discussion in text and equations (9) and (13), which will always hold if there is a unique solution to the firm's input allocation problem, as per through Lemma 1. Note that uniqueness

 $^{^{30}}$ Roughly speaking, Assumption 7 is satisfied if all goods are weak substitutes for each other (demand for each good j is non-decreasing in the price of all goods $k \neq j$), and some goods are strict substitutes for one another (demand for good j strictly increases if the price of good k rises). See Berry et al. (2013) and Berry and Haile (2014) for more details.

³¹Note that this form of collusion requires that firms only cooperate in output markets, i.e. through pricing first-order conditions, *not* through input markets, i.e. firm's cannot engage in labour or capital sharing.

of the input allocation problem is guaranteed for standard homogenous production functions such as Cobb-Douglas and CES, as it is straightforward to verify that (6) implies that $X_{it}^j = S_{it}^j X_{it} \ \forall j \in \mathbb{Y}_{it}$.

3.2.1 Theorem 1 In Practice: Examples

While Theorem 1 is formulated in fairly general terms, one can obtain some useful insights by applying it to some special cases that are often used in the applied literature. For example, models of perfect competition with price-taking firms imply that all firms choose quantities such that $P_{it}^j = MC_{it}^j$. Substituting this into (9) yields: $S_{it}^j = \frac{P_{it}^j Y_{it}^j}{\sum_{k \in \mathbb{Y}_{it}} P_{it}^k Y_{it}^k}$, i.e the optimal input allocations are simply revenue shares.

While perfect competition is a highly unrealistic assumption for a model where firms produce differentiated varieties, note that revenue share input allocation rule will also hold whenever multiplicative markups $\mu_{it}^j \equiv \frac{P_{it}^j}{MC_{it}^j}$, are constant within a firm. Equation (9) then implies:

$$S_{it}^{j} = \frac{\frac{P_{it}^{j}}{\mu_{it}} Y_{it}^{j}}{\sum_{k \in \mathbb{Y}_{it}} \frac{P_{it}^{k}}{\mu_{it}} Y_{it}^{k}} = \frac{P_{it}^{j} Y_{it}^{j}}{\sum_{k \in \mathbb{Y}_{it}} P_{it}^{k} Y_{it}^{k}} \quad \text{if } \mu_{it}^{j} = \mu_{it} \ \forall j \in \mathbb{Y}_{it}.$$
 (14)

The fact that constant within-firm markups allows one to use revenue shares to allocate inputs, was previously noted by Collard-Wexler and De Loecker (2015), although their derivation of this property only proceeded after making the restriction $\omega_{it}^j = \omega_{it} \ \forall j \in \mathbb{Y}_{it}$. Note that, as the derivation of (9) makes clear, a revenue share input allocation rule will be valid with constant within-firm markups even if there is within-firm TFP heterogeneity.

Interestingly, it turns out that equilibrium markups will be constant within a firm if variety-level demand is CES, i.e. $Q_{it}^{j}(\vec{P_t}) = \frac{(P_{it}^{j})^{-\sigma}}{\sum_{l=1}^{N_t} \sum_{k \in \mathbb{Y}_{lt}} (P_{lt}^{k})^{1-\sigma}}$, even if firms are not atomistic, a property of CES demand systems previously noted by Feenstra and Ma (2007). In particular, it is straightforward to show the pricing first-order conditions (11) for this demand system imply the following marginal cost inversion:³²

$$MC_{it}^{j} = \frac{P_{it}^{j}}{\left(1 + \frac{1}{(\sigma - 1)(1 - RS_{it})}\right)} \quad \text{if } Q_{it}^{j}(\vec{P}_{t}) \text{ is CES}, \tag{15}$$

where $RS_{it} \equiv \frac{\sum_{j \in \mathbb{Y}_{it}} P_{it}^{j} Q_{it}^{j}}{\sum_{l=1}^{N_{t}} \sum_{k \in \mathbb{Y}_{lt}} P_{lt}^{k} Q_{lt}^{k}}$ is the revenue share of all products produced by firm i. Note that as long as the number of competitors in the market is finite, so $RS_{it} > 0$ for at least some i, there will be heterogeneous markups across firms, but since RS_{it} does not vary with $j \in \mathbb{Y}_{it}$, markups will constant within firm, and therefore according to (14) input allocations will follows a simple revenue share rule.³³ Hence, in practice, if a researcher believes that demand for a particular industry can be

³³Interestingly, note that this revenue sharing rule is valid for any ownership matrix \mathbb{O}_t . This means that revenue share allocations can be used with CES demand functions, even if firms are colluding, and the sets of colluding firms are unknown.

well approximated by CES, then no other information, besides revenue at the firm-product level, is necessary to estimate input allocations for multi-product firms.

Note, however, that CES demand for all products is a very strong restriction. If demand is non-CES, the allocation rule described in Theorem 1 will depend on price elasticities, which will have to be estimated. For example, if there is a logit demand system for each product, $Q_{it}^j(\vec{P}_t) = \frac{\exp(\eta_{it}^j - \alpha P_{it}^j)}{\sum_{l=1}^{N_t} \sum_{k \in \mathbb{Y}_{lt}} \exp(\eta_{it}^k - \alpha P_{it}^k)}$, where η_{it}^j can be interpreted as product-specific quality, then (11) implies the following marginal cost inversion:

$$MC_{it}^{j} = P_{it}^{j} - \frac{1}{\alpha \left(1 - QS_{it}\right)} \quad \text{if } Q_{it}^{j}(\vec{P_t}) \text{ is Logit}, \tag{16}$$

where $QS_{it} \equiv \frac{\sum_{j \in \mathbb{Y}_{it}} Q_{it}^j}{\sum_{l=1}^{N_t} \sum_{k \in \mathbb{Y}_{lt}} Q_{lt}^k}$ is the quantity-share of firm i.

Substituting (16) into (9), yields the following input allocation rule for simple logit demand systems:

$$S_{it}^{j} = \frac{\left(P_{it}^{j} - \frac{1}{\alpha(1 - QS_{it})}\right) Y_{it}^{j}}{\sum_{k \in \mathbb{Y}_{it}} \left(P_{it}^{k} - \frac{1}{\alpha(1 - QS_{it})}\right) Y_{it}^{k}} \quad \text{if } Q_{it}^{j}(\vec{P_t}) \text{ is Logit.}$$

$$(17)$$

Note that the allocation rule (17) depends on the demand parameter α , which governs the own and cross-product elasticities. Hence, to determine the input allocations from price and quantity data, the demand parameter α must first be estimated.

In practice, researchers may wish to specify richer demand systems, such as the nested logit model of McFadden (1978), or the random coefficients demand system described in Berry et al. (1995). These will require estimation of more demand side parameters to determine the exact functional form (13) takes. Whether the added computational burden associated with the estimation of richer demand systems is appropriate will vary with the particular application a researcher has in mind. Nevertheless, note that as long as the demand system satisfies Assumption 7, which is the case for nested logit as well as random coefficient models, an appropriate inversion mapping will exist.

3.3 Discussion and Extensions

The key significance of Theorem 1 is that under the maintained assumptions, demand side information (prices and quantities), can be used to infer the allocations of inputs across production lines, as long as demand derivatives and ownership structures are known. This immediately suggests a straightforward strategy for estimating TFP in a dataset with multiproduct firms when prices and quantities are observed. First, use the price and quantity data to estimate the shape of the demand function. This is a standard problem in industrial organization, where a number of standard techniques have been developed just for this purpose. Having estimated the demand function, the researcher can then obtain estimates of demand derivatives at the product level, and then apply Theorem 1 to obtain the allocations of inputs across production lines within a firm. This provides the researcher with estimates of input use at the firm product level, which can then be used for the purpose of production function

estimation, using standard techniques from the literature.

Before describing this approach in detail, which I turn to in Section 5, it is useful to examine the role played by some of the stronger assumptions used in Theorem 1, and the cost of relaxing these restrictions.

3.3.1 General Function Forms

To examine the role the functional form restrictions in Assumption 1 and 2 play, suppose instead that $Y_{it}^j = \exp(\omega_{it}^j) F^j(\vec{X}_{it}^j)$, i.e. $F^j(.)$ does depend on j, and, in principal, is neither homogeneous or quasi-concave. One can then modify (6) to allow the production function to vary with j, and then divide this expression by $Y_{it}^j = \exp(\omega_{it}^j) F^j(\vec{X}_{it}^j)$ and rearrange, yielding:

$$X_{it}^{j} = \frac{\theta^{j} \left(\vec{X}_{it}^{j} \right) \lambda_{it}^{j} Y_{it}^{j}}{\nu_{it}^{X}}, \tag{18}$$

where $\theta^j\left(\vec{X}_{it}^j\right) \equiv \frac{\partial F^j\left(\vec{X}_{it}^j\right)}{\partial X} \frac{X_{it}^j}{F^j\left(\vec{X}_{it}^j\right)}$ is the *output elasticity* for input $X \in (\mathbb{K}, \mathbb{M})$. Summing (18) over all $j \in \mathbb{Y}_{it}$, and dividing (18) by this new expression, yields, after using the envelope theorem and marginal cost inversion results described in Section 3.2:

$$X_{it}^{j} = \frac{\theta^{j} \left(\vec{X}_{it}^{j} \right) g_{it}^{j} \left(\vec{Q}_{t}, \vec{P}_{t}, \partial_{t}, \mathbb{O}_{t} \right) Y_{it}^{j}}{\sum_{k \in \mathbb{Y}_{it}} \theta^{k} \left(\vec{X}_{it}^{k} \right) g_{it}^{k} \left(\vec{Q}_{t}, \vec{P}_{t}, \partial_{t}, \mathbb{O}_{t} \right) Y_{it}^{k}} \sum_{j \in \mathbb{Y}_{it}} X_{it}^{j}.$$

$$(19)$$

Note that equilibrium inputs shares, if they exist, will form a fixed point of the system of equations described by (19). If such a solution exists, note that this mapping does not depend on the unobservable TFP terms, ω_{it}^{j} , implying that that input shares are separately identified from TFP if F^{j} is known, even if Assumptions 1 and 2 do not hold.³⁴

Note, however, that while Assumptions 1 and 2 are unnecessary if $F^{j}(.)$ is known, in general the production function must also be estimated. This severely restricts the practicality of (19) for applied work, since, if one wishes to use multi-product firms to estimate the production function, one would have to already know the production technology to determine their input allocations.³⁵ Moreover, in practice,

³⁴Note that a fixed-point is guaranteed to exist in (19) for the case of heterogeneous Cobb-Douglas production functions, i.e. $F^j\left(\vec{X}_{it}^j\right) = \prod_{X \in (\mathbb{K}, \mathbb{M})} \left(X_{it}^j\right)^{\beta_X^j}$. Since output elasticities are constant for each input with Cobb-Douglas, i.e. $\theta^j\left(\vec{X}_{it}^j\right) = \beta_X^j$, (19) simplifies to:

$$X_{it}^{j} = \frac{\beta_X^{j} g_{it}^{j} \left(\vec{Q}_t, \vec{P}_t, \partial_t, \mathbb{O}_t \right) Y_{it}^{j}}{\sum_{k \in \mathbb{Y}_{it}} \beta_X^{k} g_{it}^{k} \left(\vec{Q}_t, \vec{P}_t, \partial_t, \mathbb{O}_t \right) Y_{it}^{k}} \sum_{j \in \mathbb{Y}_{it}} X_{it}^{j}, \tag{20}$$

i.e. X^j_{it} will be known as long as the various production function parameters, demand elasticities, ownership structures, and $\sum_{j \in \mathbb{Y}_{it}} X^j_{it}$ are also known. Imposing Assumption 5 on top of this, then (20) implies that input shares $S^{jX}_{it} \equiv \frac{X^j_{it}}{X_{it}}$, will generally depend on the identity of the input, i.e. $S^{jX}_{it} \neq S^{jZ}_{it}$, which means that Assumption (2) will often be necessary for restriction (DGKP 2) to hold.

³⁵Note that there exist approaches to estimation that can deal with this circularity problem. For example, one could simply estimate the production function parameters using single product firms, as in De Loecker et al. (2016), although

most of the applied literature estimates production function by *industry* rather than by *product*, largely due to the fact that most datasets do not have sufficient observations to calculate production functions at the product level *even if input allocations were known*. Hence, for the purposes of most applied work, I regard Assumptions 1 and 2 as relatively weak, in the sense that most work in this area implicitly makes such assumptions, and moreover, fairly reasonable from a pragmatic point of view, as they can allow the researcher to include multi-product producers in a straightforward and transparent manner.

3.3.2 Public Inputs

It is also worth considering whether Assumption 5 is appropriate, i.e. all inputs are attributable to each production line, so that $X_{it}^j = S_{it}^{jX} X_{it}$. While this assumption is made implicitly by basically all the literature that uses input allocation rules to deal with multi-product firms, note that it is restrictive in the sense that it rules out public inputs, which may generate economies of scope. Note, however, that a variant of the identification result discussed in the main text carries through, even with public inputs.

Suppose that the use of some input X_{it} within a firm can be divided into a *public* or *common* component, X_{it}^C and a *rivalrous* component, X_{it}^R . Rivalrous inputs X_{it}^R can only be allocated to a single production line, as in Assumption 5, so that $X_{it}^{jR} = S_{it}^{jXR} X_{it}^R$ with $\sum_{j \in \mathbb{Y}_{it}} S_{it}^{jXR} = 1$. Common inputs are allocated to *every* production line automatically. Hence, the quantity of *effective* inputs allocated to each production lines is given by $X_{it}^j = X_{it}^C + X_{it}^{jR}$.

Suppose further that the public component of X_{it} is a constant fraction of total inputs owned by the firm, i.e. $X_{it}^C = \kappa^X X_{it}$ and $X_{it}^R = (1 - \kappa^X) X_{it}$, and the production technology is Cobb-Douglas, so that $F\left(\vec{X}_{it}^j\right) = \prod_{X \in (\mathbb{K}, \mathbb{M})} \left(X_{it}^j\right)^{\beta_X}$. Firms will then allocate their private inputs X_{it}^R across production lines to minimize static production costs conditional on aggregate dynamic inputs. It is straightforward to show, following the derivation in the text, that this slight modification implies that effective inputs will satisfy the following:

$$X_{it}^{j} = \frac{g_{it}^{j}(.)Y_{it}^{j}}{\sum_{k \in \mathbb{Y}_{it}} g_{it}^{k}(.)Y_{it}^{k}} \sum_{j \in \mathbb{Y}_{it}} X_{it}^{j} = \frac{g_{it}^{j}(.)Y_{it}^{j}}{\sum_{k \in \mathbb{Y}_{it}} g_{it}^{k}(.)Y_{it}^{k}} \left(1 + (J_{it} - 1)\kappa^{X}\right) X_{it}, \tag{21}$$

where $J_{it} \equiv |\mathbb{Y}_{it}|$ is the number of products produced by firm i^{36}

While the fractions of public inputs, κ^X , are unobservable, and hence the level of input useage will

this approach requires potentially strong restrictions on the process governing selection into producing multiple products, and will be less efficient as one will have to drop a large numbers of firms for estimation. Moreover, GMM based estimators only require a set of orthogonality conditions with some set of unobservables (E.g. TFP), and these unobservables need only be expressed as a function of data and parameters to be estimated. Hence, a GMM estimator based on orthogonality of some instruments to TFP, could be used after substituting (19) into $Y_{it}^j = \exp\left(\omega_{it}^j\right) F^j(\vec{X}_{it}^j)$, which would allow the researcher to express TFP as a function of parameters to be estimated and data, as Valmari (2016) does in the case of heterogeneous Cobb-Douglas technologies. Note, however, that this approach may suffer from identification problems, since the model would be inherently non-linear, and therefore verification of the asymptotic identification conditions, i.e. that the parameters to be estimated are uniquely pinned down by the assumed moment conditions, would be quite difficult to verify in practice.

³⁶Implicitly, this derivation assumes that rivalrous inputs are chosen to be non-negative, and this constraint is not binding, i.e. the firm does not wish to move common inputs from one production line to another.

not be identified in this framework, note that under Cobb-Douglas, (21) implies that the unobservable component of these input allocations, $(J_{it} - 1) \kappa^X$, are observationally equivalent to a TFP shifter received by multiproduct firms. To see this, substitute (21) into the production function, yielding:

$$Y_{it}^{j} = \exp(\omega_{it}^{j} + SC_{it}) \prod_{X \in (\mathbb{K}, \mathbb{M})} \left(\widehat{X}_{it}^{j}\right)^{\beta_{X}}, \tag{22}$$

where \widehat{X}_{it}^{j} is the level of input useage obtained from Theorem 1, and SC_{it} is the economies of scope shifter, given by $SC_{it} \equiv \sum_{X \in (\mathbb{K}, \mathbb{M})} \beta_X \ln \left(1 + (J_{it} - 1) \kappa^X\right)$.

Hence, when public inputs take this form, they are observationally equivalent to TFP shifters that depend on the number of products produced by a firm. As a result, one can deal with the complications introduced by public inputs by controlling for the number of products in the production function estimation routine.³⁷

Note, however, that under these assumptions the production function residual, $\widehat{\omega}_{it}^{j}$ will be composed of both a "pure" TFP component and an economy of scope shifter, SC_{it} , i.e. $\widehat{\omega}_{it}^{j} = \omega_{it}^{j} + SC_{it}$. This means that multi-product firms may have higher measured TFP either due to selection, i.e. firms with high ω_{it}^{j} terms are more likely to produce many products, as emphasized by Bernard et al. (2010) and Mayer et al. (2014), or due to economies of scope, i.e. multi-product production scales up input effectiveness due to public inputs. Since I do not know of a clean source of exogenous variation that would randomly allocate plants across multi-product firm status, I do not attempt to separately identify these two different mechanisms which could potentially explain a "multi-product production premium." Instead, I simply note that both mechanisms can be present in my data, and can lead to higher measured TFP in multi-product firms.

4 Data

The primary data set used in this paper comes from the 2000-2007 Indian Annual Survey of Industries (ASI), provided by by the Indian Ministry of Statistics.³⁸ The sample frame for the survey is all manufacturing plants in India that employ more than 10 workers. Plants with more than 100 workers ("census" firms) are surveyed every year, while smaller plants are randomly sampled each year. The data contains consistent plant-level identifiers across years for both census and non-census plants, allowing me to construct plant-level panels for both types of firms.³⁹ As described in Martin et al. (2017), the panel

 $[\]overline{^{37}}$ More generally, one could allow the fraction of public inputs, κ^X , to depend on the *sets* of outputs produced by the firm, in which case one would have to include product set fixed effects.

³⁸Years in the ASI are recorded from April 1 to March 31. While the Ministry of Statistics refers to years by the end year, I will refer to years by the start year since the majority of production time takes place in that year. I follow this convention when matching the data to other datasets (e.g. trade data).

³⁹Since the unit of observation is a plant-item, rather than a firm-item, I will consider separate plants as separate firms in my empirical analysis, and generally use these terms interchangeably. This may be problematic for my empirical strategy, as multi-plant firms may choose prices in a coordinated way that would violate the Bertrand-Nash assumption at the plant-level assumed by my model. Roughly speaking, this will only matter if within-firm and across-plant cannibalization effects are large. Note, however, that many of the observations are likely to approximate the decision making unit, as firms have the option of filing a joint return to the census for all of their factories located within the same state. 13% of

data is fairly high quality, and covers a much larger subset of Indian producers than other comparable datasets for the country, such as Prowess.

I focus on a single industry in my empirical application, Machinery, Equipment, and Parts, the details of which are described in Appendix B. I focus on this industry for two reasons. *First*, I wish to focus on an industry where it is appropriate to think of inputs as being directly allocated to different production lines, as per Assumption 5. Unfortunately, this is not the case for two of India's largest industries, sugar and textiles, as many plants in these industries produce groups of products that are *by-products*. In particular, most refined sugar producers also produce molasses, which is generated by the refining process, while many cotton producers also produce cotton waste, a by-product of cotton production that is often resold for further production purposes. Unfortunately, this means that Theorem 1 is unlikely to apply for these firms. ⁴⁰ Machinery manufacturing, on the other hand, does not generate many by-products that are also sold on the market by the same plant, making Theorem 1 more likely to apply.

Second, I wish to focus on industries where there are many multi-product firms, but the output sets produced by a firm are not driven by vertical integration, as per Assumption 6. If some of the outputs produced by a firm are also inputs in a vertically integrated production line, then Theorem 1 will not apply, as the shadow value of inputs will vary depending on where they are being allocated along the vertical production line. ⁴¹ Unfortunately, this is not the case for many other industries. After constructing an input-output table, the details of which I describe in Appendix B, I find that more than 50% of the observed revenue in industries such as steel, food, and synthetic textiles, are produced by multi-product firms that are potentially vertically integrated, in the sense that one of their outputs is likely to be an input for another of their production lines. While there are many firms in the Machinery, Equipment, and Parts industry that also produce product sets that may indicate vertical integration, this is less of a problem than in other industries, with just over 20% of firm-year observations belonging to firms that are potentially vertically integrated (See Table 2). ⁴²

They key variables used in this study are described in Tables 1 and 2. Each plant lists the revenues and quantities produced and sold for up to ten different products produced within the plant. Associated with each product entry is a 5-digit ASI Commodity Classification, or ASICC, code, with just over 1000

all the large census plant-year observations within the 2001-2008 ASI panel reported these joint returns. Hence, for plants to approximate firms, I implicitly assume that the firms that operate plants in multiple states decentralize the pricing decision to local managers, or that with-firm, across-plant cannibalization effects are small enough to be safely ignored. Note that across state cannibalization effects could be safely ignored if I defined the market at the state, rather than country level. However, since many plants may ship goods across state borders, I define the market at the country-year level, rather than the state-year level, to allow for price competition across states.

⁴⁰Note that by-products may be thought of as an extreme case of public inputs (i.e. all inputs into cotton are automatically allocated to cotton waste), which my approach can be extended to handle, as discussed in Section 3.3. Note, however, that this approach requires that public inputs enter all production lines symmetrically, which is problematic if a multi-product firm also produces other outputs that are not by-products, e.g. jute.

⁴¹For example, labour allocated to a downstream production line will be less costly (in terms of shadow values), since if it was instead allocated to an upstream production line, it would also be indirectly producing downstream output. This means that input allocation rules will, in general, depend on the details of the vertically integrated production process, which is beyond the scope of this paper.

⁴²I drop potentially vertically integrated plants whenever I require information on the estimated within-plant input allocations, since these are unlikely to be measured correctly.

unique item codes belonging to the Machinery, Parts, and Equipment industry. Each ASICC code is associated with a particular unit of quantity, such as kilograms, tonnes, or units sold, which allows one to use the information on revenues and quantities to construct a within product code consistent unit price, which I take to be item-level prices. While each item produced by a plant is assigned a product code, approximately 12 % of census plant-year observations report multiple entries for the same product code. These are, according to the ASI documentation, not to be regarded as duplicates, indicating that plants also report separate product lines within a 5-digit ASICC code as well. In my empirical analysis, I consider each *entry* as a separate variety of the same general product class, rather than aggregating to the 5-digit ASICC code level.

Table 1: Plant-Product-Year Summary Statistics: Machinery, Equipment, and Parts

Variable	Obs	Mean	Std. Dev.	Min	Max	Median
Log Revenue (r_{it}^j)	64109	15.96	2.59	1.39	25.14	15.97
Log Quantity Sold (q_{it}^j)	64109	7.69	4.19	-6.91	24.17	7.5
Log Prices (p_{it}^j)	64109	8.07	3.58	-3	21.27	8.05
Log Quantity Produced (y_{it}^j)	64109	7.83	4.2	-3.44	24.19	7.69
Multi-product	64109	.76	.43	0	1	1
Multi-product \times Single Industry	64109	.62	.48	0	1	1
Multi-product \times Single Industry \times No VI	64109	.43	.5	0	1	0

Notes: Multi-Product, Single Industry, and No VI are all dummy variables. Single industry refers to produce product produced by plants that only produce products belonging to ASICC codes 74-78. No VI refers to plants that do not produce output sets that I classify as potentially vertically integrated, using information on input use by single product plants. See Appendix B for more details.

For the purpose of estimation, I take the inputs of the production function to be labour, L_{it} , capital, K_{it} , and materials, M_{it} , as is standard in the literature. I then measure labour input L_{it} by the number of man-days worked, and capital, K_{it} , as net-value of fixed assets deflated by the yearly capital deflator used in Allcott et al. (2016).⁴⁵ Since I observe information on the price and quantity of various inputs at the 5-digit ASICC code, I generate a Cobb-Douglas quantity index to measure material inputs, M_{it} , that is not subject to the input price bias discussed by De Loecker and Goldberg (2014) and De Loecker et al. (2016).⁴⁶

⁴³These product codes correspond to all product codes that belong to the 2-digit ASICC categories 74-78. See Appendix B for more details and examples of the 5-digit codes.

 $^{^{44}}$ Note that some product codes do not have quantity information, although these are relatively rare, with only 15% of the item-plant level observations containing no quantity information, and just over 9% of plant-item level revenues involving no quantity information.

⁴⁵I only include worker hours in L_{it} , rather than managerial hours, in part because this measure is less likely to be a public input.

⁴⁶This index is created using information on the value and quantity of up to ten major inputs that are purchased from domestic firms, and up to five material inputs that are imported from firms abroad. I use this information to construct firm-varying materials input prices, W_{it}^k , by dividing the purchase values by observed purchase quantities. I then construct

a Cobb-Douglas price index for materials, given by $\prod_{k} \left(\frac{W_{it}^{k}}{\gamma_{it}^{k}}\right)^{\gamma_{it}^{k}}$, where γ_{it}^{k} is the fraction of observed materials input expenditure going into material k. Aggregate materials, M_{it} , is then determined by dividing total observed materials

Before I move on to discussing estimation, there are two features of the ASI data that require some minor tweaks to the baseline model discussed in Section 3. First, note that the ASI contains information on both quantity sold and quantity manufactured in each year (See Table 1). While these variables are generally quite similar, with just over 75 % of observations involving quantities sold and produced that are within 5 % of each other, there are some cases where quantity sold and quantity manufactured differ by a significant margin. Since this is due to inventory management, I augment the model described in Section 3 to account for inventories by modifying the output market clearing condition to be $Q_{it}^j = Y_{it}^j - \Delta INV_{it}^j$, where ΔINV_{it}^j is the change in inventories of product j at time t, while I take Q_{it}^j to be quantity sold and Y_{it}^j to be quantity produced. In practice, accounting for inventories in this way changes the model very little, except for slight differences in notation. In particular, inventories now become a state variable for the firm's problem, and marginal costs should be interpreted as marginal costs conditional on some desired change in inventories.⁴⁷

Table 2: Plant-Year Summary Statistics: Machinery, Equipment, and Parts

Variable	Obs	Mean	Std. Dev.	Min	Max	P50
Log labour hours (l_{it})	35475	9.19	1.54	3.26	15.38	8.95
Log capital stock (rupees) (k_{it})	35475	15.74	2.34	.69	23.82	15.54
Log materials (Cobb-Douglas aggregator) (m_{it})	35475	7.08	3.27	-3.91	22.99	6.63
Number of Varieties (J_{it})	35475	2.52	2.06	1	10	2
Multi-product	35475	.57	.5	0	1	1
$Multi-product \times No VI$	35475	.46	.5	0	1	0

Notes: Summary statistics only reported for plants which only produce products belonging to ASICC codes 74-78. Labour hours refers to mandays worked by workers, i.e. excluding managerial workers. Net Closing Value is the net closing value for all fixed assets reported in Block C of the ASI, deflated by the yearly capital deflator used in Allcott et al. (2016). Cobb-Douglas materials are constructed according to the following formula: $M_{it} = \frac{\sum_k \cot_i^k}{\sum_{it}^k \cot_i^k}$, where \cot_i^k is total expenditure on input k by firm i, W_{it}^k is the unit-value (price) of input k, $\prod_k \left(\frac{W_{it}^k}{\gamma_{it}^k}\right)_{it}^{\gamma_{it}^k}$, where \cot_i^k is total expenditure on input k by firm i, i, i is the unit-value (price) of input k,

and γ_{it}^k is the fraction of observed materials input expenditure going into material k. Multi-product and No VI are dummy variables, where Multi-product equals 1 if a plant produces multiple products, while No VI equals 1 if the plant does not produce an output sets that I classify as potentially vertically integrated. See Appendix B for more details.

The second feature of that the data that I have to account for is the fact that the ASI records

expenditures by the plant level materials input price index. Note that this formulation of the Cobb-Douglas price index implies that different plants may use Cobb-Douglas materials aggregators that have different input elasticities, γ_{it}^k . However, cost minimization by the firm implies that the observed materials cost shares will be equal to these heterogeneous input elasticities. Note that using such a Cobb-Douglas index to measure input quantities may generate issues in the comparability of TFP across firms, as plants using different types of inputs may not be in comparable units. These differences in units may then show up in one's TFP estimates, which will generate measured differences in productivity that have nothing to do with "real" productivity differences. To deal with this concern, I also re-estimate all of my main TFP regressions with materials expenditures in place of the materials quantity aggregator, to verify that my results are not being driven by materials aggregator issues. These results can be found in Appendix E.

⁴⁷Note that the conditional cost-minimization problem remains conditional on $Y_{it}^j = Q_{it}^j + \Delta I N V_{it}^j$ in this case, which means the conditional cost function for (10) becomes $C\left(\vec{K}_{it}, \vec{Q}_{it}\left(\vec{P}_{t}\right) + \Delta I N V_{it}^j, \vec{\omega}_{it}, \vec{A}_{it}\right)$, which, in practice, changes very little except for the *interpretation* of marginal costs. Note that I will treat inventory changes as exogenous, since they are not the focus of this paper. However, one could certainly augment the model to allow inventory changes to be a firm-level choice variable.

information on the amount of tax paid by each item sold. Since taxes generate an *observable* wedge between the price paid by consumers and the price received by producers, I modify the pricing model in Section 3 to account for differences in *ad-valorem* tax rates that must be paid by consumers across products.⁴⁸ This generates a minor modification to the pricing first-order conditions, which I discuss in detail in Appendix C2, where I work out the details of the marginal cost inversion for my empirical model.

Finally, since I wish to examine the effect of import competition on within-plant specialization, I supplement the ASI data with information on aggregate imports and tariff rates by 4-digit HS code from UN Comtrade, and the UNCAD TRAINS database, respectively. I then match this information to the 5-digit ASICC codes I observe in the ASI, using a series of public crosswalks, as well as information on the value of imported inputs reported in the ASI, to deal with many-to-one matches of product codes. See Appendix B3 for details.

5 Estimation

While Theorem 1 applies to a fairly general demand and production functions, to make estimation feasible for the given sample size, I specify the functional form of the demand and production function up to small number of unknown parameters, as is standard in most applied work. In the following section, I first describe the specification of the demand system, present my identification strategy for identifying the demand parameters, and then present the estimates. I then describe the specification and estimation of the production function, and present my estimated production function parameters.

5.1 Demand Estimation

I specify the aggregate demand system using a variant of the discrete choice models incorporating continuous quantity choice described in Björnerstedt and Verboven (2013). While more details are described in Appendix C, this demand system is essentially a generalization of the standard nested logit model described in McFadden et al. (1978) or Berry (1994), that allows consumers to make continuous quantity choices.⁴⁹ As in nested logit, the consumer choice problem can be represented as a sequential choice problem, with consumers first choosing a product-nest, which I will consider to be separate 5-digit ASICC codes, and then a variety within that product nest, corresponding to a particular product produced by an individual plant. Let Λ_t^g denote the set of varieties belonging to product-nest g, including imported varieties. Demand for product $j \in \Lambda_t^g \in \Omega_t^h$, where Ω_t^h is the set of varieties belonging to 3-digit

 $^{^{48}}$ I measure the *ad-valorem* tax rate as the ratio of total taxes recorded, over gross revenues minus total taxes. Note that most of the tax rates are quite small, with a median tax rate of around 4%.

⁴⁹Note that this demand model is also equivalent to a two-stage nested CES demand function, as originally shown by Verboven (1996).

ASICC code h, is then given by:

$$Q_{it}^{jg} \left(\vec{P}_t \right) = \frac{E_t^h}{P_{it}^{jg}} \frac{\exp\left(\frac{\delta_{it}^{jg}}{\sigma}\right) \left(\sum_{k \in \Lambda_t^g} \exp\left(\frac{\delta_{mt}^{kg}}{\sigma}\right) \right)^{\sigma - 1}}{\sum_{\Lambda_t^l \in \Omega_t^h} \left(\sum_{k \in \Lambda_t^l} \exp\left(\frac{\delta_{mt}^{kl}}{\sigma}\right) \right)^{\sigma}}, \tag{23}$$

where $\alpha > 0$ and $\sigma \in (0,1]$ are demand parameters, $\delta_{it}^{jg} \equiv \eta_{it}^{jg} - \alpha p_{it}^{jg}$ is the mean-utility of product j, and E_t^h is total expenditure by consumers in the market h at time t.⁵⁰ In this formulation of the demand system, I consider different 3-digit ASICC codes to be different markets, the sense that they correspond to different *choice-sets*, Ω_t^h .⁵¹

The general approach to demand estimation is based on the mean utility inversion approach outlined in Berry (1994), where a simple linear estimating equation is generated by the *inverse mapping* between market shares, and the mean-utility levels, δ_{it}^{jg} . To obtain this mapping, I need to take a stance on the *outside option*, which in this application corresponds to products within the market that are *not* within the ASI's sample frame, or product codes that are within consumers choice sets but are not fully accounted for within the given 3-digit ASICC code. Let $0 \in \Omega_t^h$ index this outside option for any market h, and normalize $\delta_t^0 \equiv 0$, since discrete choice models are only identified up to such a normalization. It is then straightforward to show that mean-utilities are revealed by the following function of *revenue-shares*:⁵²

$$\delta_{it}^{jg} = \frac{rs_{it}^{jg}}{rs_{it}^{0h}} - (1 - \sigma) \, rs_{it}^{j|g},\tag{24}$$

where $rs_{it}^{jg} \equiv \ln\left(\frac{R_{it}^{jg}}{\sum_{\Lambda_t^g \in \Omega_t^h} \sum_{l \in \Lambda_t^g} R_{mt}^{kl}}\right)$ is the log of product j's revenue share, rs_t^{0h} is the natural log of the revenue share of the outside option in market h, and $rs_{it}^{j|g} \equiv \ln\left(\frac{R_{it}^{jg}}{\sum_{k \in \Lambda_t^g} R_{mt}^{kg}}\right)$ is the natural log of the revenue share of variety $j \in \Lambda_t^g$ within 5-digit ASICC code g. Note that I consider imported goods to be part of each 5-digit ASICC nest, and hence include total imported revenue within Λ_t^g .⁵³

⁵⁰Recall that lower case letters correspond to natural logs, so $p_{it}^j \equiv \ln \left(P_{it}^j \right)$.

⁵¹See Appendix B2 for a list of 3-digit ASICC codes. While one could certainly define choice sets in alternative ways, this particular formulation was chosen so cannibalization effects within multi-product firms are appropriately estimated. In particular, note that if all products within Machine, Parts, and Equipment are considered part of the same choice set, most firms would have approximately zero market shares, which will generate zero cross-product-code elasticities within this demand system (See Appendix C). While I regard Machine, Equipment, and Parts as fairly similar objects from the production side, on the consumption side (specifically, from the point of view of downstream firms), not all items within these sets are likely to substitute towards one another. Since the 3-digit ASICC codes appear to grouped in terms of downstream uses, such as Agricultural and Forestry Equipment (761) vs Food, Beverage and Tabacco machinery (762), these provide a relatively straightforward way to group products that should substitute towards one another.

⁵²The derivation of the mean-utility inversion exactly follows the derivation for the nested logit presented in Berry (1994), with revenue shares replacing quantity shares. In particular, note that (23) can be rewritten as $RS_{it}^{jg} = \frac{1}{2} \left(\frac{1}{2} \frac$

 $[\]frac{\exp\left(\frac{jg}{it}\right)\left(\sum_{k\in\Lambda_t^g}\exp\left(\frac{\delta_{mt}^{kg}}{\sigma}\right)\right)^{\sigma-1}}{\sum_{\Lambda_t^g\in\Omega_t^h}\left(\sum_{k\in\Lambda_t^l}\exp\left(\frac{\delta_{mt}^{kl}}{\sigma}\right)\right)^{\sigma}}, \text{ where } RS_{it}^{jg}\equiv\frac{P_{it}^{jg}Q_{it}^{jg}}{E_t^h}. \text{ Note that this is identical to the expression for the nested logit}$

demand model in Berry (1994), except with revenue shares replacing quantity shares.

⁵³See Appendix B3 for details on the mapping between 5-digit ASICC codes and 4 digit HS codes used to determine total imports by ASICC code.

Equation (24) defines the inverse mapping between market shares and mean utilities which makes linear IV estimation feasible. To see this, substitute into $\delta_{it}^{jg} \equiv \eta_{it}^{jg} - \alpha p_{it}^{jg}$ into (24), yielding:

$$\frac{rs_{it}^{jg}}{rs_t^{0h}} = (1 - \sigma) r s_{it}^{j|g} - \alpha p_{it}^{jg} + \eta_{it}^{jg}.$$
 (25)

Since this estimating equation comes from a discrete choice model, the two demand parameters that I wish to estimate in (25), (α, σ) , can be interpreted as density parameters governing the distribution of idiosyncratic taste shocks of machinery purchasers in this market, as noted by Berry (1994). In particular, as I show in Appendix C, where I derive (23) from a discrete choice microfoundation, α governs the *variance* of taste shocks across all product codes and all varieties in the market, with larger values of α implying smaller variance of the idiosyncratic taste shocks. If there is less idiosyncratic taste variation, this means that horizontal product differentiation will be much less important in the aggregate, and therefore price elasticities will generally become larger for all varieties.

The parameter σ , on the other hand, governs the correlation of taste shocks within a product code. In particular, low values of σ imply that individual consumers draw a vector of idiosyncratic taste shocks that are strongly correlated within a product code. As a result, each consumer tends to, on average, prefer consuming varieties within one product code over all the others, leading to larger cross-price elasticities within product code, compared to across product codes. On the other hand, as σ approaches 1, this generates zero correlation in the taste shocks within product codes. Note that examining how a product's within-product code revenue share, $rs_{it}^{j|g}$ leads to variation in it's overall market share rs_{it}^{jg} , helps identify this consumer taste correlation parameter, with the special case of no correlation corresponding to the case where $rs_{it}^{j|g}$ drops out of (25).

Equation (25) can be estimated using linear instrumental variables methods, with unobserved quality, η_{it}^{jg} , functioning as the structural residual.⁵⁴ Note that instrumental variables are needed because plants choose their prices with knowledge of their quality levels, η_{it}^{jg} . As a result, plants that produce higher quality products will tend to charge higher prices, meaning that the OLS estimates of α are likely to be downward biased. Furthermore, the within group revenue shares, $rs_{it}^{j|g}$, are also endogenous, implying that the estimates of $1-\sigma$ obtained by OLS would be inconsistent.

To find a suitable instrument for prices in (25), one generally needs a cost-shifter, such as input prices, that are excluded from the demand function. While plant-varying input prices are observable in the ASI, the key difficulty in the differentiated product context considered in this paper is that the

 $^{^{54}}$ Note that the right hand side of (25) is only observable if potential market size, E_t^h , is known, in which case the outside option can be constructed by subtracting observed ASI revenue in each period from potential market size. Since India experienced significant growth over the sample period, it is important to allow E_t^h to grow over time. To accomplish this, I set potential market size in 2001 for each three-digit ASICC code, E_{2001}^h , to two times observed revenue in the ASI, for the year 2001, where total industry revenue is obtained by summing up all observed revenues in the ASI for the given three-digit ASICC codes, multiplied by the the firm-level multipliers for plants that are randomly sampled, plus imports. Potential market size in subsequent years is then calculated using $E_t^h = GR_tE_{t-1}^h$, where GR_t is the growth factor in total observed revenue for all plants I observed in the Machinery, Equipment, and Parts industry plus imports, from year t-1 to t. This formulation of potential market size means that 3-digit ASICC codes that experienced slower than average revenue growth will have larger shares of consumers choosing the outside option. Note that since this formula led to occasionally negative outside option shares for a small number of 3-digit ASICC codes, I instead scaled the initial levels of these codes by three times observed revenue in 2001, to guarantee that the outside option share is always positive.

structural residual, η_{it}^{jg} , embodies quality differences across products. As a result, a standard instrument that may be valid in a homogeneous goods context, i.e. plant-varying input prices, are unlikely to be valid, since differences in input prices across firms may reflect differences in the product-level quality choices, as in, for example, Verhoogen (2008), Kugler and Verhoogen (2012) and De Loecker et al. (2016). Specifically, if plants who wish to produce higher quality outputs need to purchase higher quality inputs, input quality differences across firms are likely to be reflected in these prices, thus violating the required exclusion restriction for input prices to function as valid instruments.

To address this concern, I generate an input-price instrument that only harnesses variation in input prices in *other* output markets, thereby harnessing cost-variation that should be uncorrelated with demand or quality shocks in the Machinery, Equipment, and Parts industry. To construct these instruments, let \mathbb{I}^g denote the set of 5-digit input product codes that I observe being used by *single product* producers of product code g, and let \mathbb{F}^{kg}_t denote the set of firms observed in the ASI at time t who purchase an input with product code $k \in \mathbb{I}^g$, who do not sell any outputs in the Machinery, Equipment, and Parts Industry. Letting W^k_{it} denote the current price of input-code k paid by firm i, I define the following input price instrument which varies at the product-code and year level:

$$Z_{it}^{jg} = Z_t^g = \sum_{k \in \mathbb{I}^g} \gamma^{kg} \times \ln\left(\frac{\sum_{m \in \mathbb{F}_t^{kg}} W_{mt}^k}{|\mathbb{F}_t^{kg}|}\right),\tag{26}$$

where γ^{kg} is an input weight, given by the overall cost share of input $k \in \mathbb{I}^g$ in the production of product-code g by single product firms.⁵⁵

Roughly speaking, Z_t^g simply uses average input prices in other output markets, and will function as a valid instrument as long as time-variation in the price of inputs paid by firms operating in other output markets are primarily driven by demand and supply shocks that are specific to those industries, rather than demand shocks or quality changes in the Machinery, Equipment and Parts industry. Put differently, I require that this input price variation be driven by demand and supply shocks in other industries, that, on average, are orthogonal to machinery demand shocks. Roughly speaking, this requires that average input prices not be driven by machinery industry demand, which will obviously not be satisfied in input markets where the machinery industry is the primary downstream consumer. To deal with this concern, I exclude any input codes k from $k \in \mathbb{I}^g$ if more than 30% of the revenue I observe going into purchases of k comes from Machinery, Equipment, and Parts producers.⁵⁶

Note that one limitation inherent in \mathbb{Z}_t^g is that it does not vary across plants that produce the same

⁵⁵These weights are chosen to put greater weight on the price changes of inputs that are more important for the production of product code g. Note that in practice I trim the 95th and 5th percentiles of these prices by product code to limit the influence of extreme outliers, and demean the value of this instrument by product code so as to only take advantage of across time variation in input prices for each product code, rather than variation in the levels. See also footnote 56.

⁵⁶Since I am excluding some inputs from the construction of the input price instrument, this means that the input weights, γ^{kg} , do not necessarily sum to one for each output code g, and therefore levels of the input price instrument are not necessarily informative. To deal with this, I demean the value of each instrument by product code so as to only take advantage of the across time variation in the price shocks. While one could instead rescale the instrument so that the weights summed to one for the non-excluded input codes, this strategy would artificially put greater weight on cost shocks for inputs with small cost shares, γ^{kg} , in product codes where an important input was excluded from $k \in \mathbb{I}^g$, making this instrument less likely to scale the cost shocks correctly.

product codes. While one could define an analogous instrument at the firm-level, taking advantage of the different input sets used by different plants, I do not pursue this strategy since the choice of different inputs is almost certainly correlated with product quality. Instead, I construct a second instrument that takes advantage of the variation in the *output sets* produced by different multi-product plants. In particular, this instrument is based on the average value of Z_t^g taken by *other* products produced within the same firm:

$$Z_{it}^{-jg} = \frac{\left(\sum_{k \in \mathbb{Y}_{it}} Z_{it}^{kl}\right) - Z_{it}^{jg}}{J_{it} - 1},\tag{27}$$

where $J_{it} = |\mathbb{Y}_{it}|$. Note that this instrument will be correlated with price either through withinplant cannibalization effects, or through cost-shocks that affect common inputs that are used in each production line within a plant.

Table 3: Demand Estimates

	(1)	(2)
	OLS	IV
p_{it}^{jg}	0.00682	-0.233
	(0.00210)	(0.111)
$rs_{it}^{j g}$	0.949	0.843
	(0.00354)	(0.214)
Observations	60,066	60,066

Standard errors clustered by plant and product code

Notes: Dummies for product code, year, state, census status, age, urban, organization and ownership type, and number of products sold also included in all regressions.

The results of this estimation strategy can be found in Table 3, above.⁵⁷ As expected, OLS estimation of (25) generates a price coefficient of the wrong sign, since prices tend to be positively correlated with quality. The instrumental variables strategy, on the other hand, appears to fix this bias, generating point estimates that imply an average own-price elasticity of approximately 2.4. Note that the coefficient on $rs_{it}^{j|g}$ is quite close to one, which, given the discrete choice formulation underlying this demand system, means that consumer level taste shocks are strongly correlated within a product code, and as a result, cross-product substitution will tend to primarily occur within product codes, rather than across product

⁵⁷Note that the regressions include a set of dummies for various plant-level observables, including state, urban versus rural, census status, as well as organization and ownership type, as an alternative to including plant fixed effects. This allows me to use the price variation of the randomly sampled plants in my demand estimates, which I would have to drop if I included plant fixed effects.

codes. As a result, cannibalization effects will tend to be strongest in plants producing multiple varieties of the same product code, rather than plants producing multiple product codes.

To alleviate potential weak instrument concerns, I also report the first stage estimates in Table 4. Note that the input-price instruments are strongly correlated with the endogenous variables, with the first stage F-statistics taking values of around 24 and 12. Moreover, note that the first-stage coefficients generally take the sign one would intuitively expect, with increased input prices being associated with output price increases.⁵⁸

Table 4: First Stage Estimates

	$(1) \\ p_{it}^{jg}$	$(2) \\ rs_{it}^{j g}$
Z_t^g	0.328	0.157
	(0.109)	(0.0448)
Z_{it}^{-jg}	0.353	-0.401
	(0.267)	(0.145)
Observations	60,066	60,066
AP F-Stat	23.74	11.74

Standard errors clustered by plant and product code

Notes: Dummies for product code, year, state, census status, age, urban, organization and ownership type, and number of products sold also included in all regressions.

5.2 Production Function Estimation

As is standard in much of the productivity literature, I assume that all firms in the industry use a Cobb-Douglas production technology, $Y_{it}^j = \exp\left(\omega_{it}^{jg}\right) \left(L_{it}^{jg}\right)^{\beta_L} \left(K_{it}^{jg}\right)^{\beta_K} \left(M_{it}^{jg}\right)^{\beta_M}$. To obtain an estimate of the within-firm input allocations, I apply Theorem 1, using the estimated demand parameters $(\widehat{\alpha}, \widehat{\sigma})$ to determine the appropriate mapping between observables and estimated input allocations, $(\widehat{L}_{it}^{jg}, \widehat{K}_{it}^{jg}, \widehat{M}_{it}^{jg})$. Having solved for the input allocations, production function estimation can proceed

⁵⁸The fact that the input price instrument is *positively* correlated with the within-product code revenue shares is perhaps surprising, since one would expect input price increases to decrease market shares. Note, however, that since this instrument does not vary within a product code, this is likely due decreased output (or exit) of competitors, which will mechanically increase $rs_{it}^{j|g}$ within a product code. On the other hand, Z_{it}^{-jg} being negatively associated with $rs_{it}^{j|g}$ makes sense, in so far as cost shocks that affect common inputs within a plant lead to price increases, and therefore loses in market shares.

⁵⁹Note that since I measure output in *quantity* units, ω_{it}^{jg} will measure TFPQ, rather than TFPR.

⁶⁰See Appendix C2 for more details on the appropriate mapping given the assumed demand system (23). Note that since I have only estimated the production function for the Machinery, Equipment, and Parts industry here, I drop

using standard production function estimation tools, as outlined below. To generate a simple estimating equation to guide this discussion, take logs of the production function after substituting in the estimated input allocations, yielding:

$$y_{it}^{jg} = \beta_L \hat{l}_{it}^{jg} + \beta_K \hat{k}_{it}^{jg} + \beta_M \hat{m}_{it}^{jg} + \omega_{it}^{jg}. \tag{28}$$

As discussed earlier, a key identification problem inherent in estimating models like (28) is the transmission bias problem that has been highlighted by much of literature on production function estimation.⁶¹ Since the firm chooses input quantities with knowledge of their own productivity, each of the inputs is likely to be correlated with the structural residual, ω_{it}^{jg} , in (28). As a result, one cannot simply estimate (28) by OLS.

A popular approach to circumventing the transmission bias problem uses the proxy-variable techniques developed in Olley and Pakes (1996), Levinsohn and Petrin (2003) and Ackerberg et al. (2015), which have fruitfully been applied to answer a number of different questions related to productivity.⁶² The key insight exploited in these papers is that input demands (capital investment in Olley and Pakes (1996), materials in Levinsohn and Petrin (2003)), directly depend on productivity when firms are profit maximizing. As a result, we can use information on input demands to proxy for TFP(Q), thereby controlling for the endogeneity problem created by unobservable productivity shocks.⁶³

While a proxy-variable approach could be used to estimate (28) in principle, there are a number of complications introduced by multi-product firms that make these estimators intractable in this context. First, as discussed by Ackerberg et al. (2015), the Olley and Pakes (1996) and Levinsohn and Petrin (2003) approach will only identify the labour input elasticity if there is optimization error in labour input demand, or under variable timing assumptions where labour and materials are chosen with different information sets. These extra assumptions would invalidate the assumption underlying Thereom 1, and thus are not particularly useful in this context.

On the other hand, while the proxy-variable estimator proposed by Ackerberg et al. (2015) could still be used, note that the semi-parametric approach they propose to estimate TFP(Q) is unlikely to be tractable for multi-product firms. In particular, a key assumption maintained in Ackerberg et al. (2015) is that there exists a one-to-one mapping between current TFPQ levels and firm-level state variables. This mapping, called the control function, is estimated non parametrically in a first stage regression of outputs on inputs.⁶⁴ Note, however, that the state-space for multi-product firms is the entire vector of product-level TFPQ terms, ($\vec{\omega}_{it}$). While in principle the general approach can be augmented to account for multiple unobservables, see Ackerberg et al. (2007), this approach will quickly run into dimensionality problem in my context, since the vector of unobserved quality terms, $\vec{\eta}_{it}$, should also be

multi-industry plants in the subsequent regressions, as I do not have the necessary information to determine their input allocations.

⁶¹See Griliches and Mairesse (1995) and Ackerberg et al. (2007).

⁶²See, for example, Pavcnik (2002), De Loecker (2007), and Topalova and Khandelwal (2011).

⁶³Note that TFP here should be regarded as TFPQ, since I y_{it}^j is in quantity units.

⁶⁴Technically, the control function will not actually be identified until the second-stage is estimated, since one cannot separate production function terms from control function terms in their first stage regression. Nevertheless, the key idea is that *some* unknown function of inputs and firm-level state variables must be accurately estimated in the first stage.

state variables, as well as the particular *set* of product codes produced by the firm.⁶⁵ Even if one could guarantee that a well-defined control function exists, given the large state space, it is unlikely that one could approximate it accurately in a first-stage regression.

To avoid the issues associated with adapting the semi-parametric control function to a multi-product firm context, I instead consider a dynamic-panel style estimator, as in Anderson and Hsiao (1982), Arellano and Bond (1991), Blundell and Bond (1998), Blundell and Bond (2000) and Bond and Söderbom (2005), for the purposes of production function estimation. This approach does not require the use of a control function, and thus will not run in to the dimensionality problems discussed above. Moreover, since this does not require the existence of a control function, it places less restrictions on the econometrician's information set, since there no longer needs to be a one-to-one mapping between observables and productivity. For example, there can be firm-varying cost shocks to to all inputs that are unobserved by the econometrician, which proxy-variable methods generally cannot allow.

To obtain the estimating equation for the production function, I assume that productivity follows an AR(1) process, i.e.:

$$\omega_{it}^{jg} = \rho_0^g + \rho \omega_{i,t-1}^{jg} + \xi_{it}^{jg}, \tag{29}$$

where ρ_0^g is the mean TFP of varieties belonging to product-code g.

Substituting this law of motion for productivity into (28), yields $y_{it}^{jg} = \rho_0^g + \beta_L \hat{l}_{it}^{jg} + \beta_K \hat{k}_{it}^{jg} + \beta_M \hat{m}_{it}^{jg} + \rho \omega_{i,t-1}^{jg} + \xi_{it}^j$. Once can then quasi or " ρ -difference" this equation, by subtracting the lagged value of (28) multiplied by ρ from this expression, yielding:

$$y_{it}^{jg} = \rho_0^g + \rho y_{i,t-1}^{jg} + \beta_L \left(\hat{l}_{it}^{jg} - \rho \hat{l}_{i,t-1}^{j} \right) + \beta_K \left(\hat{k}_{it}^{jg} - \rho \hat{k}_{i,t-1}^{jg} \right) + \beta_M \left(\hat{m}_{it}^{jg} - \rho \hat{m}_{i,t-1}^{jg} \right) + \xi_{it}^{jg}, \quad (30)$$

where the innovation to productivity, ξ_{it}^{j} , becomes the structural residual. Since the above model is nonlinear in parameters, due to the interactions between ρ and the production function parameters, (30) is estimated using a nonlinear GMM estimation procedure.

Two related points are worth emphasizing concerning estimating equation (30). First, the structure of this estimating equation is very similar to that used in Ackerberg et al. (2015), with y_{it}^j essentially taking the place of their first stage predicted values.⁶⁶ This relationship between dynamic panel estimators and their estimator is also noted by Ackerberg et al. (2015), with the key tradeoff between the two approaches being that the dynamic panel approach relies more heavily on the linearity of the productivity process, while proxy-variable approaches generally allows productivity to follow a non-linear Markov process.⁶⁷ Second, and perhaps more importantly, the key difference between the specification of the production function (28) that I use to generate (30), compared to the approach in Ackerberg

⁶⁵The set of product codes being a state variable leads to most significant dimensionality problem. Roughly speaking, this means one must estimate a different control function for *every* combination of product codes observed in the data.

⁶⁶Or $\widehat{\Phi}_{it}^{jg}$, adapting their notation to my context.

⁶⁷Strictly speaking, one can allow for a non-linear Markov process using a dynamic panel approach, as long as one is willing to abstract from measurement error in output. Note, however, that the linear AR(1) structure that I use allows me to account for differences in average TFPQ across product codes in a more parsimonious manner; i.e. by demeaning within product codes, or by including a series of product code fixed effects.

et al. (2015), is that there are no productivity shocks that are unobserved by the firm. This is a more restrictive assumption on a firm's information set, relative to Ackerberg et al. (2015), which the reader should bear in mind. Note that while one can modify the above estimating equation to incorporate these extra productivity shocks, as Ackerberg et al. (2015) do when comparing the dynamic panel method to their approach, this would be problematic in a multi-product firm context because shocks that are unobserved by the firm would break the link between observed outputs, and the input allocations firm chose with knowledge of their TFPQ levels, which was used to derived the optimal allocation rule (9).⁶⁸ Hence, while acknowledging that this approach does require that I make some stronger assumptions on the firm's information set, relative to proxy-variable approaches, note that these extra restriction are only likely to be of quantitative importance if the unobserved component of productivity is relatively large compared to the component known by firms.

I now turn to the nonlinear instrumental variables strategy I use to estimate (30). As is standard in much of the production function literature, I assume that materials is a *static* input, while capital is a *dynamic and predetermined* input. In particular, I assume one-period time to build in capital, as in Olley and Pakes (1996), Levinsohn and Petrin (2003) and Ackerberg et al. (2015), so that firm-level capital follows the simple law of motion $K_{i,t+1} = l^K(K_{it}, I_{it}^K)$. This law of motion implies that K_{it} cannot react to any news that arrives during period t. Since ξ_{it}^{jg} , the "innovation" to productivity, functions as the structural residual in (30), both k_{it} and $k_{i,t-1}$ can be used as instruments to identify β_K , since they will both be uncorrelated with ξ_{it}^{jg} , as long as one accounts for the product-code specific intercepts, ρ_0^g , with a series of product-code fixed effects.⁶⁹

Similar to Ackerberg et al. (2015), I also assume that labour is also a dynamic input, in the sense that it faces dynamic adjustment costs, such as hiring or firing costs due due labour market regulation and union activity, which are fairly substantial in many Indian states, see Besley and Burgess (2004). This allows me to use lagged labour as an instrument to identify β_L , since lagged labour inputs $l_{i,t-1}$, will tend to be correlated with the quasi-differences in labour, $\hat{l}_{it}^{jg} - \rho \hat{l}_{i,t-1}^{jg}$.

While it is fairly standard to also use the lagged value of static inputs in both proxy-variable and dynamic panel approaches to production function estimation, note that only using this type of variation to identify β_M is problematic. In particular, it is not entirely obvious why the lagged value of a static input should be correlated with differences in static input usage, except through productivity or demand changes, as has recently been pointed out by Gandhi et al. (2016). While the fact that lagged inputs

⁶⁸It is easiest to think of this problem in the case where these unobserved productivity shocks occur *after* production. In this case, firm's choose expected output, rather than realized output. As a result, the link between their chosen level of output, and observable output which I use in (9), would be broken. While a "first-stage" estimating equation might allow the researcher to identify the difference between expected productivity and realized productivity, these first-stage estimating equations would run into the dimensionality problems discussed earlier, and multi-product firms would not be able to be included in the estimation procedure, as the existence of productivity shocks would make the input allocation rule no longer hold exactly.

⁶⁹Note, however, that since I assume that capital is perfectly transferable across uses within the firm during each period, the allocation of capital input across production lines, k_{it}^{jg} , will generally be correlated with the current productivity shocks, since they are chosen in the current period. Hence k_{it} can be used as an instrument, while k_{it}^{jg} cannot. Moreover, as I mention in the main text, it is preferable to use firm-level inputs as instruments, rather than the estimated input allocations, since this will deal with attenuation bias due to the measurement error generated by the fact that the input allocations are *estimated*.

may be weak instruments has been widely acknowledged in the dynamic-panel literature for a while, see Blundell and Bond (2000), note that this problem is most likely to be pronounced with static inputs.

To deal with this concern, I also use the current and lagged values of the input price instruments described in Section 5.1, Z_t^g and Z_{t-1}^g for any $j \in \Lambda_t^g$, to identify β_M . As long as these instruments are valid for demand, meaning that variation in the input-price instrument is driven by demand and supply shocks in *other* output markets, which affect the input prices of goods used to produce product code g, then they will function as valid instruments for estimating (30) as well.

Altogether, I use the following set of moments to identify the parameters of the production function:

$$\mathbb{E} \begin{bmatrix} \begin{cases} y_{i,t-1}^{jg} \\ l_{i,t-1} \\ k_{it} \\ k_{it} \\ m_{i,t-1} \\ Z_t^g \\ Z_{t-1}^{g} \\ \tilde{C}_{it}^{jg} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{for } j \in \Lambda_t^g, \tag{31}$$

where \vec{C}_{it}^{jg} is a vector of 5-digit product code fixed effects, as well as number of product fixed effects, which are included to account for the product-code specific intercepts, ρ_0^g , as well as the economy of scope effects discussed in Section 3.3. Note that I prefer to use firm-level input useage as instruments, x_{it} , rather than the estimated input useage by product line, \hat{x}_{it}^{jg} for $x \in (l, k, m)$, since using the estimated shares as instruments should generate finite sample attenuation bias, due to the fact that these shares are estimated from a first-stage regression, and thus, measured with error.

Estimation is done using the standard nonlinear IV estimator, and standard errors are obtained using the block bootstrap, where I re-sample with replacement over plant identifiers, and re-estimate the demand and production function parameters using the described two-step estimation procedure.⁷⁰ The OLS and GMM results are presented below, in Table 5.

The estimated production function parameters are fairly reasonable, with materials having the largest input elasticity, as is commonly the case with gross-output production function. Interestingly, these estimates allow me to reject constant returns to scale at the 5 % level. While constant returns to scale often cannot be rejected in many studies based on *revenue* production functions, actual returns to scale will tend to be biased downwards when one uses a revenue production function, as prices have to *fall* when a firm increases their output.⁷¹ Hence, the machinery industry appears to be characterized by significant scale economies, which one would indeed expect in large scale manufacturing.

⁷⁰For computational simplicity, I first de-mean the data within each 5-digit product code, so I do not need to actually estimate the entire vector of product-fixed effects. Since (30) is linear conditional of $(\beta_L, \beta_K, \beta_M)$, I concentrate out the remaining parameters when minimizing the criterion function, so I only need to solve for $(\beta_L, \beta_K, \beta_M)$ non-linearly.

⁷¹See Klette and Griliches (1996).

Table 5: Production Function Estimates

	OLS	GMM
eta_L	0.527	0.374
	(0.0607)	(0.151)
eta_K	0.245	0.0819
	(0.0390)	(0.0739)
eta_M	0.263	0.780
	(0.0227)	(0.128)
$\beta_L + \beta_K + \beta_M$	1.04	1.24
	(0.0327)	(0.0643)
Observations	3545	3545

Block bootstrapped standard errors in parentheses

6 Results

Having estimated the demand and production parameters for the Machinery, Equipment, and Parts industry, I use this information to construct plant-product specific estimates of log TFPQ, defined as the estimated residual from (28), as well as plant-product specific quality, defined as residual from (25). I then use these estimates to quantify the extent to which within-plant reallocations matter for explaining within-plant productivity upgrading in response to trade liberalization.

6.1 Product-Level Heterogeneity Correlations

I begin by presenting some simple correlations between the various sources of product-level heterogeneity. Table 6, below, shows the correlation between estimated quality, TFPQ, log prices, and markups, after being demeaned within their respective product code and year so I will not be making apples to oranges comparisons, due to differences in quantity units across product codes. Many of these correlations have the expected sign, with prices being negatively correlated with TFPQ but positively correlated with quality, and markups being positively correlated with quality.

One potentially surprising finding in Table 6 is that quality and TFPQ are negatively correlated. Note that this is a fairly strong negative correlation, and holds both across single-product firms, as well as across products within multi-product firms, as one can see from Figure 1, where I report a number of scatter plots between TFPQ and quality at the product level. Note that this sort of correlation appears to contradict the intuition, considered in Kugler and Verhoogen (2012) as well as Garcia-Marin and Voigtländer (2017), that TFPQ and quality are complements, in the sense that high TFPQ firms will wish to produce high quality products. Rather, it appears that high TFPQ products tend to be lower quality. A similar negative correlation between cost-side advantages and demand side advantages has

also been found by Forlani et al. (2016), and Jaumandreu and Yin (2016), who consider the separate estimation of TFPQ and quality, primarily in a single-product firm context. These correlations may imply that these different sources of product advantages are simply substitutes, in the sense that high quality producers do not have an incentive to cut costs, since overall demand for high quality products is high enough to cover high costs. On the other hand, this correlation could also be due to technological constraints, where high-quality goods simply require more inputs (e.g., worker hours), and hence, will tend to have lower measured TFPQ.

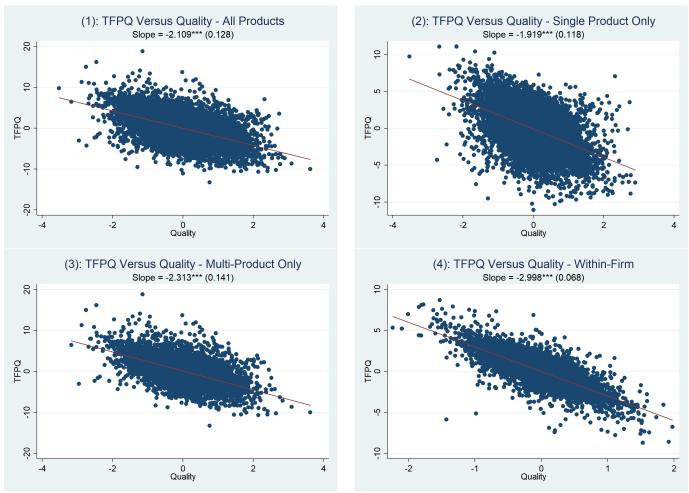


Figure 1: Negative Correlation between TFPQ and Quality

Notes: Standard errors of slopes, adjusted for two-way clustering by plant and product code, reported in parentheses. Quality refers to the year-product code demeaned residual from (25), while TFPQ refers to the year-product code demeaned residual from (28). Panel (1) includes all products, Panel (2) only includes single-product plants, and Panels (3) and (4) only includes products produced by multi-product plants. Panel (4) plots within-firm demeaned values of TFPQ and quality.

While determining the exact reason for this negative correlation is beyond the scope of this paper, note that negatively correlated quality and TFPQ has important implications for how we think about firm-heterogeneity and specialization at the plant-level. In particular, there are, roughly speaking, two classes of products within this market: *cheap-goods*, which are low quality but easy to produce (i.e. high TFPQ), and *high-quality goods*, which generate more demand for a given price, but generally are costly to produce and therefore have relatively low TFPQ levels. In the following section, I consider in

a bit more detail how to think about plant-level heterogeneity when there are two negatively correlated sources of variety-level heterogeneity.

Variables	Quality	TFPQ	$\ln\left(P_{it}^{jg}\right)$	Markup
Quality (η_{it}^{jg})	1.000			
TFPQ (ω_{it}^{jg})	-0.532	1.000		
Price $\left(\ln\left(P_{it}^{jg}\right)\right)$	0.874	-0.596	1.000	
Markups $\left(\frac{P_{it}^{jg}}{MC_{it}^{jg}}\right)$	0.258	-0.034	0.087	1.000

Notes: All variables demeaned within their respective product code and year. Quality refers to the estimated residual from (25), and TFPQ refers to the estimated residual from (28). Markups defined as the ratio of price to marginal costs, where marginal costs are estimated estimated using the marginal cost inversion described in Appendix C2.

6.2 Measuring Plant-level Productivity

While I have obtained estimates of plant-product level productivity in terms of quantity-productivity, or TFPQ, measuring plant-level productivity using this object has two major shortcomings. First, as documented in the previous section, TFPQ and quality are negatively correlated, which means that differences in TFPQ differences will partly measure quality differences, rather than pure efficiency differences. Moreover, since different product codes are measured in different units, there are serious difficulties with aggregating plant-product level TFPQ into a coherent notion of plant-level productivity.

To deal with these concerns, I follow Klette and Griliches (1996) and De Loecker (2011), and use the structure of demand and production function to generate a revenue production function, which relates plant-product level revenues to plant-product level input use. As described in further detail in Appendix E, given the demand function (23) and production function (28), then log revenues at the product level are related to input use as follows:

$$\ln(R_{it}^{jg}) = \frac{\frac{\alpha}{\sigma}}{1 + \frac{\alpha}{\sigma}} \left(\beta_l l_{it}^{jg} + \beta_k k_{it}^{jg} + \beta_m m_{it}^{jg} \right) + \underbrace{\widetilde{\omega}_{it}^{jg} + \widetilde{\eta}_{it}^{jg}}_{\equiv h_{it}^{jg}}, \tag{32}$$

where:

$$\widetilde{\eta}_{it}^{jg} \equiv \frac{\eta_{it}^{j}}{\alpha + \sigma} + \frac{1}{1 + \frac{\alpha}{\sigma}} \ln\left(E_{t}^{h}\right) \\
- \frac{(1 - \sigma)}{1 + \frac{\alpha}{\sigma}} \ln\left(\sum_{k \in \Lambda_{t}^{g}} \exp\left(\frac{\delta_{mt}^{kg}}{\sigma}\right)\right) \\
- \frac{1}{1 + \frac{\alpha}{\sigma}} \ln\left(\sum_{l \in \Omega_{t}^{h}} \left(\sum_{k \in \Lambda_{t}^{l}} \exp\left(\frac{\delta_{mt}^{kl}}{\sigma}\right)\right)^{\sigma}\right),$$
(33)

$$\widetilde{\omega}_{it}^{jg} \equiv \frac{\frac{\alpha}{\sigma}}{1 + \frac{\alpha}{\sigma}} \omega_{it}^{jg},\tag{34}$$

Note that the production function coefficients in (32) are each scaled down by $\frac{\alpha}{1+\alpha}$, since increases in input use will require that prices fall for quantity produced to still equal quantity demanded, resulting in smaller returns to scale than one would find when examining the quantity production function (28). More importantly, the residual from (32), h_{it}^{jg} which I call (log) revenue productivity, or TFPR, solves many of the issues related to using TFPQ when measuring plant-level productivity. In particular, the residual is now measured in units of revenues over inputs, and therefore is directly comparable across firms and products. Moreover, variation in TFPR incorporates both TFPQ and quality variation, and corresponds more directly to the notion of productivity measured in other studies of trade liberalization on productivity, as these studies generally do not have access quantity data.

Of course, one cost of working with TFPR, rather than TFPQ, is that this residual incorporates more than just TFPQ and quality differences- in particular, this object will also capture product-code level variation in overall demand, prices, as well as product space congestion effects, i.e. variety levels will tend to fall as the number of competitors in the market rise.⁷⁴ Note, however, that these extra effects do not vary within product-code, and can thus, for certain purposes, be purged from the model by demeaning TFPR (or $\widetilde{\omega}_{it}^{jg}$ and $\widetilde{\eta}_{it}^{jg}$) within product code. In particular, note that by construction:

$$h_{it}^{jg} = h_t^g + \widetilde{\omega}_{it}^{j|g} + \widetilde{\eta}_{it}^{j|g}, \tag{35}$$

where $h_t^g \equiv \frac{1}{|\Lambda_t^g|} \sum_{k \in \Lambda_t^g} h_{it}^{kg}$ is average TFPR by 5-digit product code at time period, and $x_{it}^{j|g} = x_{it}^{jg} - x_t^g$ for $x \in \{\widetilde{\omega}_{it}^{jg}, \widetilde{\eta}_{it}^{jg}\}$ is the variety specific deviation from that average. Note that since $\widetilde{\omega}_{it}^{j|g}$ and $\widetilde{\eta}_{it}^{j|g}$ are demeaned within product code, differences in demand levels, prices, and product space congestion effects, and well as quantity units, have been differenced out, and thus, can be directly compared across products. Moreover, since $\widetilde{\omega}_{it}^{j|g}$ and $\widetilde{\eta}_{it}^{j|g}$ are both mean zero within each market, we can interpret these variables as measuring a variety's relative position within TFPQ (cost) or quality (demand) space, respectively. As a result, I will refer to $\widetilde{\omega}_{it}^{j|g}$ as a variety's cost comparative advantage and $\widetilde{\eta}_{it}^{j|g}$ as a

⁷²Note that my notation of TFPR is slightly different from that use by other authors. For example, Foster et al. (2008) and Hsieh and Klenow (2009) generally refer to TFPR as residual from the *quantity* production function where output quantities have been replace by (industry-deflated) revenues. As a result, plant-specific output prices will mechanically show up in their notion of TFPR. Note, however, that the *revenue* production function takes into account the fact that quantities vary with price, and therefore the impact of firm-specific prices on revenues is captured by *input-use*, rather than through the residual. As a result, the residual of the revenue production function captures TFPQ and demand shifters, rather than TFPQ and price.

⁷³Note that it is crucial that all firms being compared used the same quantity production function (up to TFPQ shifters, as in Assumption 2), since otherwise one would also need to make adjustments to account for the different input indexes used by different firms.

⁷⁴In particular, note that $\frac{(1-\sigma)}{\alpha+\sigma}\ln\left(\sum_{k\in\Lambda_t^g}\exp\left(\frac{\delta_{it}^{kg}}{\sigma}\right)\right)$ and $\frac{1}{\alpha+\sigma}\ln\left(\sum_{l\in\Omega_t^h}\left(\sum_{k\in\Lambda_t^l}\exp\left(\frac{\delta_{it}^{kl}}{\sigma}\right)\right)^{\sigma}\right)$, will increase whenever there is entry of new competitors, leading to an overall decrease in $\widetilde{\eta}_{it}^{jg}$ due to product space congestion effects. On the other hand, product-code level increases in prices tend to decrease $\frac{(1-\sigma)}{\alpha+\sigma}\ln\left(\sum_{k\in\Lambda_t^g}\exp\left(\frac{\delta_{it}^{kg}}{\sigma}\right)\right)$ and $\frac{1}{\alpha+\sigma}\ln\left(\sum_{l\in\Omega_t^h}\left(\sum_{k\in\Lambda_t^l}\exp\left(\frac{\delta_{it}^{kl}}{\sigma}\right)\right)^{\sigma}\right)$, since increases in prices decrease each δ_{it}^{jg} terms, and therefore tend to increase $\widetilde{\eta}_{it}^{jg}$.

variety's demand comparative advantage, with $h_{it}^{j|g} \equiv \widetilde{\omega}_{it}^{j|g} + \widetilde{\eta}_{it}^{j|g}$ measuring a variety's overall comparative advantage in revenue space. The remaining component of variety specific TFPR, h_t^g , the product code specific average, then captures variation in overall demand for a product code, average difficulty of producing a particular product code, as well as differences in the level of competition across products.⁷⁵

To determine how much of the variation in variety specific TFPR is driven by these three terms, I follow Hottman et al. (2016), and sequentially regress each element on the right-hand side of (35) on variety level TFPR, h_{it}^{jg} . By properties of OLS, the coefficient on each of these regressions will sum to one, and can be interpreted as as a measure of how much of the variance in overall variety-level TFPR is due to variation in each of the individual components of (35). The results of these regressions can be found in Table 7, below.

Table 7: Variety-level TFPR Variance Decompositions

	h_t^g	$\widetilde{\omega}_{it}^{j g}$	$\widetilde{\eta}_{it}^{j g}$
h_{it}^{jg}	0.326	0.281	0.393
	(0.006)	(0.008)	(0.008)
Observations	30,816	30,816	30,816

Notes: The above reports OLS regressions of h_t^g , $\widetilde{\omega}_{it}^{j|g}$, and $\widetilde{\eta}_{it}^{j|g}$, on $h_{it}^{jg} \equiv \widetilde{\omega}_{it}^{jg} + \widetilde{\eta}_{it}^{jg}$, as defined in the main text. Standard errors, which take the demand and production function estimates as data, and are clustered by plant, reported in parentheses.

It is reassuring to note that overall TFPR comparative advantage, given by the sum of the effects of demand and cost comparative advantage, account for just under 70% of the TFPR variation, implying that most of the variation in TFPR is governed by TFPQ and quality, rather than product space congestion effects and changes in averages prices, which are partly embodied in the component of TFPR explained by the product specific averages. Interestingly, demand comparative advantage accounts for the largest portion of the variation in TFPR, accounting for just under 40 % of the variance. On the other hand, the product specific averages and cost comparative advantages account for approximately 30 % of the variation each.

Given that h_{it}^{jg} is primarily capturing TFPQ and quality variation, variety-level TFPR provides an ideal metric for product-level performance. I use this measure to then define an index of plant-level log TFPR, defined as the sum of plant-product level TFPR, weighted by plant-product level input shares S_{it}^{jg} .76

⁷⁵Note that one can further decompose h_t^g into TFPQ and demand specific averages, although these are difficult to compare on their own, due to the fact that they also capture differences in quantity codes across products.

⁷⁶Note that while this is a seemingly arbitrary notation of plant-level log TFPR, I show in Appendix I that this definition of plant-level log TFPR is strongly correlated with the standard notion of plant-level TFPR, i.e. $\ln \left(\sum_{j \in \mathbb{Y}_{it}} P_{it}^{jg} Q_{it}^{jg} \right) - \ln \left(\frac{\frac{\alpha}{\sigma}}{1+\frac{\alpha}{\sigma}} F(X_{it}) \right)$. In fact, these objects will deliver identical estimates of mean TFPR variables, although will slightly different forms for term measuring plant-level specialization. I use (36) to define plant-level productivity in the main text,

$$h_{it} \equiv \sum_{j \in \mathbb{Y}_{it}} S_{it}^{jg} h_{it}^{jg}. \tag{36}$$

Following Olley and Pakes (1996), I can then decompose overall plant-level TFPR into average and allocative efficiency components:⁷⁷

$$h_{it} = \frac{1}{J_{it}} \sum_{j \in \mathbb{Y}_{it}} h_{it}^{jg} + \sum_{j \in \mathbb{Y}_{it}} \left(h_{it}^{jg} - \frac{1}{J_{it}} \sum_{j \in \mathbb{Y}_{it}} h_{it}^{jg} \right) \left(S_{it}^{jg} - \frac{1}{J_{it}} \sum_{j \in \mathbb{Y}_{it}} S_{it}^{jg} \right)$$

$$= \underbrace{\bar{h}_{it}}_{\text{Average TFPR}} + \underbrace{OP_{it} \left(\vec{S}_{it}, \vec{h}_{it} \right)}_{\text{Specialization}}.$$
(37)

The second term in (37), sometimes referred to as the "OP" covariance after Olley and Pakes (1996), increases as a plant allocates a greater portion of its input towards high TFPR varieties, and therefore functions as a measure of allocative efficiency within a plant.⁷⁸ Now, however, that since (36) is additive h_{it}^{jg} , I can use the simple TFPR decomposition in (35) to further decompose plant-level log TFPR as follows:

$$h_{it} = \underbrace{\overline{h}_{it}^{g}}_{(1): \text{ Average}} + \underbrace{\overline{\psi_{it}}^{j|g}}_{(2): \text{ Average Cost}} + \underbrace{\overline{\eta_{it}}^{j|g}}_{(3): \text{ Average Demand}}$$

$$\underbrace{OP_{it}\left(\vec{S}_{it}, \vec{h}_{it}^{g}\right)}_{\text{Comparative Advantage}} + \underbrace{OP_{it}\left(\vec{S}_{it}, \widetilde{\omega}_{it}^{j|g}\right)}_{(5): \text{ Specialization in Cost}} + \underbrace{OP_{it}\left(\vec{S}_{it}, \widetilde{\eta}_{it}^{j|g}\right)}_{\text{Comparative Advantage}} + \underbrace{OP_{it}\left(\vec{S}_{it}, \widetilde{\eta}_{it}^{j|g}\right)}_{\text{Comparative Advantage}}$$

$$(38)$$

The decomposition in (38) shows that there are six key margins that can affect a plant's TFPR. First, the average value of TFPR the product codes produced will vary across plants, due to the different sets of product codes produced by different manufacturing units. This variation in TFPR will be driven by differences in the average difficulty of producing different products, average demand levels, as well as differences in the level of competition across product codes. The second and third margins capture more standard notions of firm heterogeneity, i.e. differences in comparative advantage in cost and demand space, respectively. Plants with large values of these two terms will, on average, produce varieties that are relatively more efficient in terms of TFPQ, or are relatively higher quality. The following three margins capture the various ways in which a plant specialize their inputs across production lines across production lines to increase overall TFPR. In particular, they can allocate more inputs towards high TFPR product codes, which is captured by the fourth term, or by allocating more inputs towards varieties for which they have a cost or demand comparative advantage, as is captured by the fifth and

as this formulation of of productivity allows to further decompose plant-level productivity in specialization into demand and cost comparative advantage specialization terms. However, Appendix I shows that I obtain similar results when using $\ln\left(\sum_{j\in\mathbb{Y}_{it}}P_{it}^{jg}Q_{it}^{jg}\right)-\ln\left(\frac{\frac{\sigma}{\sigma}}{1+\frac{\alpha}{\sigma}}F(X_{it})\right)$ to define plant-level productivity.

⁷⁷Derivation exactly follows the derivation provided in Olley and Pakes (1996).

⁷⁸See, for example, Bartelsman et al. (2013) and Melitz and Polanec (2015), who use this terminology in the context of across plant reallocations.

sixth terms.

Table 8: Plant-Level TFPR Variance Decompositions

	(1)	(2)	(3)	(4)	(5)	(6)
	\overline{h}_{it}^g	$\overline{\omega_{it}}^{j g}$	$\overline{\eta_{it}}^{j g}$	$OP_{it}\left(\vec{S}_{it}, \vec{h_{it}}^{g}\right)$	$OP_{it}\left(\vec{S}_{it}, \widetilde{\omega}_{it}^{\vec{j} g}\right)$	$OP_{it}\left(\vec{S}_{it}, \widetilde{\eta}_{it}^{\vec{j} g}\right)$
A: All Plants						
h_{it}	0.319	0.287	0.394	0.005	-0.019	0.014
	(0.006)	(0.007)	(0.007)	(0.001)	(0.002)	(0.002)
B: Multi-product Plants						
h_{it}	0.348	0.309	0.357	0.019	-0.081	0.049
	(0.010)	(0.013)	(0.013)	(0.007)	(0.009)	(0.009)
				(→ →)	(- 71-)	(a = 3.)

Notes: The above reports OLS regressions of \overline{h}_{it}^g , $\overline{\omega_{it}}^{j|g}$, and $\overline{\eta_{it}}^{j|g}$, $OP_{it}\left(\vec{S}_{it}, h_{it}^{\vec{g}}\right)$, $OP_{it}\left(\vec{S}_{it}, \widetilde{\omega}_{it}^{j|g}\right)$, and $OP_{it}\left(\vec{S}_{it}, \widetilde{\eta}_{it}^{j|g}\right)$,

In Table 8, I run the same variance decomposition as Table 7, applied to (38). Panel A considers all plants, while Panel B only considers multi-product plants, since single product plants always have values of zero for the specialization terms. Overall, I find the effect of average product codes, cost, and demand comparative advantage explain similar proportions of the variance of plant-level TFPR, as was the case when I examine the decomposition for variety-level TFPR, particularly when I examine the entire sample. Moreover, most of the variation is driven by the average value of variety-level heterogeneity, rather specialization. Once I restrict attention to multi-product product plants, so that the specialization terms can have an effect for all observations, I still find that the mean effects explain most of the variation, although I do find that quality specialization alone can explain around 5% of the variation in revenues. On the other hand, since quality and TFPQ are negatively correlated, it appears that plants that choose to specialize in quality, also specialize away from high TFPQ products- as a result, the specialization in cost comparative advantage term ends up being negatively correlated with TFPR. These results imply that most of the variation in plant-level TFPR are explained by average values of TFPR, TFPQ, and quality, rather than specialization per se. In the next section, I examine whether these patterns also hold when examining productivity upgrading in response to increased import competition.

6.3 Decomposing Within-Plant Productivity Improvements due to Chinese Import Competition

A sizable literature has documented how trade liberalization often leads to within-firm productivity improvements, including Pavcnik (2002), Amiti and Konings (2007), Fernandes (2007), De Loecker (2011), Topalova and Khandelwal (2011), and Bloom et al. (2016). While many papers frame these improvements as being driven by increased innovation in response to increased import competition,

on $h_{it}^{jg} \equiv \tilde{\omega}_{it}^{jg} + \tilde{\eta}_{it}^{jg}$, as defined in the main text. Panel A has 19,261 plant-year observations. Panel B has plant-year 3,852 observations. Standard errors, which take the demand and production function estimates as data, and are clustered by plant, reported in parentheses.

access to new markets, or lower costs of intermediate inputs, Eckel and Neary (2010), Bernard et al. (2011), Mayer et al. (2014) and Mayer et al. (2016) have shown that within-firm productivity gains from trade can operate through a reallocation margin. Specifically, multi-product firms that face greater import competition should be more specialized, either by dropping low quality or low efficiency varieties, or by focusing more of their resources towards their highest performing products. In this section, I use my estimates of within and across plant heterogeneity to quantify the relative contribution of within-plant reallocations in response to an import competition shock, compared to the standard margin of improving the average productivity of all varieties.

For this purpose, I take advantage of the emergence of China during the 2000s as a major trading partner, as a potentially exogenous source of variation in import competition experience by Indian plants. As discussed in Autor et al. (2013), the emergence of China as a major player in international markets following their ascension to the WTO in 2001 was largely driven by efficiency improvements within China, implying that a significant portion of Chinese export growth is plausibly exogenous to industry performance in India. More importantly, as I show in Figure 2, the increase in Chinese import competition experienced within the Machinery, Equipment, and Parts industry in India was extremely large. While India imported just over 5% of their machinery, equipment, and parts, from China in the early 2000s, by the the end of my sample period close to 25% of all imports in this industry come from China. Moreover, Chinese imports increased by approximately a factor of 30 in this industry, which is roughly three times the increase in Chinese imports experienced in other Indian industries.⁷⁹

To examine whether this sizeable increase in Chinese import competition led to increased plant-level efficiency, I begin by estimating the following model:

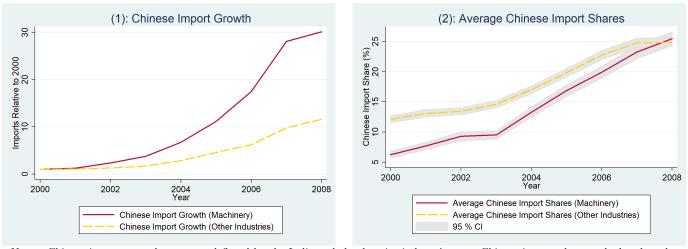
$$\Delta h_{it} = \beta \sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\text{IM}_{\text{IND,CHN},t}^m \right) + \gamma X_{it} + \epsilon_{it}, \tag{39}$$

Where $IM_{IND,CHN,t}^m$ is imports from China to India by 3-digit ASICC code m, $\alpha_{i,t-1}^m$ is the revenue share of product codes belonging to 3-digit ASICC code m at time t-1 in plant i, X_{it} is a vector of various controls, such as year dummies, dummies for the current and lagged number of products, state, plant age, census status, rural versus urban, type of organization and ownership structure, lagged TFPR levels, and input and output tariffs, and Δ is the first difference operator.⁸⁰ Note that when I first-difference imports, I hold the set of products produced by the plant fixed, so as to not have the effect contaminated by firms endogenously switching to other product codes in response to the change in Chinese imports.

⁷⁹While these rates of import growth are enormous, one should bear in mind that both China and India experienced massive GDP growth over this time period, with India's GDP more than doubling and China's increasing by a factor of 4, while more than half of Chinese export growth in the 2000s was driven by machinery exports (Berger and Martin (2013)).

⁸⁰Total imports at the three digit ASICC code level are obtained by first matching 5-digit ASICC codes to HS6 codes, as described in Appendix B3, and then aggregating to the 3 digit level. Note that I measure Chinese imports at the 3-digit level, rather than the 5-digit, largely to accommodate my instrumental variable strategy- see Appendix G for a discussion of the results at the 5 digit level, as well as discussion of some of the difficulties of this approach.

Figure 2: Chinese Imports in India: Machinery, Equipment, and Parts vs. Other Industries



Notes: Chinese import growth rates are deflated by the Indian wholesale price index. Average Chinese import shares calculated as the simple average of import shares over each 5-digit ASICC codes. See Data Appendix B3 for details on the construction of imports by product-code.

Bloom et al. (2016) have previously documented that increases in Chinese import competition generally raised innovative activity of European firms, both through increased patenting, as well as increased measured TFPR. As a result, the expected sign of β in (39) is positive. Note, of course, that simply estimating (39) by OLS is likely to result in severely downward biased estimates of the effect of Chinese imports on TFPR, as Chinese firms are most likely to increase their exports in product codes where productivity is growing the *least*, due to comparative advantage forces. In particular, under the null that Chinese import competition has *no* effect on productivity, one would generally expect to find a negative coefficient on β , as consumers in India will generally prefer to import goods from more efficient countries.

To deal with this source of bias, I estimate (39) by 2SLS, instrumenting Chinese imports to India with total imports from China in other low to middle income countries *besides* India.⁸¹ More formally, for each plant I construct the instrument:

$$Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i:t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t}^m \right) \right), \tag{40}$$

Where $\mathbb{C}^{\text{No India}}$ denotes the set of low to middle income countries excluding India. Note that Z_{it}^{CHN} will function as a valid instrument as long as Chinese import growth in other countries is not driven by factors determining productivity growth in India.

Table 9 presents the results of estimating (39) by OLS, while Table 10 shows the IV results, with Table 11 showing the corresponding first stage regressions. Each table presents four sets specifications-

⁸¹I only include low to middle income countries in this set as Chinese trade to high income countries is likely to depend on very different factors than those determining trade between other low and middle income countries, which would tend to decrease instrument strength. This is analogous to the strategy in Autor et al. (2013), who when constructing a similar instrument for Chinese import exposure in the United States, only include Chinese exports to other high income countries. The list of WTO low and middle income countries can be found at http://wits.worldbank.org/referencedata.html.

first, the baseline regressions which only control for year fixed effects, as well as dummies for the currrent and lagged number of products, state, plant age, type of organization, and ownership structure of each plant. I then control for lagged TFPR levels in column (2), to allow for persistence of TFPR shocks over time. In Column 3, I control for differences in input tariffs across time, since previous studies (e.g. Amiti and Konings 2007, Topolova and Khandelwahl 2011), have found input market liberalization to be a significant source of TFP gains.⁸² Finally, in column 4 I control for differences in the average tariff rate for all Indian trading partners, to allow for import competition effects from other countries.⁸³

Table 9: Effect of Chinese Import Competition on TFPR: OLS

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	TFPR	TFPR	TFPR	TFPR
Change in Chinese Imports	0.253	0.322	0.301	0.298
	(0.0807)	(0.0811)	(0.0843)	(0.0849)
Lagged TFPR level		-0.298	-0.298	-0.298
		(0.0164)	(0.0164)	(0.0164)
Change in World Input Tariffs			-0.0313	-0.0332
			(0.0258)	(0.0269)
Change in World Output Tariffs				0.00834
				(0.0158)
Observations	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports OLS regressions Δh_{it} on the described variables. All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in Chinese Imports measured as $\sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \ln \left(\mathrm{IM}^m_{\mathrm{IND,CHN},t} \right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \tau^m_t$, where τ^m_t is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Across the OLS specifications, I find that Chinese import competition has a fairly robust, and fairly precisely estimated, positive effect on plant-level TFPR. Given the negative bias of OLS due to endogenous imports, this provides fairly convincing evidence that Chinese imports are having *some* positive effect on plant-level TFPR. Turning to the IV results, I find that correcting for the endogeneity of Chinese imports results in much larger effects of Chinese import competition on plant-level productivity,

⁸²Input tariffs are constructed using the input-output table described in Appendix B. In particular, for each 5-digit output ASICC code, I calculate the average value of the tariffs applied to the inputs that are used to produce that 5-digit product code. I then take the average value of the input tariffs by 3-digit ASICC code. The changes in plant-level input tariffs are then calculated as the lagged revenue-share weighted difference in 3-digit ASICC tariffs. As with my measure of plant-level Chinese import exposure, I hold the set of products produced by a plant at their lagged value when taking first differences.

⁸³As with my input tariff controls, plant-level output tariffs are the first difference between lagged-revenue share weighted average of output tariffs applied to each 3-digit ASICC code produced by the plant. I hold output sets fixed at their lagged value.

Table 10: Effect of Chinese Import Competition on TFPR: IV

	(1) Change in	(2) Change in	(3) Change in	(4) Change in
VARIABLES	TFPR	TFPR	TFPR	TFPR
Change in Chinese Imports	1.271 (0.989)	1.460 (0.981)	1.333 (0.734)	1.314 (0.705)
Lagged TFPR level	(====)	-0.305	-0.304	-0.304
Change in World Input Tariffs		(0.0172)	(0.0169) 0.0243 (0.0500)	(0.0169) 0.0249 (0.0509)
Change in World Output Tariffs			(0.0000)	-0.00795
				(0.0196)
Observations	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports IV regressions of Δh_{it} on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t}^m \right) \right)$ instrumenting the change in Chinese imports. All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in Chinese imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\text{IM}_{\text{IND,CHN},t}^m \right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \tau_t^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistics are 16.80, 16.74, 27.73, and 30.57, for columns (1) through (4), respectively. Standard errors treat demand and production function estimates as data.

although these are less precisely estimated, as is generally the case with IV estimators. The first stage estimates of the exports from China to other countries appears to have the right sign, and is strongly correlated within Chinese imports for India, generating first stage F-stats of around 16 to 30. The point estimate implies that that 1 % increase in Chinese import leads to plant-level increases in TFPR of around 1.3 %, a fairly large increase. Note that cutting world import tariffs also appears to have a positive effect on plant-level TFPR, as has been found in other studies, while I am not able to obtain precise enough estimates of the effect of inputs tariffs on TFPR to conclude whether they have an effect or not.

Note, however, that the results in Tables 9 and 10 are themselves not all that novel, in that many other studies have documented how import competition tends to increase plant-level productivity. However, since I have previously estimated within-plant productivity heterogeneity, I can now break down these effects into the portion of TFPR growth due to mean TFPR changes and reallocation effects. In particular, I make use of the following decomposition of the growth rate of plant-level TFPR

$$\underbrace{\Delta h_{it}}_{\text{TFPR Growth}} = \underbrace{\Delta \overline{h}_{it}^{jg}}_{\text{Average}} + \underbrace{\Delta OP_{it} \left(\vec{S}_{it}, \vec{h}_{it}^{g} \right)}_{\text{Specialization in High}} + \underbrace{\Delta OP_{it} \left(\vec{S}_{it}, \widetilde{\omega}_{it}^{j|g} \right)}_{\text{Specialization in Cost}} + \underbrace{\Delta OP_{it} \left(\vec{S}_{it}, \widetilde{\omega}_{it}^{j|g} \right)}_{\text{Specialization in Cost}} + \underbrace{\Delta OP_{it} \left(\vec{S}_{it}, \widetilde{\eta}_{it}^{j|g} \right)}_{\text{Specialization in Demand Comparative Advantage}}, \tag{41}$$

Where (41) is simply the first difference of (38), although I have combined all of the mean TFPR effects into a single term, largely because these terms are very hard to interpret on their own in first

differences. In particular, the mean comparative advantage terms can be difficult to interpret, since across time variation in these variable may be entirely driven by differences in the composition of average products within group across time periods.⁸⁴

Table 11: First Stage

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	Chinese Imports	Chinese Imports	Chinese Imports	Chinese Imports
Chinese Import Instrument	0.163 (0.0399)	0.163 (0.0398)	0.212 (0.0402)	0.220 (0.0399)
Observations	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports OLS regressions of the Change in Chinese Imports, $\sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \ln \left(\mathrm{IM}^m_{\mathrm{IND,CHN},t} \right)$, on the Chinese Import Instrument, $Z^{\mathrm{CHN}}_{it} = \sum_{m \in \Lambda^m_{i,t-1}} \left(\alpha^m_{i,t-1} \Delta \ln \left(\sum_{k \in \mathbb{C}^{\mathrm{No\ India}}} \mathrm{IM}^m_{k,\mathrm{CHN},t} \right) \right)$. All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, and ownership structure. Column (2) also controls for lagged h_{it} levels, while column (3) adds changes in world input tariffs, and column (4) adds controls for world output tariffs, respectively. World input and output tariffs measured as $\sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \tau^m_t$, where τ^m_t is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

To determine which element of (41) is primarily responsible for the trade induced productivity gains, I estimate (39) by OLS and IV separately for each element of the right-hand side of (41). Since (41) is an identity, by properties of OLS and IV, these coefficients will sum to the coefficient obtained by using Δh_{it} as the dependent variable. For this purpose, I use the specification (4) from Tables 9 and 10 which contain the most controls, while instead including lags for each element of the right hand side of (41), rather than only the lag of h_{it} .⁸⁵ The results of the OLS and IV regressions can be found in Tables 12 and 13, below.

The most striking result in all Tables 12 and 13 is that essentially all of the TFPR increase is due to mean TFPR effects- In fact, within plant reallocations in response to Chinese import competition appear to, on net, have approximately no effect on TFPR, often generating a negative point estimate overall. However, while mean TFPR increases appear to explain most of the increase in plant-level productivity, note that there is also a sizable reallocation within multiproduct plants towards their demand comparative advantage products.⁸⁶ In particular, a one percent increase in Chinese imports appears to lead plants to reallocate inputs towards their highest quality good in a manner that increases

⁸⁴These decompositions, however, which are consistent with mean quality upgrading, can be found in Appendix F, as well as a more detailed discussion concerning difficulties in interpretation.

⁸⁵Other specifications can be found in Appendix F, as well as the coefficients on the individual lags, which are suppressed in the main text in the interest of space.

⁸⁶Note that this pattern also holds under different definitions of import competition (equal, or cost share, weights), as well as when potentially vertically integrated plants are included in the sample. See Appendix F- in particular, these regressions also show that essentially all of TFPR upgrading effect is due to mean TFPR improvements, rather than reallocations, and also provide evidence for quality reallocations in reponse to increases in Chinese import competition. However, it is worth noting that in most of these cases, the precision of the estimates suffer, although the general magnitude of the estimates is not all that different.

plant level TFPR by just under 0.4 percent. However, these reallocations tend to not be (revenue) productivity improving overall since high quality goods tend to have low TFPQ, likely in part because quality is difficult to produce.

Table 12: Decomposition of Effect of Chinese Import Competition on TFPR: OLS

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	0.306	0.308	0.0117	-0.0186	0.00524
	(0.0852)	(0.0855)	(0.0107)	(0.0164)	(0.0178)
Change in World Input Tariffs	-0.0324	-0.0321	-0.00270	-0.000143	0.00257
	(0.0269)	(0.0273)	(0.00361)	(0.00509)	(0.00464)
Change in World Output Tariffs	0.00794	0.00912	-0.000818	0.00629	-0.00665
	(0.0158)	(0.0159)	(0.00166)	(0.00276)	(0.00300)
Observations	5,236	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports OLS regressions of the various elements of (41) on the described variables. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\bar{h}_{i,t-1}^{jg}$ $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \bar{h}_{i,t-1}^g\right)$, $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \widetilde{\omega}_{i,t-1}^{j|g}\right)$, and $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \widetilde{\eta}_{i,t-1}^{j|g}\right)$. Coefficients on the lags can be found in Appendix F. Change in Chinese Imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln\left(\mathrm{IM}_{\mathrm{IND,CHN},t}^m\right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \tau_{i,t-1}^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Table 13: Decomposition of Effect of Chinese Import Competition on TFPR: IV

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	1.346	1.366	-0.153	-0.242	0.375
	(0.710)	(0.714)	(0.0941)	(0.135)	(0.182)
Change in World Input Tariffs	0.0271	0.0284	-0.0121	-0.0129	0.0237
	(0.0512)	(0.0516)	(0.00635)	(0.00932)	(0.0112)
Change in World Output Tariffs	-0.00881	-0.00792	0.00183	0.00989	-0.0126
	(0.0197)	(0.0199)	(0.00244)	(0.00384)	(0.00485)
Observations	5,236	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports IV regressions of the various elements of (41) on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \operatorname{IM}_{k,\text{CHN},t}^m\right)\right)$ instrumenting the change in Chinese imports. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\overline{h}_{i,t-1}^{jg}$ $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \vec{h}_{i,t-1}^g\right)$, $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \widetilde{\omega}_{i,t-1}^{j|g}\right)$, and $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \widetilde{\eta}_{i,t-1}^{j|g}\right)$. Coefficients on the lags can be found in Appendix F. Change in Chinese imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\operatorname{IM}_{\text{IND,CHN},t}^m\right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\prod_{i,t-1}^m \Delta \tau_{i,t-1}^m\right)$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistic is 30.41. Standard errors treat demand and production function estimates as data.

One key takeaway from these results is that within-plant responses to Chinese import competition in this industry are not so much productivity increasing, as has been emphasized by much of theoretical literature such as Bernard et al. (2011), and Mayer et al. (2014), Mayer et al. (2016), but rather involve reallocations towards a plant's relatively highest quality goods. As a result, within-plant reallocations in response Chinese import competition appear to primarily affect a plant's location in quality/cost space. This can likely be explained by Chinese imports being relatively lower quality, cheap goods, and thus plants are able to partly shield themselves from this type of competition by relying more on their higher quality goods. On the other hand, the standard productivity improving effect of import competition appears to primarily be driven by mean TFPR improvements.

7 Conclusion

In this this paper, I have developed a new approach to estimating product-level TFP for multiproduct firms in data sets where the allocation of inputs across production lines is unknown. The approach, based on a standard differentiated goods model of price competition, only requires information on firm-product level prices and quantities to be implemented. Since this information is becoming more widely available in many firm and plant level datasets, the techniques developed here should be able to help researchers interested in firm level productivity tackle a wider class of problems.

I then applied the approach to a panel of manufacturing plants in India from 2000-2007, finding new evidence for the within-firm allocative gains from trade, emphasized by Eckel and Neary (2010), Bernard et al. (2011), Mayer et al. (2014) and Mayer et al. (2016). In particular, I found that increased Chinese import competition generated increased specialization in quality, where firms reallocated a greater portion or their inputs towards relatively higher quality goods, but away from their relatively high TFPQ goods. These findings provide new evidence for the quality upgrading effects of import competition, previously explored by Amiti and Khandelwal (2013), although through a new margin: the within-plant reallocation of inputs across high and low quality production line. Since these reallocations towards quality are costly, in the sense that TFPQ is relatively lower for these varieties, overall these reallocations tended to decrease plant level TFPQ. As a result, while these reallocations did not generate sizeable productivity gains, they generally implied large gross changes in the composition of a plant's output towards higher quality goods. On the other hand, within-plant improvements in their mean levels of TFPR appear to be responsible to pretty much all of the previously documented productivity improvements in response to increase trade competition.

The findings of this paper point towards a number of interesting paths for future research. For example, the negative correlation between quality and TFPQ uncovered in this paper, which is also documented in Forlani et al. (2016) and Jaumandreu and Yin (2016), is a relatively surprising finding worthy of further analysis. Whether this reflects an explicit choice by the plants, i.e. explicitly aiming to cut the costs of products without widespread consumer appeal, or a technological constraint, such as high quality goods simply requiring more inputs, has implications for the patterns of specialization I observe at the plant level. In particular, if the pattern reflects an explicit choice by plants, it would be

useful to determine if there are ways to change firms incentives to engage in both quality upgrading and productivity upgrading. These sorts of policies could then augment the quality-focusing gains from trade uncovered in this paper, through subsequent efficiency gains. On the other hand, if this relationship is entirely technological in nature, within-firm allocative gains, as well as the across-firm allocative gains uncovered in other papers, may be limited by the quality-cost tradeoff uncovered in this paper.

Similarly, while the negative correlation between TFPQ and quality was uncovered using a very simple demand system, it would be useful to examine if one can uncover this relationship with more flexible demand structures, that allow different types of consumers to value quality and low-prices differently (e.g. low versus high income consumers). While the demand system considered in this paper cannot allow for these sort of effects, the observed movement towards specialization in quality, could have something to do with producers trying to capture different subsets of the market, and may have implications for income inequality that are worthy of further analysis.

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8 Appendix

8.1 Appendix A: Proof of Lemma 1

Consider the following conditional cost-minimization problem (CM) for firms possessing an arbitrary production technology $Y_{it}^j = \exp(\omega_{it}^j) F\left(\vec{X}_{it}^j\right)$:

$$\underset{\mathbb{X}_{it}}{\text{Min}} \qquad \sum_{M \in \mathbb{M}} \sum_{j \in \mathbb{Y}_{it}} \mathbf{W}^{M} \left(M_{it}, A_{it}^{M} \right) M_{it}^{j}$$
subject to $\exp(\omega_{it}^{j}) F\left(\vec{X}_{it}^{j} \right) \geq Y_{it}^{j}, \ \forall j \in \mathbb{Y}_{it}$

$$\sum_{j \in \mathbb{Y}_{it}} X_{it}^{j} = X_{it}, \ \forall X \in \mathbb{K},$$
(CM)

where $M_{it} = \sum_{j \in \mathbb{Y}_{it}} M_{it}^j$.

Any solution to (CM), will consist of an aggregate input vector for static inputs \vec{M}_{it} , as well as an input-share allocation for each input $j \in \mathbb{Y}_{it}$, \vec{S}_{it}^j , where S_{it}^j is a $1 \times P$ vector of input shares going into production line j. Let \vec{S}_{it} denote the matrix of all input shares, with typical element (j, X) of \vec{S}_{it} being given by S_{it}^{jX} . Note that using this notation, $\vec{X}_{it}^j = \vec{S}_{it}^j \circ \vec{X}_{it}$, where \circ denotes element-by-element multiplication of two vectors.

To prove Lemma 1, it is easier to break down the above problem into two parts. First consider the following transformation function problem (TF):

where **1** is a $1 \times P$ vector of ones, $r \in \mathbb{Y}_{it}$ is an arbitrary good within firm i's output set, $\mathbb{Y}_{it}^{-r} \equiv \mathbb{Y}_{it} \setminus \{r\}$, and \vec{S}_{it}^{-r} is the matrix of input shares obtained after excluding the row corresponding to output r.

The objective function of (TF) evaluated at an optimal solution yields the transformation function for output r, $Y_{it}^r = \tilde{T}(\vec{Y}_{it}^{-r}, \vec{X}_{it}, \vec{\omega}_{it})$, which corresponds to the maximal level of output firm i can obtain, given a vector of aggregate inputs \vec{X}_{it} and some predetermined level of other outputs that firm i wishes to produce, \vec{Y}_{it}^{-r} .⁸⁷

Rather than directly solving (CM) to obtain the conditional cost function, we may instead obtain the cost function by first solving (TF), and the following problem:

$$\underset{\vec{M}_{it}}{\text{Min}} \qquad \sum_{M \in \mathbb{M}} \mathbf{W}^{M} \left(M_{it}, A_{it}^{M} \right) M_{it}$$
subject to $Y_{it}^{r} = \widetilde{T}(\vec{Y}_{it}^{-r}, \vec{X}_{it}, \vec{\omega}_{it}).$
(CM2)

⁸⁷Note that the transformation function essentially describes the set of efficient aggregate-input and output vectors that firm i may produce with the TFP vector $\vec{\omega}_{it}$: see Diewert (1973).

With this in mind, the input shares implied by (TF) will solve (CM) as long as the level of static inputs in (TF) is chosen to solve (CM2). I use this property to prove Lemma 1:

Lemma 1. If Assumptions 1 through 6 hold, then there exists a solution to the firm's conditional cost minimization problem satisfying $X_{it}^j = S_{it}^j X_{it} \quad \forall X \in (\mathbb{K}, \mathbb{M}), \text{ where } S_{it}^j \in [0, 1] \text{ and } \sum_{j \in \mathbb{Y}_{it}} S_{it}^j = 1.$

Proof. Consider the Lagrangian associated with (TF)

$$L = \exp(\omega_{it}^r) F\left(\left(\mathbf{1} - \sum_{j \in \mathbb{Y}_{it}^r} \vec{S}_{it}^j\right) \circ \vec{X}_{it}\right) + \sum_{j \in \mathbb{Y}_{it}^r} \mu_{it}^j \left(\exp(\omega_{it}^j) F\left(\vec{S}_{it}^j \circ \vec{X}_{it}\right) - Y_{it}^j\right). \tag{42}$$

Suppose that the vector of static inputs, \vec{M}_{it} , is chosen to solve (CM2), which guarantees that at an optimum $\mathbf{1} - \sum_{j \in \mathbb{Y}_{it}^r} \vec{S}_{it}^j > \mathbf{0}$. The set of first-order necessary conditions for a solution to (TF) where all production function constraints bind $(\mu_{it}^j > 0 \ \forall j \in \mathbb{Y}_{it}^r)$ are then given by:

$$FOC_{it}^{jX}\left(\vec{S}_{it}, \vec{\mu_{it}}\right) \equiv \exp(\omega_{it}^{r}) \frac{\partial F(\vec{X}_{it}^{r})}{\partial X} X_{it} - \mu_{it}^{j} \exp(\omega_{it}^{j}) \frac{\partial F(\vec{X}_{it}^{j})}{\partial X} X_{it} = 0, \tag{43}$$

$$P^{j}\left(\vec{S}_{it}, \vec{\mu_{it}}\right) \equiv Y_{it}^{j} - \exp(\omega_{it}^{j}) F\left(\vec{X}_{it}^{j}\right) = 0, \tag{44}$$

where I am making the substitution $\vec{X}_{it}^j = \vec{S}_{it}^j \circ \vec{X}_{it}$. Note that the set of first-order conditions are satisfied if $FOC_{it}^{jX}\left(\vec{S}_{it}, \vec{\mu_{it}}\right) = 0 \ \forall j \in \mathbb{Y}_{it}$ and $\forall X \in (\mathbb{K}, \mathbb{M})$, and $P^j\left(\vec{S}_{it}, \vec{\mu_{it}}\right) = 0 \ \forall j \in \mathbb{Y}_{it}$.

Consider the following allocation \vec{S}_{it}^* and set of Lagrangian multipliers, $\vec{\mu}_{it}^*$:

$$S_{it}^{jX*} = S_{it}^{j*} = \frac{\left(\mu_{it}^{j*} \exp(\omega_{it}^{j})\right)^{\frac{1}{1-\phi}}}{\left(\exp(\omega_{it}^{r})\right)^{\frac{1}{1-\phi}} + \sum_{k \in \mathbb{Y}_{t}^{r}} \left(\mu_{it}^{k*} \exp(\omega_{it}^{k})\right)^{\frac{1}{1-\phi}}} \quad \forall j \in \mathbb{Y}_{it}^{r}, \forall X \in (\mathbb{K}, \mathbb{M}),$$
(45)

$$\mu_{it}^{j*} = \exp(\omega_{it}^r - \omega_{it}^j) \left(\frac{\left(\frac{Y_{it}^j}{\exp(\omega_{it}^j)}\right)^{\frac{1}{\phi}}}{\left(\left(F(\vec{X}_{it})\right)^{\frac{1}{\phi}} - \sum_{k \in \mathbb{Y}_{it}^r} \left(\frac{Y_{it}^k}{\exp(\omega_{it}^k)}\right)^{\frac{1}{\phi}}\right)} \right)^{1-\phi} \quad \forall j \in \mathbb{Y}_{it}^r.$$

$$(46)$$

I now show that $FOC_{it}^{jX}\left(\vec{S}_{it}^*, \vec{\mu}_{it}^*\right) = P^j\left(\vec{S}_{it}^*, \vec{\mu}_{it}^*\right) = 0 \ \forall j \in \mathbb{Y}_{it}^r \text{ and } \forall X \in (\mathbb{K}, \mathbb{M}).$

Since F is homogenous of degree ϕ , $\frac{\partial F}{\partial X}$ is homogenous of degree $\phi - 1$. Moreover, note that (45) implies $S_{it}^{jX*} = S_{it}^{j*} \quad \forall X \in (\mathbb{K}, \mathbb{M})$, which implies $\vec{X}_{it}^j = S_{it}^{j*} \vec{X}_{it} \; \forall j \in \mathbb{Y}_{it}$. Substituting these expressions into (43) yields:

$$FOC_{it}^{jX}\left(\vec{S}_{it}^{*}, \vec{\mu}_{it}^{*}\right) = \exp(\omega_{it}^{r}) \left(S_{it}^{r*}\right)^{\phi-1} \frac{\partial F(\vec{X}_{it})}{\partial X} X_{it} - \mu_{it}^{j*} \exp(\omega_{it}^{j}) \left(S_{it}^{j*}\right)^{\phi-1} \frac{\partial F(\vec{X}_{it})}{\partial X} X_{it}, \tag{47}$$

where S_{it}^{j*} has been taken out of $\frac{\partial F}{\partial X}$ since $\frac{\partial F}{\partial X}$ is homogeneous of degree $\phi - 1$. We can then write:

⁸⁸In particular, note that (45) implies
$$S_{it}^{rX*} = S_{it}^{r*} = \frac{(\exp(\omega_{it}^r))^{\frac{1}{1-\phi}}}{\left(\exp(\omega_{it}^r)\right)^{\frac{1}{1-\phi}} + \sum_{k \in \mathbb{Y}_{it}^r} \left(\mu_{it}^{k*} \exp(\omega_{it}^k)\right)^{\frac{1}{1-\phi}}} \quad \forall X \in (\mathbb{K}, \mathbb{M}).$$

$$FOC_{it}^{jX}\left(\vec{S}_{it}^{*}, \vec{\mu}_{it}^{*}\right) = \frac{\partial F(\vec{X}_{it})}{\partial X} X_{it} \left(\exp(\omega_{it}^{r}) \left(S_{it}^{r*}\right)^{\phi-1} - \mu_{it}^{j*} \exp(\omega_{it}^{j}) \left(S_{it}^{j*}\right)^{\phi-1}\right). \tag{48}$$

Note that (45) implies that $s_{it}^{rX*} = s_{it}^{r*} = \frac{\left(\exp(\omega_{it}^r)\right)^{\frac{1}{1-\phi}}}{\left(\exp(\omega_{it}^r)\right)^{\frac{1}{1-\phi}} + \sum_{k \in \mathbb{Y}_{it}^r} \left(\mu_{it}^{k*} \exp(\omega_{it}^k)\right)^{\frac{1}{1-\phi}}} \quad \forall X \in (\mathbb{K}, \mathbb{M}).$ Substituting this expression and (45) into (48) yields, after cancelling and collecting like terms:

$$FOC_{it}^{jX}\left(\vec{S}_{it}^{*}, \vec{\mu}_{it}^{*}\right) = \frac{\partial F(\vec{X}_{it})}{\partial X} X_{it} \left((\exp(\omega_{it}^{r}))^{\frac{1}{1-\phi}} + \sum_{k \in \mathbb{Y}_{it}^{r}} \left(\mu_{it}^{k*} \exp(\omega_{it}^{k}) \right)^{\frac{1}{1-\phi}} \right)^{1-\phi} (1-1) = 0.$$

Hence, $FOC_{it}^{jX}\left(\vec{S}_{it}^{*}, \vec{\mu}_{it}^{*}\right) = 0 \ \forall j \in \mathbb{Y}_{it}^{r} \text{ and } \forall X \in (\mathbb{K}, \mathbb{M}).$

Next I show the same holds for all $P^j\left(\vec{S}_{it}^*, \vec{\mu}_{it}^*\right) = 0$. Substitute $\vec{X}_{it}^j = S_{it}^{j*} \vec{X}_{it}$ into (44), and use the fact that F is homogeneous of degree ϕ , yielding:

$$P^{j}\left(\vec{S}_{it}, \vec{\mu}_{it}\right) = Y_{it}^{j} - \exp(\omega_{it}^{j}) \left(S_{it}^{j*}\right)^{\phi} F\left(\vec{X}_{it}\right). \tag{49}$$

To obtain an expression for S_{it}^{j*} in terms of exogenous parameters, multiply (46) by $\exp(\omega_{it}^j)$, and then take this equation to the power of $\frac{1}{1-\phi}$, yielding:

$$\left(\mu_{it}^{j*} \exp(\omega_{it}^{j})\right)^{\frac{1}{1-\phi}} = \left(\exp(\omega_{it}^{r})\right)^{\frac{1}{1-\phi}} \frac{\left(\frac{Y_{it}^{j}}{\exp(\omega_{it}^{j})}\right)^{\frac{1}{\phi}}}{\left(\left(F(\vec{X}_{it})\right)^{\frac{1}{\phi}} - \sum_{k \in \mathbb{Y}_{it}^{r}} \left(\frac{Y_{it}^{k}}{\exp(\omega_{it}^{k})}\right)^{\frac{1}{\phi}}\right)}.$$
 (50)

Sum (50) over all $k \in \mathbb{Y}_{it}^r$, yielding:

$$\sum_{k \in \mathbb{Y}_{it}^r} \left(\mu_{it}^{k*} \exp(\omega_{it}^k) \right)^{\frac{1}{1-\phi}} = \left(\exp(\omega_{it}^r) \right)^{\frac{1}{1-\phi}} \frac{\sum_{k \in \mathbb{Y}_{it}^r} \left(\frac{Y_{it}^k}{\exp(\omega_{it}^j)} \right)^{\frac{1}{\phi}}}{\left(\left(F(\vec{X}_{it}) \right)^{\frac{1}{\phi}} - \sum_{k \in \mathbb{Y}_{it}^r} \left(\frac{Y_{it}^k}{\exp(\omega_{it}^k)} \right)^{\frac{1}{\phi}} \right)}.$$
 (51)

Adding $(\exp(\omega_{it}^r))^{\frac{1}{1-\phi}}$ to both sides of (51) yields:

$$(\exp(\omega_{it}^r))^{\frac{1}{1-\phi}} + \sum_{k \in \mathbb{Y}_{it}^r} \left(\mu_{it}^{k*} \exp(\omega_{it}^k) \right)^{\frac{1}{1-\phi}} = (\exp(\omega_{it}^r))^{\frac{1}{1-\phi}} \frac{\left(F(\vec{X}_{it}) \right)^{\frac{1}{\phi}}}{\left(\left(F(\vec{X}_{it}) \right)^{\frac{1}{\phi}} - \sum_{k \in \mathbb{Y}_{it}^r} \left(\frac{Y_{it}^k}{\exp(\omega_{it}^k)} \right)^{\frac{1}{\phi}} \right)}.$$
 (52)

Dividing (50) by (52) yields:

$$\frac{\left(\mu_{it}^{j*} \exp(\omega_{it}^{j})\right)^{\frac{1}{1-\phi}}}{(\exp(\omega_{it}^{r}))^{\frac{1}{1-\phi}} + \sum_{k \in \mathbb{V}^{r}} \left(\mu_{it}^{k*} \exp(\omega_{it}^{k})\right)^{\frac{1}{1-\phi}}} = \left(\frac{Y_{it}^{j}}{\exp(\omega_{it}^{j})F(\vec{X}_{it})}\right)^{\frac{1}{\phi}}.$$
 (53)

Since
$$\frac{\left(\mu_{it}^{j*}\exp(\omega_{it}^{j})\right)^{\frac{1}{1-\phi}}}{\left(\exp(\omega_{it}^{r})\right)^{\frac{1}{1-\phi}} + \sum_{k \in \mathbb{Y}_{it}^{r}} \left(\mu_{it}^{k*}\exp(\omega_{it}^{k})\right)^{\frac{1}{1-\phi}}} = S_{it}^{j*}, \text{ we can substitute (53) into (49), yielding:}$$

$$P^{j}\left(\vec{S}_{it}, \vec{\mu}_{it}\right) = Y_{it}^{j} - \exp(\omega_{it}^{j}) \left(\left(\frac{Y_{it}^{j}}{\exp(\omega_{it}^{j})F(\vec{X}_{it})}\right)^{\frac{1}{\phi}} \right)^{\phi} F\left(\vec{X}_{it}\right) = Y_{it}^{j} - Y_{it}^{j} = 0.$$

Hence, $FOC_{it}^{jX}\left(\vec{S}_{it}^*, \vec{\mu}_{it}^*\right) = P^k\left(\vec{S}_{it}^*, \vec{\mu}_{it}^*\right) = 0 \ \forall j \in \mathbb{Y}_{it}^r \text{ and } \forall X \in (\mathbb{K}, \mathbb{M}), \text{ which means } \vec{S}_{it}^* \text{ and } \vec{\mu}_{it}^* \text{ satisfy the first-order necessary conditions for (TF). Moreover, since } F \text{ is quasi-concave, the first-order necessary conditions are sufficient. Therefore } \vec{S}_{it}^* \text{ and } \vec{\mu}_{it}^* \text{ are a solution to (TF). The proposition follows by noting } \vec{S}_{it}^* \text{ satisfies } S_{it}^{jX} = S_{it}^j \ \forall X \in (\mathbb{K}, \mathbb{M}) \text{ by construction, and the shares implied by (TF) will also solve (CM) when evaluated at the level of static inputs implied by (CM2).}$

8.2 Appendix B: Data Appendix

8.2.1 Appendix B1: Sample Selection and Industry Details

The core sample used in this paper is all firms producing products belonging to the two-digit ASICC codes 74-78, which I refer to as the Machinery, Equipment and Parts industry. Details on these product codes are described in Table 14, below. This choice of codes was in part driven by observed similarities (see Table 14), as well as data driven. In particular, while I regard ASICC 77: ELECTRICAL & ELECTRONIC MACHINERY & EQUIPMENT INCL PARTS as the "core" industry, note that 45 % of the multi-product observations for ASICC 77 are produced by plants that produce products belonging to multiple 2-digit AISICC codes. Since focusing only of ASICC 77 firms would require that I drop close to half of the multi-product observations, I also add ASICC codes 74, 75, 76, and 78 to the sample, because they are relatively similar categories, and together account for more than 80 % of the multi-product variety-year observations for firms producing products belonging to ASICC 77.

As discussed in the main text, this industry was also chosen in large part because vertical integration is a smaller problem than in other industries. In particular, as described in Table 15, below, the Machinery, Equipment, and Parts industry has a relatively low level of vertical integration.

To determine firms which may be vertically integrated, I construct an input-output table, using information on inputs purchased by single product firms. For each output 5-digit ASICC code j, I collect the set of 5-digit ASICC codes k that are listed as inputs for single-product firms that produce j. I consider a multi-product firm potentially vertically integrated if they produce a pair of outputs, j and k, where at least 10 % of single-product producers of j also buy input k. While the 10% threshold is somewhat arbitrary, note more restrictive rules may result in many non-vertically integrated firms being classified as vertically integrated. For example, if one instead considers a firm potentially vertically integrated if any producer of j is observed using k, then more than 75 % of multi-product firms in the ASI, i.e. including all industries, are considered vertically integrated. Since it seems, at face value,

 $^{^{89}}$ Two digit ASICC codes from the 2009 version of the ASICC codes, which I match to the observed 2000-2008 ASICC codes in my data.

implausible that this many firms are actually producing multiple outputs for vertical integration reasons, I restrict attention to input-output pairs that are somewhat common, to remove the influence of outlier (rare) production sets. The 10% threshold results in just over 40% of all ASI multiproduct firms being classified as potentially vertically integrated. While this approach still classifies a fairly large percentage of multi-product firms as potentially vertically integrated, note that more lenient thresholds (e.g 20%) resulted in some obvious examples of vertical integration (e.g. batteries and battery plates) not being classified as vertically integrated.

As discussed in the main text, unless otherwise stated, I drop potentially vertically integrated plants in any regressions requiring information on the allocation of inputs across production lines, since these are unlikely to be measured correctly. I do, however, include these firms in the demand estimation regressions, since input shares do not need to be known for their prices do be information. Similarly, while I include multi-industry firms in the demand regressions, since they can aid in identification, I only consider single industry firms in all subsequent regressions, i.e. firms who only produce products from ASICC codes 74-78, since these regressions generally require estimates of within-firm input allocations, which I cannot know without knowledge of the production function used in other industries. These regressions also do not include plants with ad-valorem tax rates below 0 or above 1, as they are likely plagued by data entry errors. Similarly, all regressions drop plants with negative or missing prices and quantities, who do not record any values for any of the inputs used in production function estimation, who did not operate for 12 months, and whose entries appeared to have some major data entry errors, including a small number of firms who listed their initial production year as being past the end of the sample period, reported input quantity codes that did not equal the standard quantity code for the given ASICC code, as well some firms who had entries for a particular firm-type that was not included in any of the ASI code books.

Table 14: Machinery, Equipment, and Parts

ASICC	ASICC Description	Observations	Multi-product Multi-product Revenue Observation Share (%) Share (%)	Multi-product Observation Share (%)	Example 5-digit Products
74	MISC. MANUFACTURE OF BASE METALS, N.E.C	10,035	99	73	-Cylinders -Wheels -Rims
75	NON-ELECTRICAL MACHINE TOOLS & GEN- ERAL PURPOSE MACHINERIES AND COMPO- NENTS AND PARTS THEREOF	14,903	74	22	-Gears -Ball Bearings -Valves
92	NON-ELECTRICAL INDUSTRY SPECIFIC EQUIP- MENT / MACHINERIES INCL PARTS THEREOF	12,317	80	22	-Agriculture Implements -Textile Machinery -Drilling Machines
2.2	ELECTRICAL & ELECTRONIC MACHINERY & EQUIPMENT INCL PARTS (EXCL MEDICAL & NON-CONVENTIONAL ENERGY EQUIPMENT)	22,032	71	92	-Transformers -Control Equipment -Batteries
82	ELECTRONICS EQUIPMENT & PARTS EXCL BIO- MEDICAL EQUIPMENT	4,822	29	77	-Printed Circuit Plate -T.V. Set -Personal Computer
	Notes: Observations refers to number of variety-vear observations.	o number of variety-	vear observations.		

Notes: Observations refers to number of variety-year observations.

Table 15: Potentially Vertically Integrated Firms: Top 10 2-Digit ASICC Codes

15 BEVERAGES, TOBACCO AND PAN MASALA 64 SYNTHETIC (MAN-MADE) AND MIXED TEXTILES 74 MISC. MANUFACTURE OF BASE METALS, N.E.C [NO SUB-GROUP FORMATION] 76 NON-ELECTRICAL INDUSTRY SPECIFIC EQUIPMENT/MACHINERIES INCL PARTS THEREOF 75 PLASTIC, PVC ARTICLES INCL PACKAGING PRODUCTS AND FOOTWEAR PLASTIC OR PVC 76 NON-ELECTRICAL MACHINE TOOLS & GENERAL PURPOSE MACHINERIES AND 77 COMPONENTS AND PARTS THEREOF 77 ELECTRICAL & HON-CONVENTIONAL ENERGY EQUIPMENT INCL PARTS (EXCL 76 MEDICAL & NON-CONVENTIONAL ENERGY EQUIPMENT) 77 MEDICAL & NON-CONVENTIONAL ENERGY EQUIPMENT) 63 COTTON, COTTON YARN AND FABRICS 71 IRON & STEEL (INCL STAINLESS STEEL) & ARTICLES THEREOF 72 FRUITS, VEGETABLES, CEREALS & PULSES AND OTHER VEGETABLE PRODUCES 73 TIKE LAC. CHIM FTC. AND PREPAR ATTON THEREOF	Observations I.	Potentially Vertically integrated Revenue Share (%)	Potentially Vertically Potentially Vertically Integrated Revenue Share $(\%)$ Integrated Observation Share $(\%)$
	890'8	20	22
	9,592	52	29
	1	28	18
	INERIES INCL PARTS THEREOF 12,317	12	14
	D FOOTWEAR PLASTIC OR PVC 12,545	43	20
	MACHINERIES AND 14,903	26	19
	NCL PARTS (EXCL 22,032	31	31
71 IRON & STEEL (INCL STAINLESS STEEL) & ARTICLES THEREOF FRUITS, VEGETABLES, CEREALS & PULSES AND OTHER VEGETABLE PRODUCES 12 TIKE LAC GIM FTC AND PREPARATION THEREOF	26,565	36	36
FRUITS, VEGETABLES, CEREALS & PULSES AND OTHER VEGETABLE PRODUCES 12 THE LAC GIM FTC AND PREPARATION THEREOF	EOF 32,619	55	29
LINE LINE (CON E.C. MILE I DELINEATION THEREOF	GETABLE PRODUCES 34,616	57	62

Notes: Observations refers to number of variety-year observations. Potentially vertically integrated firms are firms that produce at least one 5-digit ASICC code that is classified as a potential input for another product they product. Product code j is classified as a potential input for product code k is at least 10% of observed single product k producers list item code j as an input.

8.2.2 Appendix B2: 3-digit ASICC codes

3-digit ASICC codes

ASICC code	Description
740	MISC. MANUFACTURE OF BASE METALS, N.E.C
751	NON-ELECTRICAL MACHINE TOOLS & GENERAL PURPOSE MACHINERIES AND
751	COMPONENTS AND PARTS THEREOF
761	AGRICULTURAL & FORESTRY MACHINERIES/PARTS THEREOF
762	FOOD, BEVERAGES & TOBACCO PROCESSING MACHINERIES & PARTS
763	MININGS, QUARRYING & METALLURGICAL MACHINERIES/PARTS
764	CONSTRUCTION/CEMENT MACHINERIES & PARTS
765	TEXTILE, LEATHER & RUBBER PROCESSING, PAPER PRINTING MACHINERIES
765	& PARTS THEREOF
766	NON-ELECTRICAL DOMESTIC/OFFICE APPLIANCES & PARTS
767	CHEMICAL/PLASTIC/GLASS/WEAPON/AMMUNITION MACHINERIES AND PARTS THEREOF
768	LIFT AND LIFTING EQUIPMENT, FIXED OR MOBILE & PARTS THEREOF
769	MISC NON-ELECTRICAL MACHINERIES AND PARTS THEREOF, N.E.C
771	ELECTRICAL MACHINERY/EQUIPMENT
772	ELECTRICAL MOTORS, GENERATORS, TRANSFORMER, POWER PACK [THIS INCL PUMP
112	SET FITTED WITH ELECTRIC MOTOR]
773	SWITCH, SWITCH-GEAR, CONTROL PANEL, CIRCUIT BREAKERS ETC AND PARTS
119	THEREOF
774	LAMP, FILAMENT, ELECTRODES/ANODES/CONNECTORS, FITTINGS & PARTS
775	MEASURING/CONTROLLING/REGULATING INSTRUMENTS
776	BATTERY, ACCUMULATORS, CELLS AND PARTS THEREOF
777	DOMESTIC AND OFFICE ELECTRICAL EQUIPMENT
778	ELECTRO MAGNET, FANS, ARMATURE, COILS & ELECTRO-MAGNETIC EQUIPMENT
779	ELECTRICAL EQUIPMENT, PARTS AND ACCESSORIES, N.E.C
781	TELEPHONE/TELECOMMUNICATION/TRANSMISSION EQUIPMENT
782	AUDIO/VIDEO/SOUND APPARATUS & PARTS
783	COMPUTER & COMPUTING EQUIPMENT & PERIPHERALS & PARTS
784	ELECTRONIC VALVES/TUBES & COMPONENTS
785	ELECTRONIC CARDS & ITS
100	COMPONENTS
789	OTHER ELECTRONIC COMPONENTS & PARTS

Notes: All 3-digit ASICC codes at the from 2009 ASICC. I map three digit ASICC codes to 5-digit ASICC codes from 2001-2008 using a custom made concordance that matches product descriptions at the 5-digit level.

8.2.3 Appendix B3: Trade Data Appendix

As described in the main text, I use data on imports and tariffs from UN Comtrade and TRAINS at the 4-digit HS-96 level to construct estimates of imports and tariffs for each 5-digit ASICC code. To generate the concordance between 5-digit ASICC codes, I first create a concordance between the observed

ASICC codes from 2001 to 2008, to the 2009 5-digit ASICC codes.⁹⁰ I then match 5-digit ASICC codes to the 7-digit 2011 National Product Classification for Manufacturing Sector (NPCMS) codes using a concordance created by the Indian Ministry of Statistics.⁹¹ Since the first five digits of NPCMS-2011 are equivalent to the international Central Product Classification 2.0 (CPC2), I then match 5-digit NPCMS-2011/CPC2 codes to 4-digit HS-2007 codes using a United Nations Statistics Division created concordance.⁹² Finally, I match 4-digit HS-2007 to 4-digit HS-96 using the concordance which can be found at https://unstats.un.org/unsd/trade/classifications/correspondence-tables.asp.

This process generates a crosswalk between 5-digit ASICC codes and 4-digit HS-96 codes. ⁹³ Note, however, that many HS codes map to multiple ASICC codes. This does not generate any real difficulties for calculating tariff levels across products, as I can simply use the average tariff rate across all matching HS codes to obtain a estimate of the average rate of protection. However, for imports this can generate problems due to double counting imports across items. To deal with this, I allocate imports from each HS-96 code that links to multiple ASI codes proportionally to the overall value of observed materials imports in the ASI production data. Specifically, since each plant records the value of imported inputs for up to five 5-digit ASICC codes each year, I take the total observed value of imports for each ASICC code g, I_{ASICC}^g , and allocate observed imports from HS code h, I_{HSt}^h , to ASICC code g according the formula Imports $g^g = \frac{I_{ASICC}^g}{\sum_{l \in \mathbb{H}^h} I_{ASICC}^l} I_{HSt}^h$, where Imports g^g is value of imports for product code g during time g^g that I use in my empirical model, and g^g is the set of 5-digit ASICC codes that map to HS code g^g .

8.3 Appendix C: Demand Appendix

8.3.1 Appendix C1: Derivation of Demand System from Underlying Preferences

The formulation of consumer preferences follows the "constant expenditure share" formulation of the discrete choice demand system described in of Björnerstedt and Verboven (2013).⁹⁴ During each market-period t, a mass S_t of consumers decide whether they wish to buy a *single* item $j \in \Lambda_t^g \subset \Omega_t$. Conditional on choosing to consume j, utility of consumer c is given by:

$$U_{ct}(j, Q_{cit}^j, \epsilon_{cit}^{jg}) = \left(Q_{cit}^{jg} \exp\left(\frac{\eta_{it}^{jg} + \epsilon_{cit}^{jg}}{\alpha}\right)\right)^{\beta_I} (Q_{c0})^{1-\beta_I}, \tag{54}$$

where Q_{cit}^{jg} is the quantity of good $j \in \Lambda_t^g$, which is produced by firm i, currently bought by consumer c, Q_{c0} is the quantity of some composite outside good purchased by consumer c, while η_{it}^{jg} is the mean

⁹⁰I create this concordance by matching on the given item-code descriptions between ASICC 2001-2008 and 20009, which results in a match for the vast majority of the codes.

 $^{^{91} \}mathrm{This}$ can be found at http://www.csoisw.gov.in/cms/En/1027-npcms-national-product-classification-for-manufacturing-sector.aspx

⁹²This can be found at which can be found at https://unstats.un.org/unsd/cr/registry/cpc-2.asp.

⁹³A small number of ASICC codes (13) could not be matched to trade data. These account for less than one percent of the overall sample, and are thus dropped, as I cannot guarantee to be measuring market size correctly for these observations.

⁹⁴While Björnerstedt and Verboven (2013) work with the indirect utility function directly, one can start from the utility function as a do here, similar to Anderson et al. (1987).

quality of good $j \in \Lambda_t^g$, ϵ_{cit}^{jg} is an idiosyncratic taste shock for product j, and $\alpha > 0$ and $\beta_I \in (0,1)$ are preference parameters.

I assume that conditional on j, consumers choose Q_{ct}^j and Q_{c0} to maximize their utility, given the unit price of good j, P_t^j , and the price of the composite composite commodity, which I normalize to 1, subject to some exogenous income level Y_{ct} . Since (54) is a standard Cobb-Douglas utility function, this leads to equilibrium conditional consumer demands $Q_{ct}^{j*g} = \beta_I \frac{Y_{ct}}{P_{it}^{jg}}$ and $Q_{c0}^* = (1 - \beta_I)Y_{ct}$. Substituting these expressions into (54) yields the conditional indirect utility function:

$$V_{ct}(j, \epsilon_{cit}^{jg}, Y_{ct}) = C_{ct}Y_{ct} \left(\exp\left(\frac{\beta_I}{\alpha} \eta_{it}^{jg} - \beta_I p_{it}^{jg} + \frac{\beta_I}{\alpha} \epsilon_{cit}^{jg} \right) \right), \tag{55}$$

where $C_{ct} \equiv (\beta_I)^{\beta_I} (1 - \beta_I)^{1 - \beta_I}$ is an arbitrary constant, and $p_t^j \equiv \ln(P_t^j)$.

Given the conditional indirect utility function (55), consumers will choose the $j \in \Omega_t$ that provides the largest indirect utility. Note, however, that it will prove useful to rescale the indirect utility function by first taking it's natural log, and then multiplying by $\frac{\alpha}{\beta_t}$, yielding:

$$\widetilde{V}_{ct}(j, \epsilon_{cit}^{jg}, Y_{ct}) = \widetilde{c}_{ct} + \frac{\alpha}{\beta_I} y_{ct} - \alpha p_t^j + \eta_t^j + \epsilon_{ct}^j, \tag{56}$$

where $y_{ct} \equiv \ln(Y_{ct})$, and \tilde{c}_{ct} is an arbitrary constant.

Since positive monotonic transformations of (55) will not affect the ordinal ranking of the products in Ω_t , the consumers problem can instead be formulated as choosing the $j \in \Omega_t$ that maximizes the transformed indirect utility function (56). As is standard in discrete choice models of demand, following McFadden (1978), instead of solving this problem on a consumer by consumer basis, one models the distribution of consumer taste shocks, ϵ_{cit}^{jg} , so that one may determine the probability that each product $j \in \Omega_t$ is chosen by a randomly chosen consumer within the general population. In particular, I assume the vector of idiosyncratic taste terms for each consumer c, $\vec{\epsilon}_{ct} = \left(\epsilon_{ct}^{1g}, \epsilon_{ct}^{2g}, ..., \epsilon_{ct}^{1l}, \epsilon_{ct}^{2l}, ...\right)$, can be modeled as an i.i.d. draw from a generalized extreme-value distribution with cdf:

$$F(\vec{\epsilon}_{ct}) = \exp\left(-\sum_{g=0}^{G} \left(\sum_{j \in \Lambda_t^g} \exp\left(-\frac{\epsilon_{cit}^{jg}}{\sigma}\right)\right)^{\sigma}\right),\tag{57}$$

where G is the number of product groups (i.e. 5-digit product codes) on the market.

Note that σ governs the within-product code correlation of the idiosyncratic taste terms. In the special case where $\sigma = 1$, (57), there is no correlation of the idiosyncratic taste shocks within a product group, and (57) simplifies to a standard Type-1 extreme value distribution. However, as σ decreases, the correlation between the taste shocks within a product group increases-see Train (2009).

 $^{^{95}}$ Since in my empirical application, "consumers" of Machinery, Equipment, and Parts, are perhaps more appropriately thought of as downstream producers, I can adapt this framework to such a setting by regarding (54) as a Cobb-Douglas materials aggregator function used by a particular downstream producer c. In this case, producers choose quantities to maximize effective materials per rupee spent, which is an implication of cost-minimization. While the portion of expenditure allocated to material expenditures, Y_{ct} , is endogenous in the downstream firms overall profit maximization problem, note that as long as upstream producers take the downstream output-pricing game as given, total expenditure on materials, Y_{ct} is pinned down by downstream output prices and markups, as I discuss in footnote 97, below.

It is well known that if each $\vec{\epsilon}_{ct}$ is an i.i.d. draw from (57), then, given the indirect utility function described by (56), the probability a randomly chosen consumer chooses to consume item $j \in \Lambda_t^g$ has the following closed-form expression:

$$Prob_{it}^{jg} = \frac{\exp\left(\frac{\delta_{it}^{jg}}{\sigma}\right) \left(\sum_{k \in \Lambda_t^g} \exp\left(\frac{\delta_{it}^{kg}}{\sigma}\right)\right)^{\sigma-1}}{\sum_{l=0}^{G} \left(\sum_{k \in \Lambda_t^l} \exp\left(\frac{\delta_{it}^{kl}}{\sigma}\right)\right)^{\sigma}},\tag{58}$$

where $\delta_{it}^{jg} \equiv \eta_{it}^{jg} - \alpha p_{it}^{jg}$ is the mean utility for choosing to consume variety $j \in \Lambda_t^g$.

Since there are a continuum of consumers within the market, each of whom, with probability $Prob_{it}^{jg}$, purchases $Q_{cit}^{jg*} = \beta_I \frac{Y_{ct}}{P_{it}^{jg}}$ units of item j, the total aggregate demand for product $j \in \Lambda_t^g$, produced by firm i, is given by:

$$Q_{it}^{jg} = \frac{E_t}{P_{it}^{jg}} \frac{\exp\left(\frac{\delta_{it}^{jg}}{\sigma}\right) \left(\sum_{k \in \Lambda_t^g} \exp\left(\frac{\delta_{mt}^{kg}}{\sigma}\right)\right)^{\sigma-1}}{\sum_{l=0}^G \left(\sum_{k \in \Lambda_t^l} \exp\left(\frac{\delta_{mt}^{kl}}{\sigma}\right)\right)^{\sigma}},\tag{59}$$

where $E_{t} = \mathbb{E}\left(Y_{ct}\right)$ is expected expenditure by all consumers on the items within this market.

Note that in the special case where $\sigma=1$, (59) simplifies to $Q_{it}^{jg}=\frac{E_t \exp(\eta_{it}^j) \left(P_{it}^{jg}\right)^{-(1+\alpha)}}{\sum_{l=1}^G \sum_{k\in\Lambda_l^l} \exp(\eta_{mt}^{jl}) \left(P_{mt}^{kl}\right)^{-\alpha}}$, which is a quality-adjusted variation of the CES demand system that is commonly used in the international trade literature. While this specification of the demand function is often used in applied work for it's simplicity, note that setting $\sigma=1$ results in conditional choice probabilities that satisfy the independence of irrelevant alternatives (IIA) property, which means that following an increase in the price of any item j, choice probabilities for all varieties $k\neq j$ will increase by the exact same proportion. ⁹⁶ Given the Cobb-Douglas form of the consumer-level conditional demand function, this also implies a proportionate increase in overall demand for all items $k\neq j$. While these substitution patterns may be appropriate for sufficiently similar products, note that most firms produce products that belong to multiple product classes, such as, for example, rubber insulated cables (ASICC 77475) and PVC insulated Cables (ASICC 77471). As a result, to use this simplified demand system, one must be willing to assume that crossprice elasticities between these two types of cables are the same as the cross-price elasticities between different varieties of a particular class of cable, which is unlikely to hold in practice.

To get around the unrealistic cross-price elasticities generated by the IIA property, one introduces correlation in the individual-level tastes shocks by allowing $\sigma \in (0,1)$. As long as taste shocks are correlated within product groups, this means that, in the aggregate, some consumers generally prefer rubber cables to PVC cables, and vice-versa. This then leads to cross-price elasticities that are *larger* within product groups than across product groups, since a change in the price of a particular variety of PVC cable will primarily lead to product substitution by the consumers who prefer PVC cables, rather than rubber cables.

⁹⁶To see this, note that relative choice probabilities, $\frac{Prob_t^j}{Prob_t^k}$, only depend on the relative payoffs between the two items, rather than all items in the consumers choice set, as $\frac{Prob_t^j}{Prob_t^k} = \exp\left(\delta_t^j - \delta_t^k\right)$.

8.3.2 Appendix C2: Input Share Inversion Details

To obtain the mapping between demand-side observables and marginal costs for the demand system used in the main text, I need the demand derivatives for (23). These derivatives are given by:⁹⁷

$$\frac{\partial Q_{it}^{jg}}{\partial P_{it}^{jg}} = -\frac{Q_{it}^{jg}}{P_{it}^{jg}} \left(1 + \frac{\alpha}{\sigma} - \frac{(1 - \sigma)\alpha R S_{it}^{j|g}}{\sigma} - \alpha R S_{it}^{jg} \right) \quad \text{if } j \in \Lambda_t^g, \tag{60}$$

$$\frac{\partial Q_{it}^{jg}}{\partial P_{it}^{kl}} = \begin{cases}
\frac{Q_{it}^{jg}}{P_{it}^{kg}} \left(\frac{(1-\sigma)\alpha RS_{it}^{k|g}}{\sigma} + \alpha RS_{it}^{kg} \right) & \text{if } (j,k) \in \Lambda_t^g \\
\frac{Q_{it}^{jg}}{P_{it}^{kl}} \alpha RS_{it}^{kl} & \text{if } j \in \Lambda_{gt}, \ k \in \Lambda_t^l, l \neq g,
\end{cases}$$
(61)

where $RS_{it}^{jg} \equiv \frac{P_{it}^{jg}Q_{it}^{jg}}{E_t^h}$ is the overall revenue market share of product $j \in \Lambda_t^g \subset \Omega_t^h$, while $RS_{it}^{j|g} \equiv \frac{P_{it}^{jg}Q_{it}^{jg}}{\sum_{k \in \Lambda_t^g} P_{it}^{kg}Q_{it}^{kg}}$ is the within-nest revenue share of product j.

I then use these demand derivatives, combined with the firm's pricing first-order conditions, to find the marginal cost inversion necessary to obtain the within-firm input allocations. Note, however, that to account for product specific taxes, which I observe in my data, I need to make a small adjustment to the baseline model discussed in the main text. Let τ_{it}^j denote the ad-valorem tax rate on product $j \in \mathbb{Y}_{it}$, which I assume takes the form of sales tax that consumers must pay on top of the listed price. The per-unit price that consumers must pay for some good j overall is then $P_{it}^{jC} = P_{it}^j(1 + \tau_{it}^j)$, where P_{it}^j is the price set (and received) at the plant level. Since $P_{it}^{jC} \neq P_{it}^j$, the firm's first order pricing conditions become:

$$Q_{it}^j + (1 + \tau_{it}^j) \sum_{k \in \mathbb{Y}_{it}} \frac{\partial Q_{it}^k}{\partial P_{it}^{jC}} \left(P_{it}^k - M C_{it}^k \right) = 0, \tag{62}$$

where the $(1 + \tau_{it}^j)$ in (62) reflects the fact that $\frac{\partial Q_{it}^k}{\partial P_{it}^j} = \frac{\partial Q_{it}^k}{\partial P_{it}^{jC}} \frac{\partial P_{it}^{jC}}{\partial P_{it}^j} = \frac{\partial Q_{it}^k}{\partial P_{it}^{jC}} (1 + \tau_{it}^j)$. Stacking the above in vector notation:

$$\vec{\tilde{Q}}_{it} + \Delta_{it} \left(\vec{P}_{it} - \vec{MC}_{it} \right) = 0, \tag{63}$$

⁹⁷ Note that if we interpret consumers as downstream purchases of aggregate materials, as described in footnote 95, then these demand derivatives assume that Machinery, Equipment, and Parts producers take downstream expenditure levels on materials as exogenous. This can be justified by treating the downstream output market equilibrium as given. In particular, suppose that downstream production is done using a (potentially unknown) Cobb Douglas production function, with (55) corresponding to the Cobb-Douglas materials aggregator for some downstream firm c. Further suppose that downstream firms also compete in the manner described by (4) in the main text. Then one can use the cost-minizing condition for materials, (6), to show that $W_{ct}^M M_{ct}^{jD} = \beta_M^D \frac{P_{ct}^{jD}Q_{ct}^{jD}}{\mu_{ct}^{jD}}$, where β_M^D is the Cobb-Douglas materials coefficient for downstream firms, $P_{ct}^{jD}Q_{ct}^{jD}$ is total revenue earned by some downstream firm c in product line j, μ_{ct}^{jD} is the markup they charge on this product line, i.e. $\left(\frac{P_{ct}^{jD}}{MC_{ct}^{jD}}\right)$, and $W_{ct}^M M_{ct}^{jD}$ is total expenditure on aggregate materials for product line j in the downstream firm c. Note that total income spent on materials is pinned down by the prices, quantities, and markups charged by the downstream firms in the output market. Hence, as long as upstream firms in the Machinery, Equipment, and Parts industry take the downstream output market equilibirum as given, then each individual producer's "income", i.e. $W_{ct}^M M_{ct}^{jD}$, is exogenously given from their point of view.

where \vec{P}_{it} , and \vec{MC}_{it} are column vectors of prices and marginal costs, \vec{Q}_{it} is the vector of quantity sold over $1 + \tau_{it}^j$, i.e. $\frac{Q_{it}^j}{1 + \tau_{it}^j}$, and Δ_{it} is a square matrix of own and cross-price demand derivatives for all the $J_{it} = |\mathbb{Y}_{it}|$ products produced by firm i, i.e.:

$$\Delta_{it} = \begin{bmatrix}
\frac{\partial Q_{it}^{i_1}}{\partial P_{it}^{i_1}} & \frac{\partial Q_{it}^{i_2}}{\partial P_{it}^{i_1}} & \cdots & \frac{\partial Q_{it}^{i_{Jit}}}{\partial P_{it}^{i_1}} \\
\frac{\partial Q_{it}^{i_1}}{\partial P_{it}^{i_2}} & \frac{\partial Q_{it}^{i_2}}{\partial P_{it}^{i_2}} & \cdots & \frac{\partial Q_{it}^{i_{Jit}}}{\partial P_{it}^{i_2}} \\
\cdots & \cdots & \cdots & \cdots \\
\frac{\partial Q_{it}^{i_1}}{\partial P_{it}^{i_{Jit}}} & \frac{\partial Q_{it}^{i_2}}{\partial P_{it}^{i_{Jit}}} & \cdots & \frac{\partial Q_{it}^{i_{Jit}}}{\partial P_{it}^{i_{Jit}}}
\end{bmatrix},$$
(64)

where i_j is the jth element of \mathbb{Y}_{it} .

Since (23) satisfies Assumption 7, and therefore Δ_{it} is invertible, (63) can be rewritten as:

$$\vec{MC}_{it} = (\Delta_{it})^{-1} \vec{\tilde{Q}}_{it} + \vec{P}_{it} = g\left(\vec{P}_{it}, \vec{\tilde{Q}}_{it}, \vec{RS}_{it}, \vec{RS}_{it}^g, \alpha, \sigma\right), \tag{65}$$

where the second equality follows from (60) and (61). Hence, using (65), one can rewrite (9) as:

$$X_{it}^{j} = \frac{g_{it}^{j} \left(\vec{P}_{it}, \vec{\tilde{Q}}_{it}, \vec{RS}_{it}, \vec{RS}_{it}^{g}, \alpha, \sigma\right) Y_{it}^{j}}{\sum_{k \in \mathbb{Y}_{it}} g_{it}^{k} \left(\vec{P}_{it}, \vec{\tilde{Q}}_{it}, \vec{RS}_{it}, \vec{RS}_{it}^{g}, \alpha, \sigma\right) Y_{it}^{k}} X_{it},$$

$$(66)$$

where the equilibrium value of each g_{it}^{j} function can be determined from (65), given (α, σ) .

8.4 Appendix D: Revenue TFP

In this section I show how one can use the structure of the production function and demand function to generate a rescaled production function whose residual is a combination of productivity and demand shifters, following Klette and Griliches (1996) and De Loecker (2011).

Let
$$\exp\left(\widetilde{\eta}_{it}^{jg}\right) \equiv \left(\frac{E_t^h \exp\left(\frac{\eta_{it}^{jg}}{\sigma}\right)}{\left(\sum_{k \in \Lambda_t^g} \exp\left(\frac{\delta_{mt}^{kg}}{\sigma}\right)\right)^{1-\sigma} \sum_{l \in \Omega_t^h} \left(\sum_{k \in \Lambda_t^l} \exp\left(\frac{\delta_{mt}^{kl}}{\sigma}\right)\right)^{\sigma}}\right)^{\frac{1}{1+\frac{\alpha}{\sigma}}}$$
, and substitute this expression

into (23), yielding $Q_{it}^{jg} = \exp(\widetilde{\eta}_{it}^{jg}) \left(P_{it}^{jg}\right)^{-\left(1+\frac{\alpha}{\sigma}\right)}$. One can then use this expression to write product-level revenues as:

$$R_{it}^{jg} = \exp\left(\widetilde{\eta}_{it}^{jg}\right) \left(Q_{it}^{jg}\right)^{-\frac{\alpha}{1+\frac{\alpha}{\sigma}}}.$$
 (67)

Substituting $Q_{it}^{jg} = Y_{it}^{j} = \exp\left(\omega_{it}^{jg}\right) \left(L_{it}^{jg}\right)^{\beta_L} \left(K_{it}^{jg}\right)^{\beta_K} \left(M_{it}^{jg}\right)^{\beta_M}$ into (67), taking logs and rearranging yields:⁹⁸

⁹⁸Note that this formula does not take into accounts changes in inventories, although can be adapted to account for this.

$$\frac{R_{it}^{jg}}{\left(L_{it}^{jg}\right)^{\widetilde{\beta}_L} \left(K_{it}^{jg}\right)^{\widetilde{\beta}_K} \left(M_{it}^{jg}\right)^{\widetilde{\beta}_M}} = \exp\left(\widetilde{\eta}_{it}^{jg} + \frac{\frac{\alpha}{\sigma}\omega_{it}^{jg}}{1 + \frac{\alpha}{\sigma}}\right) \equiv TFPR_{it}^{jg},\tag{68}$$

where $\widetilde{\beta}_X \equiv \frac{\frac{\alpha}{\sigma}}{1+\frac{\alpha}{\sigma}}\beta_X$ for $X \in (L,K,M)$. Note that $TFPR_{it}^{jg}$ varies across products due to both productivity variation, $\widetilde{\omega}_{it}^{jg} \equiv \frac{\frac{\alpha}{\sigma}\omega_{it}^{jg}}{1+\frac{\alpha}{\sigma}}$, as well demand variation, $\widetilde{\eta}_{it}^{jg}$, which is driven by differences in quality, average product-appeal at the 5-digit ASICC code, as well as product-space congestion effects.⁹⁹

Note, however, that TFPR variation within a 5-digit product code and year is simply a weighted sum of the quality and TFPQ residuals from (25) and (28), respectively. To see this, take logs and demean (68) by product-code and year, yielding:

$$\ln\left(TFPR_{it}^{jg}\right) - \ln\left(\overline{TFPR_t^g}\right) = \frac{1}{\sigma + \alpha}\left(\eta_{it}^{jg} - \overline{\eta_t^g}\right) + \frac{\frac{\alpha}{\sigma}}{1 + \frac{\alpha}{\sigma}}\left(\omega_{it}^{jg} - \overline{\omega_t^g}\right) - \frac{1}{1 + \frac{\alpha}{\sigma}}RC_t^g,\tag{69}$$

where $RC_t^g \equiv \ln\left(E_t^h\right) - (1-\sigma)\ln\left(\sum_{k\in\Lambda_t^g}\exp\left(\frac{\delta_{it}^{kg}}{\sigma}\right)\right) - \ln\left(\sum_{l=1}^G\left(\sum_{k\in\Lambda_t^l}\exp\left(\frac{\delta_{it}^{kg}}{\sigma}\right)\right)^\sigma\right)$, while $\overline{\eta}_t^g$ and $\overline{\omega}_t^g$ are the average values of quality and TFPQ, respectively, within a product code-year.

⁹⁹Note that the demand shifter $\widetilde{\eta}_{it}^{jg}$ also incorporates product-space congestion effects, since increases in $\left(\sum_{k\in\Lambda_t^g}\exp\left(\frac{\delta_{it}^{kg}}{\sigma}\right)\right)^{1-\sigma}$ and $\sum_{l\in\Omega_t^h}\left(\sum_{k\in\Lambda_t^l}\exp\left(\frac{\delta_{it}^{kg}}{\sigma}\right)\right)^{\sigma}$, caused by an increase in the number of competitors at the 5-digit and 3-digit ASICC code, respectively, will scale down $\widetilde{\eta}_{it}^j$.

8.5 Appendix E: Results with Materials Measured in Value Units

All results below replicate the table and pictures in the main text, except the materials has been replaced by materials expenditures (in rupees), rather than the firm-specific Cobb-Douglas aggregator.

(2): TFPQ Versus Quality - Single Product Only Slope = -2.655*** (0.092) (1): TFPQ Versus Quality - All Products Slope = -2.75*** (0.093) 20 9 10 ŦF 0 Quality (3): TFPQ Versus Quality - Multi-Product Only (4): TFPQ Versus Quality - Within-Firm Slope = -2.859*** (0.1) Slope = -3.286*** (0.047) 20 10 TFP TFP 0

Figure 3: Negative Correlation between TFPQ and Quality: Materials in Value Units

Notes: Standard errors of slopes, adjusted for two-way clustering by plant and product code, reported in parentheses. Quality refers to the year-product code demeaned residual from (25), while TFPQ refers to the year-product code demeaned residual from (28), with materials measured in value, rather than quantity, units. Panel (1) includes all products, Panel (2) only includes single-product plants, and Panels (3) and (4) only includes products produced by multi-product plants. Panel (4) plots within-firm demeaned values of TFPQ and quality.

Table 16: Effect of Chinese Import Competition on TFPR: OLS

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	TFPR	TFPR	TFPR	TFPR
Change in Chinese Imports	0.266	0.241	0.216	0.214
	(0.0642)	(0.0643)	(0.0674)	(0.0681)
Lagged TFPR level		-0.338	-0.339	-0.339
		(0.0279)	(0.0278)	(0.0279)
Change in World Input Tariffs			-0.0367	-0.0382
			(0.0204)	(0.0214)
Change in World Output Tariffs				0.00671
				(0.0101)
Observations	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports OLS regressions Δh_{it} on the described variables. All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in Chinese Imports measured as $\sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \ln \left(\mathrm{IM}^m_{\mathrm{IND,CHN},t} \right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \tau^m_t$, where τ^m_t is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errorstreat demand and production function estimates as data.

Table 17: Effect of Chinese Import Competition on TFPR: IV

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	TFPR	TFPR	TFPR	TFPR
Change in Chinese Imports	0.719	0.720	0.759	0.754
	(0.702)	(0.689)	(0.514)	(0.493)
Lagged TFPR level		-0.336	-0.336	-0.336
		(0.0284)	(0.0283)	(0.0284)
Change in World Input Tariffs			-0.00750	-0.00734
			(0.0380)	(0.0388)
Change in World Output Tariffs				-0.00200
				(0.0134)
Observations	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports IV regressions of Δh_{it} on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t}^m \right) \right)$ instrumenting the change in Chinese imports. All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in Chinese imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\text{IM}_{\text{IND,CHN},t}^m \right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \tau_t^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistics are 16.80, 16.74, 27.73, and 30.57, for columns (1) through (4), respectively. Standard errors treat demand and production function estimates as data.

Table 18: Effect of Chinese Import Competition on TFPR: OLS

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	0.222	0.222	0.00992	-0.0148	0.00439
	(0.0680)	(0.0684)	(0.00853)	(0.0162)	(0.0176)
Change in World Input Tariffs	-0.0386	-0.0385	-0.000500	-0.00209	0.00250
	(0.0214)	(0.0218)	(0.00263)	(0.00485)	(0.00461)
Change in World Output Tariffs	0.00623	0.00750	-0.000421	0.00562	-0.00647
	(0.0101)	(0.0103)	(0.00115)	(0.00270)	(0.00300)
Observations	$5,\!235$	5,235	5,235	5,235	5,235

Standard errors clustered by plant

Notes: The above reports OLS regressions of the various elements of (41) on the described variables. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\overline{h}_{i,t-1}^{jg}$ $OP_{i,t-1}\left(\vec{S}_{i,t-1},\vec{h}_{i,t-1}^{g}\right)$, $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\omega}_{i,t-1}^{j|\vec{g}}\right)$, and $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\eta}_{i,t-1}^{j|\vec{g}}\right)$. Change in Chinese Imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\mathrm{IM}_{\mathrm{IND,CHN},t}^m\right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \tau_t^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Table 19: Effect of Chinese Import Competition on TFPR: IV

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	0.768	0.786	-0.111	-0.279	0.372
	(0.489)	(0.495)	(0.0709)	(0.138)	(0.179)
Change in World Input Tariffs	-0.00736	-0.00625	-0.00740	-0.0172	0.0235
	(0.0385)	(0.0390)	(0.00475)	(0.00950)	(0.0110)
Change in World Output Tariffs	-0.00266	-0.00168	0.00154	0.00993	-0.0125
	(0.0134)	(0.0136)	(0.00188)	(0.00386)	(0.00480)
Observations	5,235	5,235	5,235	5,235	5,235

Standard errors clustered by plant

Notes: The above reports IV regressions of the various elements of (41) on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \operatorname{IM}_{k,\text{CHN},t}^m \right) \right)$ instrumenting the change in Chinese imports. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\overline{h}_{i,t-1}^{jg}$ $OP_{i,t-1}\left(\vec{S}_{i,t-1},\vec{h}_{i,t-1}^g\right)$, $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\omega}_{i,t-1}^{j|g}\right)$, and $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\eta}_{i,t-1}^{j|g}\right)$. Change in Chinese imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\operatorname{IM}_{\text{IND,CHN},t}^m\right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta t_i^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistic is 30.41. Standard errors treat demand and production function estimates as data.

8.6 Appendix F: Supplementary Regressions

8.6.1 Coefficients on Lags For Decompositions

Table 20: Effect of Chinese Import Competition on TFPR: OLS

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
$\overline{h}_{i,t-1}^{jg}$	-0.297	-0.298	0.000547	-0.00173	0.00190
	(0.0163)	(0.0164)	(0.00164)	(0.00254)	(0.00239)
$OP_{i,t-1}\left(\vec{S}_{i,t-1},\vec{h}_{i,t-1}^g\right)$	-0.0753	0.473	0.203	-0.234	-0.517
	(0.211)	(0.220)	(0.0676)	(0.0926)	(0.0759)
$OP_{i,t-1}\left(\vec{S}_{i,t-1}, \widetilde{\omega}_{i,t-1}^{j g}\right)$	0.0251	0.588	0.201	-1.011	0.247
, ,	(0.285)	(0.299)	(0.0793)	(0.117)	(0.0944)
$OP_{i,t-1}\left(\vec{S}_{i,t-1}, \widetilde{\eta}_{i,t-1}^{j \vec{g}}\right)$	-0.343	0.161	-0.680	-0.129	0.305
, ,	(0.226)	(0.231)	(0.0789)	(0.0937)	(0.0850)
Observations	5,236	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports IV regressions of the various elements of (41) on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t}^m \right) \right)$ instrumenting the change in Chinese imports. See notes to Table 12 in the main text.

Table 21: Effect of Chinese Import Competition on TFPR: IV

-	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
$\overline{h}_{i,t-1}^{jg}$	-0.303	-0.304	0.00153	-0.000398	-0.000305
	(0.0167)	(0.0169)	(0.00179)	(0.00264)	(0.00267)
$OP_{i,t-1}\left(\vec{S}_{i,t-1},\vec{h}_{i,t-1}^g\right)$	-0.0774	0.471	0.203	-0.233	-0.518
	(0.214)	(0.223)	(0.0677)	(0.0939)	(0.0793)
$OP_{i,t-1}\left(\vec{S}_{i,t-1}, \widetilde{\omega}_{i,t-1}^{j g}\right)$	0.0531	0.617	0.196	-1.017	0.257
, ,	(0.286)	(0.301)	(0.0786)	(0.120)	(0.0986)
$OP_{i,t-1}\left(\vec{S}_{i,t-1}, \widetilde{\omega}_{i,t-1}^{j g}\right)$	-0.390	0.113	-0.672	-0.119	0.288
,	(0.230)	(0.234)	(0.0815)	(0.0931)	(0.0888)
Observations	5,236	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports IV regressions of the various elements of (41) on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t}^m \right) \right)$ instrumenting the change in Chinese imports. See notes to Table 13 in the main text.

8.6.2 Alternative Specifications for Decompositions

In this section, I show the results of the decompositions considered in the main text (Tables 12 and 13), for the more limited sets of controls considered in Tables 9 and 10 in columns (1) through (3).

Table 22: Effect of Chinese Import Competition on TFPR: OLS - Limited Controls

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	0.253	0.255	-0.00204	0.00976	-0.00965
	(0.0807)	(0.0810)	(0.0126)	(0.0225)	(0.0243)
Observations	5,236	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports OLS regressions of the various elements of (41) on the described variables. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in Chinese Imports measured as $\sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \ln \left(\mathrm{IM}^m_{\mathrm{IND,CHN},t} \right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \tau^m_t$, where τ^m_t is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Table 23: Effect of Chinese Import Competition on TFPR: IV - Limited Controls

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	1.271	1.276	-0.353	-0.344	0.693
	(0.989)	(0.994)	(0.223)	(0.228)	(0.359)
Observations	5,236	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports IV regressions of the various elements of (41) on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t}^m \right) \right)$ instrumenting the change in Chinese imports. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in Chinese imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\text{IM}_{\text{IND,CHN},t}^m \right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \tau_t^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistic is 16.80. Standard errors treat demand and production function estimates as data.

Table 24: Effect of Chinese Import Competition on TFPR: OLS - Adding Lags

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	0.330	0.332	0.0134	-0.0174	0.00228
	(0.0815)	(0.0818)	(0.0104)	(0.0152)	(0.0171)
Observations	5,236	5,236	5,236	5,236	5,236

Notes: The above reports OLS regressions of the various elements of (41) on the described variables. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\bar{h}_{i,t-1}^{jg}$ $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \bar{h}_{i,t-1}^{g}\right)$, $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \tilde{\omega}_{i,t-1}^{j|g}\right)$, and $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \tilde{\eta}_{i,t-1}^{j|g}\right)$. Coefficients on the lags can be found in Appendix F. Change in Chinese Imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln\left(\operatorname{IM}_{\mathrm{IND,CHN},t}^m\right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln_{i,t-1}^m \alpha_{i,t-1}^m \Delta \tau_t^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Table 25: Effect of Chinese Import Competition on TFPR: IV - Adding Lags

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	1.505	1.530	-0.220	-0.329	0.524
	(0.990)	(0.996)	(0.134)	(0.194)	(0.265)
Observations	5,236	5,236	5,236	5,236	5,236
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Standard errors clustered by plant

Notes: The above reports IV regressions of the various elements of (41) on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t}^m \right) \right)$ instrumenting the change in Chinese imports. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\overline{h}_{i,t-1}^{jg}$ $OP_{i,t-1}\left(\vec{S}_{i,t-1},\vec{h}_{i,t-1}^g\right)$, $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\omega}_{i,t-1}^{j}\right)$, and $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\eta}_{i,t-1}^{j}\right)$. Change in Chinese imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\text{IM}_{\text{IND,CHN},t}^m\right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \tau_t^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistic is 16.65. Standard errors treat demand and production function estimates as data.

Table 26: Effect of Chinese Import Competition on TFPR: OLS - Adding Input Tariffs

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	0.309	0.311	0.0114	-0.0165	0.00302
	(0.0846)	(0.0849)	(0.0107)	(0.0162)	(0.0177)
Change in World Input Tariffs	-0.0306	-0.0301	-0.00289	0.00127	0.00108
	(0.0258)	(0.0261)	(0.00350)	(0.00487)	(0.00450)
Observations	5,236	5,236	5,236	5,236	5,236

Notes: The above reports OLS regressions of the various elements of (41) on the described variables. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\bar{h}_{i,t-1}^{jg}$ $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \bar{h}_{i,t-1}^{g}\right)$, $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \tilde{\omega}_{i,t-1}^{j|g}\right)$, and $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \tilde{\eta}_{i,t-1}^{j|g}\right)$. Coefficients on the lags can be found in Appendix F. Change in Chinese Imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln\left(\mathrm{IM}_{\mathrm{IND,CHN},t}^m\right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \tau_t^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Table 27: Effect of Chinese Import Competition on TFPR: IV - Adding Input Tariffs

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	1.367	1.385	-0.157	-0.266	0.405
	(0.739)	(0.743)	(0.0988)	(0.143)	(0.195)
Change in World Input Tariffs	0.0264	0.0278	-0.0120	-0.0122	0.0228
	(0.0502)	(0.0507)	(0.00623)	(0.00921)	(0.0111)
Observations	5,236	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports IV regressions of the various elements of (41) on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t}^m \right) \right)$ instrumenting the change in Chinese imports. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\overline{h}_{i,t-1}^{jg}$ $OP_{i,t-1}\left(\vec{S}_{i,t-1},\vec{h}_{i,t-1}^g\right)$, $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\omega}_{i,t-1}^{jg}\right)$, and $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\eta}_{i,t-1}^{jg}\right)$. Change in Chinese imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\text{IM}_{\text{IND,CHN},t}^m\right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \tau_t^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistic is 27.57. Standard errors treat demand and production function estimates as data.

8.6.3 Decomposition of Mean TFPR Effects

The following section further decomposes $\Delta \overline{h}_{it}^{jg}$ into average product TFPR growth, average cost comparative advantage growth, and average demand comparative advantage growth. In particular, by construction

$$\Delta \overline{h}_{it}^{jg} = \underbrace{\Delta \overline{h}_{it}^{g}}_{\text{(1): Average Product }} + \underbrace{\Delta \overline{\omega_{it}}^{j|g}}_{\text{Comparative Advantage}} + \underbrace{\Delta \overline{\eta_{it}}^{j|g}}_{\text{Comparative Advantage}}$$

$$(70)$$

Note that these results are not emphasized in the main text, in large part because these results can be quite difficult to interpret, and thus may be more misleading than informative. In particular, since I am looking at the response of these variables to import competition, which only varies *across* product codes, these responses will in some sense only capture *averages responses* within product code. As a result, the average comparative advantage terms may primarily capture *differences in the average reference group*. Although it worth noting that these results are consistent with Chinese import competition leading to quality upgrading.

Table 28: Effect of Chinese Import Competition on TFPR: OLS

	(1)	(2)	(3)
	Average Product	Average Cost	Average Demand
VARIABLES	Code TFPR Growth	Comparative Advantage	Comparative Advantage
Change in Chinese Imports	0.105	0.0419	0.161
	(0.0559)	(0.0847)	(0.0767)
Change in World Input Tariffs	-0.0139	0.00449	-0.0227
	(0.0206)	(0.0275)	(0.0241)
Change in World Output Tariffs	0.00153	-0.0123	0.0198
	(0.0106)	(0.0181)	(0.0152)
Observations	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports OLS regressions of the various elements of (70) on the described variables. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\overline{h}_{i,t-1}^g$, $\overline{\omega}_{i,t-1}^{j|g}$, and $\overline{\eta}_{i,t-1}^{j|g}$. Change in Chinese Imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\mathrm{IM}_{\mathrm{IND,CHN},t}^m \right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \overline{\tau}_{i,t-1}^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

¹⁰⁰That said, these effects *could* capture actual quality and TFPQ upgrading, under some conditions. In particular, since different plants have different product sets, and therefore face different levels of import competition due to different *compositions* of products, these effects could capture actual quality or TFPQ upgrading through variation in the composition of products (and therefore, effective import competition). At this stage, I am still thinking about the right way to interpret these terms, and whether there are some robustness checks I could do to pin these type of effects down.

Table 29: Effect of Chinese Import Competition on TFPR: IV

	(1)	(2)	(3)
	Average Product	Average Cost	Average Demand
VARIABLES	Code TFPR Growth	Comparative Advantage	Comparative Advantage
Change in Chinese Imports	-0.416	0.0484	1.734
	(0.467)	(0.681)	(0.746)
Change in World Input Tariffs	-0.0437	0.00486	0.0673
	(0.0348)	(0.0488)	(0.0501)
Change in World Output Tariffs	0.00993	-0.0124	-0.00549
	(0.0130)	(0.0208)	(0.0199)
Observations	5,236	5,236	5,236

Notes: The above reports IV regressions of the various elements of (70) on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t}^m \right) \right)$ instrumenting the change in Chinese imports. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\overline{h}_{i,t-1}^g$, $\overline{\omega}_{i,t-1}^{j|g}$, and $\overline{\eta}_{i,t-1}^{j|g}$. Change in Chinese imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\text{IM}_{\text{IND,CHN},t}^m \right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \tau_{i,t-1}^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistic is 30.41. Standard errors treat demand and production function estimates as data.

8.6.4 Including Potentially Vertically Integrated Plants

This section re-estimates the main regressions in the main text, while also including the potentially vertically integrated plants. The results differ very little from those presented in the main text.

Table 30: Effect of Chinese Import Competition on TFPR: OLS

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	TFPR	TFPR	TFPR	TFPR
Change in Chinese Imports	0.193	0.273	0.248	0.248
	(0.0826)	(0.0806)	(0.0836)	(0.0841)
Lagged TFPR level		-0.305	-0.304	-0.304
		(0.0159)	(0.0159)	(0.0159)
Change in World Input Tariffs			-0.0380	-0.0384
			(0.0248)	(0.0260)
Change in World Output Tariffs				0.00157
				(0.0151)
Observations	5,677	5,677	5,677	5,677

Notes: The above reports OLS regressions Δh_{it} on the described variables. All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in Chinese Imports measured as $\sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \ln \left(\mathrm{IM}^m_{\mathrm{IND,CHN},t} \right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \tau^m_t$, where τ^m_t is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Table 31: Effect of Chinese Import Competition on TFPR: IV

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	TFPR	TFPR	TFPR	TFPR
Change in Chinese Imports	1.266	1.226	1.167	1.143
	(0.940)	(0.906)	(0.691)	(0.669)
Lagged TFPR level		-0.311	-0.311	-0.311
		(0.0167)	(0.0164)	(0.0163)
Change in World Input Tariffs			0.0122	0.0134
			(0.0480)	(0.0491)
Change in World Output Tariffs				-0.0109
				(0.0178)
Observations	5,677	5,677	5,677	5,677

Standard errors clustered by plant

Notes: The above reports IV regressions of Δh_{it} on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \operatorname{IM}_{k,\text{CHN},t}^m \right) \right)$ instrumenting the change in Chinese imports. All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in Chinese imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\operatorname{IM}_{\text{IND},\text{CHN},t}^m \right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \tau_t^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistics are 18.38, 18.48, 29.97, and 32.36, for columns (1) through (4), respectively. Standard errors treat demand and production function estimates as data.

Table 32: First Stage

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	Chinese Imports	Chinese Imports	Chinese Imports	Chinese Imports
Chinese Import Instrument	0.165	0.165	0.212	0.219
	(0.0385)	(0.0384)	(0.0387)	(0.0384)
Observations	5,677	5,677	5,677	5,677

reports OLS regressions of the Change in Chinese Imports, $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\text{IM}_{\text{IND,CHN},t}^m \right),$ Z_{it}^{CHN} ${\bf Import}$ Chinese Instrument, $\sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t}^m \right) \right) .$ All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Column (2) also controls for lagged h_{it} levels, while World input and output tariffs, and column (4) adds controls for world output tariffs, respectively. World input and output tariffs measured as $\sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \tau^m_t$, where τ^m_t is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Table 33: Effect of Chinese Import Competition on TFPR: OLS

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	0.258	0.256	0.0105	-0.00744	-0.000867
	(0.0835)	(0.0846)	(0.0129)	(0.0188)	(0.0203)
Change in World Input Tariffs	-0.0368	-0.0380	-0.00349	0.00282	0.00183
	(0.0259)	(0.0264)	(0.00399)	(0.00579)	(0.00549)
Change in World Output Tariffs	0.00168	0.00323	-0.000844	0.00766	-0.00837
	(0.0151)	(0.0152)	(0.00187)	(0.00344)	(0.00358)
Observations	5,677	5,677	5,677	5,677	5,677

Standard errors clustered by plant

Notes: The above reports OLS regressions of the various elements of (41) on the described variables. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\overline{h}_{i,t-1}^{jg}$ $OP_{i,t-1}\left(\vec{S}_{i,t-1},\vec{h}_{i,t-1}^{g}\right)$, $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\omega}_{i,t-1}^{j|g}\right)$, and $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\eta}_{i,t-1}^{j|g}\right)$. Change in Chinese Imports measured as $\sum_{m\in\Lambda_{i,t-1}^m}\alpha_{i,t-1}^m\Delta\ln\left(\mathrm{IM}_{\mathrm{IND,CHN},t}^m\right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m\in\Lambda_{i,t-1}^m}\alpha_{i,t-1}^m\Delta\tau_{i,t-1}^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Table 34: Effect of Chinese Import Competition on TFPR: IV

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	1.152	1.130	-0.159	-0.163	0.344
	(0.670)	(0.671)	(0.0984)	(0.145)	(0.184)
Change in World Input Tariffs	0.0150	0.0127	-0.0133	-0.00623	0.0218
	(0.0492)	(0.0496)	(0.00684)	(0.0104)	(0.0118)
Change in World Output Tariffs	-0.0107	-0.00888	0.00150	0.00982	-0.0131
	(0.0178)	(0.0179)	(0.00240)	(0.00419)	(0.00482)
Observations	5,677	5,677	5,677	5,677	5,677

Notes: The above reports IV regressions of the various elements of (41) on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \operatorname{IM}_{k,\text{CHN},t}^m \right) \right)$ instrumenting the change in Chinese imports. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\overline{h}_{i,t-1}^{jg}$ $OP_{i,t-1}\left(\vec{S}_{i,t-1},\vec{h}_{i,t-1}^g\right)$, $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\omega}_{i,t-1}^{j}\right)$, and $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\eta}_{i,t-1}^{j}\right)$. Change in Chinese imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\operatorname{IM}_{\text{IND,CHN},t}^m\right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \tau_t^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistic is 30.41. Standard errors treat demand and production function estimates as data.

8.6.5 Alternative Measures of Imports: Simple Averages and Cost-Share Weighting

This section re-estimates the main regressions in the main text, while considering two different measures of import competition. In particular, rather than weighting imports by a plant's lagged three-digit ASICC code revenue share, I use the unweighted average of all imports across 3-digit ASICC products produced in the previous period, as well as lagged cost-share weights. More formally, for simple averages:

Change in Chinese Imports =
$$\ln \left(\frac{1}{J_{i,t-1}} \sum_{m \in \Lambda_{i,t-1}^m} \text{IM}_{\text{IND,CHN},t}^m \right) - \ln \left(\frac{1}{J_{i,t-1}} \sum_{m \in \Lambda_{i,t-1}^m} \text{IM}_{\text{IND,CHN},t,-1}^m \right)$$

$$Z_{it}^{\text{CHN}} = \ln \left(\frac{1}{J_{i,t-1}} \sum_{m \in \Lambda_{i,t-1}^m} \sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t}^m \right) - \ln \left(\frac{1}{J_{i,t-1}} \sum_{m \in \Lambda_{i,t-1}^m} \sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t-1}^m \right), \quad (72)$$

While for cost share weighting,

Change in Chinese Imports =
$$\sum_{m \in \Lambda_{i,t-1}^m} \gamma_{i,t-1}^m \Delta \ln \left(\text{IM}_{\text{IND,CHN},t}^m \right), \tag{73}$$

and,

$$Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i-t-1}^m} \left(\gamma_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t}^m \right) \right)$$
 (74)

Where
$$\gamma_{i,t-1}^m \equiv \frac{MC_{it}^{jg}Y_{it}^{jg}}{\sum_{k \in \mathbb{Y}_{it}} MC_{it}^{kg}Y_{it}^{kg}}$$

Where $\gamma_{i,t-1}^m \equiv \frac{MC_{it}^{jg}Y_{it}^{jg}}{\sum_{k \in \mathbb{Y}_{it}} MC_{it}^{kg}Y_{it}^{kg}}$. Note that simple averages produce somewhat smaller TFPR upgrading effects in the IV specification, while cost-share weights different very little from revenue weights.

Table 35: Effect of Chinese Import Competition on TFPR: OLS (Simple Averages)

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	TFPR	TFPR	TFPR	TFPR
Change in Chinese Imports	0.258	0.329	0.310	0.307
	(0.0828)	(0.0831)	(0.0862)	(0.0869)
Lagged TFPR level		-0.298	-0.298	-0.298
		(0.0164)	(0.0164)	(0.0164)
Change in World Input Tariffs			-0.0283	-0.0306
			(0.0258)	(0.0270)
Change in World Output Tariffs				0.0102
				(0.0159)
Observations	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports OLS regressions Δh_{it} on the described variables. All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in Chinese Imports measured according to (71). Change in world input and output tariffs measured as $\frac{1}{J_{i,t-1}}\sum_{m\in\Lambda^m_{i,t-1}}\tau^m_t-\frac{1}{J_{i,t-1}}\sum_{m\in\Lambda^m_{i,t-1}}\tau^m_{t-1}, \text{ where }\tau^m_t \text{ is the average value of input or output tariffs, respectively, by 3-digit ASICC code }m. Standard errorstreat$ demand and production function estimates as data.

Table 36: Effect of Chinese Import Competition on TFPR: IV (Simple Averages)

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	TFPR	TFPR	TFPR	TFPR
Change in Chinese Imports	1.165	1.254	1.162	1.153
	(1.020)	(1.003)	(0.745)	(0.717)
Lagged TFPR level		-0.304	-0.303	-0.303
		(0.0171)	(0.0168)	(0.0168)
Change in World Input Tariffs			0.0164	0.0168
			(0.0496)	(0.0507)
Change in World Output Tariffs				-0.00408
				(0.0199)
Observations	5,236	5,236	5,236	5,236

Notes: The above reports IV regressions of Δh_{it} on the described variables, with the expression for Z_{it}^{CHN} given by (72) instrumenting the change in Chinese imports. All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in Chinese Imports measured according to (71). Change in world input and output tariffs measured as $\frac{1}{l_{i,t-1}}\sum_{m\in\Lambda_{i,t-1}^m} \tau_t^m - \frac{1}{l_{i,t-1}}\sum_{m\in\Lambda_{i,t-1}^m} \tau_{t-1}^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistics are 16.70, 16.70, 28.12, and 31.16, for columns (1) through (4), respectively. Standard errors treat demand and production function estimates as data.

Table 37: First Stage (Simple Averages)

	(1)		(3)	(4)	
	Change in	Change in	Change in	Change in	
VARIABLES	Chinese Imports	Chinese Imports	Chinese Imports	Chinese Imports	
Chinese Import Instrument	0.162 (0.0396)	0.162 (0.0395)	0.212 (0.0400)	0.221 (0.0396)	
Observations	5,236	5,236	5,236	5,236	

Standard errors clustered by plant

Notes: The above reports OLS regressions of the Change in Chinese Imports, given by (71), on the Chinese Import Instrument given by (72). All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Column (2) also controls for lagged h_{it} levels, while column (3) adds changes in world input tariffs, and column (4) adds controls for world output tariffs, respectively. World input and output tariffs measured as $\frac{1}{J_{i,t-1}}\sum_{m\in\Lambda^m_{i,t-1}}\tau^m_{t-1}$, where τ^m_t is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Table 38: Effect of Chinese Import Competition on TFPR: OLS (Simple Averages)

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	0.312	0.313	-0.00688	-0.00530	0.0110
	(0.0870)	(0.0874)	(0.00929)	(0.0125)	(0.0135)
Change in World Input Tariffs	-0.0302	-0.0293	-0.00216	-0.00160	0.00294
	(0.0269)	(0.0273)	(0.00349)	(0.00467)	(0.00435)
Change in World Output Tariffs	0.00985	0.0103	-0.000266	0.00575	-0.00597
	(0.0159)	(0.0160)	(0.00162)	(0.00266)	(0.00283)
Observations	5,236	5,236	5,236	5,236	5,236

Notes: The above reports OLS regressions of the various elements of (41) on the described variables. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\bar{h}_{i,t-1}^{jg}$ $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \bar{h}_{i,t-1}^g\right)$, $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \widetilde{\omega}_{i,t-1}^{j|g}\right)$, and $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \widetilde{\eta}_{i,t-1}^{j|g}\right)$. Change in Chinese Imports measured by (71). Change in world input and output tariffs measured as $\frac{1}{J_{i,t-1}}\sum_{m\in\Lambda_{i,t-1}^m}\tau_{t}^m-\frac{1}{J_{i,t-1}}\sum_{m\in\Lambda_{i,t-1}^m}\tau_{t-1}^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Table 39: Effect of Chinese Import Competition on TFPR: IV (Simple Averages)

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	1.189	1.201	-0.0830	-0.130	0.202
	(0.722)	(0.725)	(0.0667)	(0.108)	(0.114)
Change in World Input Tariffs	0.0189	0.0203	-0.00641	-0.00858	0.0136
	(0.0509)	(0.0512)	(0.00517)	(0.00795)	(0.00759)
Change in World Output Tariffs	-0.00497	-0.00466	0.00102	0.00785	-0.00919
	(0.0200)	(0.0201)	(0.00203)	(0.00343)	(0.00367)
Observations	5,236	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports IV regressions of the various elements of (41) on the described variables, with Z_{it}^{CHN} , given by (72), instrumenting the change in Chinese imports, given by (71). All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\overline{h}_{i,t-1}^{jg}$, $OP_{i,t-1}\left(\vec{S}_{i,t-1},\vec{h}_{i,t-1}^g\right)$, and $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\eta}_{i,t-1}^{j|\vec{g}}\right)$. Change in world input and output tariffs measured as $\frac{1}{J_{i,t-1}}\sum_{m\in\Lambda_{i,t-1}^m}\tau_t^m-\frac{1}{J_{i,t-1}}\sum_{m\in\Lambda_{i,t-1}^m}\tau_{t-1}^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistic is 30.93. Standard errors treat demand and production function estimates as data.

Table 40: Effect of Chinese Import Competition on TFPR: OLS (Cost Share Weights)

(1)	(2)	(3)	(4)
Change in	Change in	Change in	Change in
TFPR	TFPR	TFPR	TFPR
0.264	0.333	0.313	0.310
(0.0808)	(0.0813)	(0.0843)	(0.0849)
	-0.298	-0.298	-0.298
	(0.0164)	(0.0164)	(0.0164)
		-0.0296	-0.0313
		(0.0258)	(0.0269)
			0.00772
			(0.0158)
5 236	5 236	5 236	5,236
	Change in TFPR 0.264	Change in TFPR Change in TFPR 0.264 0.333 (0.0808) (0.0813) -0.298 (0.0164)	Change in TFPR Change in TFPR Change in TFPR 0.264 0.333 0.313 (0.0808) (0.0813) (0.0843) -0.298 -0.298 (0.0164) (0.0164) -0.0296 (0.0258)

Notes: The above reports OLS regressions Δh_{it} on the described variables. All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in Chinese Imports measured according to (73). Change in world input and output tariffs measured as Change in world input and output tariffs measured as $\sum_{m \in \Lambda^m_{i,t-1}} \gamma^m_{i,t-1} \Delta \tau^m_t$, where $\gamma^m_{i,t-1}$ is the lagged output times marginal cost share of 3-digit ASICC code m in firm i, and τ^m_t is the average value of input or output tariffs, respectively, by 3-digit ASICC code m, where τ^m_t is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errorstreat demand and production function estimates as data.

Table 41: Effect of Chinese Import Competition on TFPR: IV (Cost Share Weights)

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	TFPR	TFPR	TFPR	TFPR
Change in Chinese Imports	1.280	1.502	1.361	1.340
	(0.998)	(0.992)	(0.739)	(0.710)
Lagged TFPR level		-0.305	-0.305	-0.304
		(0.0172)	(0.0169)	(0.0168)
Change in World Input Tariffs			0.0268	0.0275
			(0.0503)	(0.0512)
Change in World Output Tariffs				-0.00862
				(0.0196)
Observations	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports IV regressions of Δh_{it} on the described variables, with the expression for Z_{it}^{CHN} given by (74) instrumenting the change in Chinese imports, given by (73). All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \gamma_{i,t-1}^m \Delta \tau_t^m$, where $\gamma_{i,t-1}^m$ is the lagged output times marginal cost share of 3-digit ASICC code m in firm i, and τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistics are 16.65, 16.56, 27.56, and 30.38, for columns (1) through (4), respectively. Standard errors treat demand and production function estimates as data.

Table 42: First Stage (Cost Share Weights)

	(1)	(2)	(3)	(4)	
	Change in	Change in	Change in	Change in	
VARIABLES	Chinese Imports	Chinese Imports	Chinese Imports	Chinese Imports	
Chinese Import Instrument	0.162 (0.0398)	0.162 (0.0398)	0.211 (0.0402)	0.220 (0.0398)	
Observations	5,236	5,236	5,236	5,236	

Notes: The above reports OLS regressions of the Change in Chinese Imports, given by (73), on the Chinese Import Instrument given by (74). All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Column (2) also controls for lagged h_{it} levels, while column (3) adds changes in world input tariffs, and column (4) adds controls for world output tariffs, respectively. World input and output tariffs measured as $\sum_{m \in \Lambda^m_{i,t-1}} \gamma^m_{i,t-1} \Delta \tau^m_t$, where $\gamma^m_{i,t-1}$ is the lagged output times marginal cost share of 3-digit ASICC code m in firm i, and τ^m_t is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Table 43: Effect of Chinese Import Competition on TFPR: OLS (Cost Share Weights)

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Chinese Imports	0.317	0.319	0.0127	-0.0206	0.00540
	(0.0853)	(0.0857)	(0.0107)	(0.0163)	(0.0177)
Input Tariffs	-0.0304	-0.0297	-0.00272	-0.000622	0.00267
	(0.0269)	(0.0273)	(0.00362)	(0.00508)	(0.00463)
World Output Tariffs	0.00725	0.00829	-0.000705	0.00620	-0.00654
	(0.0158)	(0.0159)	(0.00167)	(0.00278)	(0.00301)
Observations	5,236	5,236	5,236	5,236	5.236

Standard errors clustered by plant

Notes: The above reports OLS regressions of the various elements of (41) on the described variables. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\overline{h}_{i,t-1}^{jg}$ $OP_{i,t-1}\left(\vec{S}_{i,t-1},\vec{h}_{i,t-1}^{g}\right)$, $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\omega}_{i,t-1}^{j|g}\right)$, and $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\eta}_{i,t-1}^{j|g}\right)$. Change in Chinese Imports measured by (73). Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \gamma_{i,t-1}^m \Delta \tau_t^m$, where $\gamma_{i,t-1}^m$ is the lagged output times marginal cost share of 3-digit ASICC code m in firm i, and τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Table 44: Effect of Chinese Import Competition on TFPR: IV (Cost Share Weights)

	(1)	(2)	(3)	(4)	(5)
	Full Plant	Average	Specialization in	Specialization in	Specialization in
	TFPR	TFPR	High TFPR	Cost Comparative	Demand Comparative
VARIABLES	Growth	Growth	Product Codes	Advantage	Advantage
Change in Chinese Imports	1.381	1.410	-0.155	-0.242	0.368
	(0.715)	(0.720)	(0.0952)	(0.135)	(0.182)
Change in World Input Tariffs	0.0305	0.0326	-0.0123	-0.0133	0.0234
	(0.0514)	(0.0519)	(0.00647)	(0.00932)	(0.0112)
Change in World Output Tariffs	-0.00971	-0.00907	0.00197	0.00972	-0.0123
	(0.0197)	(0.0199)	(0.00245)	(0.00383)	(0.00484)
Observations	5,236	5,236	5,236	5,236	5,236

Notes: The above reports IV regressions of the various elements of (41) on the described variables, with Z_{it}^{CHN} , given by (74), instrumenting the change in Chinese imports, given by (73). All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for $\vec{h}_{i,t-1}^{jg}$, $OP_{i,t-1}\left(\vec{S}_{i,t-1}, \vec{h}_{i,t-1}^{g}\right)$,

 $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\omega}_{i,t-1}^{j\mid\vec{g}}\right)$, and $OP_{i,t-1}\left(\vec{S}_{i,t-1},\widetilde{\eta}_{i,t-1}^{j\mid\vec{g}}\right)$. Change in world input and output tariffs measured as $\sum_{m\in\Lambda_{i,t-1}^m}\gamma_{i,t-1}^m\gamma_{i,t-1}^m\Delta\tau_t^m$, where $\gamma_{i,t-1}^m$ is the lagged output times marginal cost share of 3-digit ASICC code m in firm i, and τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistic is 30.22. Standard errors treat demand and production function estimates as data.

8.7 Appendix G: Results Using Chinese Imports by 5-Digit ASICC Code

In this section, I briefly discuss the issues for my instrumental variables strategy when measuring Chinese Import Competition at the 5-digit level. Tables 45, 46, and 47, below reproduce the main TFPR regressions used in the main text, except with import competition measured at the 5-digit level. ¹⁰¹ The core issue, as should be clear from the first-stage results in Table 47, is that my instrument for Chinese import competition at this level, i.e. changes in Chinese exports to other low-to-middle income countries, do not positively predict changes in Chinese imports in India. In fact, at this level, Chinese exports appear to be negatively predicting imports. Since this is exactly the opposite sort of correlation one would expect to pick up if this instrument were truly capturing Chinese export growth due to China specific factors, while also generating a first stage well F-statistic well under 10, this approach is unlikely to deliver consistent estimates of the impact of Chinese import competition on productivity. This is likely due to some measurement error in the year-to-year imports, when measured at the 5-digit level. ¹⁰² Note, however, that the OLS results still imply that Chinese import competition has a positive effect on plant-level TFPR, although one that is much smaller in magnitude. This is also consistent with attenuation bias when measuring imports at the 5-digit level, which is solved by aggregating imports

¹⁰¹This is also true for the tariff controls.

¹⁰²Note that I obtain the same negative "first-stage", when I only examine the imports by 5-digit products without any plant-level revenue weighting, i.e. China export growth from other low to middle income countries appears to negatively predict growth in Chinese imports to India. However, this problem appears to be confined to examing growth through first differences- one obtains the expected positive sign if, instead of using first-differences, one looks at imports in levels and includes product code fixed effects (results available upon request). I take this as evidence of some measurement error in the year-to-year differences in imports.

of to the 3-digit level. In particular, since the concordance between 4-digit HS imports and 5-digit ASICC codes involved a number of imperfect matches (See Appendix B), imports at this level is note precisely measured. On the other hand, aggregating to the 3-digit level likely removes the influence of the imperfect matching at the more disaggregated level.

Table 45: Effect of Chinese Import Competition on TFPR: OLS

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	TFPR	TFPR	TFPR	TFPR
Change in Chinese Imports	0.0602	0.0547	0.0521	0.0530
	(0.0188)	(0.0180)	(0.0183)	(0.0183)
Lagged TFPR level		-0.296	-0.296	-0.296
		(0.0164)	(0.0164)	(0.0164)
Change in World Input Tariffs			-0.0262	-0.0309
			(0.0202)	(0.0205)
Change in World Output Tariffs				0.0214
				(0.00931)
Observations	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports OLS regressions Δh_{it} on the described variables. All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, and ownership structure. Change in Chinese Imports measured as $\sum_{g \in \Lambda^g_{i,t-1}} \alpha^g_{i,t-1} \Delta \ln \left(\mathrm{IM}^g_{\mathrm{IND,CHN},t} \right)$, where $\alpha^g_{i,t-1}$ is the revenue share of 5-digit ASICC code g in plant i at time t-1, and $\mathrm{IM}^g_{\mathrm{IND,CHN},t}$ is total imports from China to India by 5-digit product code g. Change in world input and output tariffs measured as $\sum_{m \in \Lambda^g_{i,t-1}} \alpha^g_{i,t-1} \Delta \tau^g_t$, where τ^g_t is the average value of input or output tariffs, respectively, by 5-digit ASICC code g. Standard errors treat demand and production function estimates as data.

Table 46: Effect of Chinese Import Competition on TFPR: IV

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	TFPR	TFPR	TFPR	TFPR
Change in Chinese Imports	-0.384	-0.510	-0.537	-0.508
	(0.358)	(0.374)	(0.389)	(0.387)
Lagged TFPR level		-0.299	-0.298	-0.299
		(0.0172)	(0.0173)	(0.0172)
Change in World Input Tariffs			-0.0612	-0.0633
			(0.0302)	(0.0297)
Change in World Output Tariffs				0.0172
				(0.0103)
Observations	5,236	5,236	5,236	5,236

Notes: The above reports IV regressions of Δh_{it} on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{g \in \Lambda_{i,t-1}^g} \left(\alpha_{i,t-1}^g \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \operatorname{IM}_{k,\text{CHN},t}^g\right)\right)$ instrumenting the change in Chinese imports, where $\alpha_{i,t-1}^g$ is the revenue share of 5-digit ASICC code g in plant i at time t-1, and $\operatorname{IM}_{k,\text{CHN},t}^g$ is total imports from China to country k by 5-digit product code g. All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in Chinese imports measured as $\sum_{g \in \Lambda_{i,t-1}^g} \alpha_{i,t-1}^g \Delta \ln \left(\operatorname{IM}_{\text{IND,CHN},t}^g\right)$. Change in world input and output tariffs measured as $\sum_{g \in \Lambda_{i,t-1}^g} \alpha_{i,t-1}^g \Delta \tau_t^g$, where τ_t^g is the average value of input or output tariffs, respectively, by 5-digit ASICC code g. First-Stage F-statistics are 5.06, 5.04, 4.81, and 4.73, for columns (1) through (4), respectively. Standard errors treat demand and production function estimates as data.

Table 47: First Stage

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	Chinese Imports	Chinese Imports	Chinese Imports	Chinese Imports
Chinese Import Instrument	-0.159	-0.159	-0.155	-0.153
	(0.0708)	(0.0709)	(0.0707)	(0.0706)
Observations	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports OLS regressions of the Change in Chinese Imports, $\sum_{g \in \Lambda^g_{i,t-1}} \alpha^g_{i,t-1} \Delta \ln \left(\mathrm{IM}^g_{\mathrm{IND,CHN},t} \right), \text{ on the Chinese Import Instrument, } Z^{\mathrm{CHN}}_{it} = \sum_{g \in \Lambda^g_{i,t-1}} \left(\alpha^g_{i,t-1} \Delta \ln \left(\sum_{k \in \mathbb{C}^{\mathrm{No \ India}}} \mathrm{IM}^g_{k,\mathrm{CHN},t} \right) \right), \text{ where } \alpha^g_{i,t-1} \text{ is the revenue share of 5-digit ASICC code } g \text{ in plant } i \text{ at time } t-1, \text{ and } \mathrm{IM}^g_{k,\mathrm{CHN},t} \text{ is total imports from China to country } k \text{ by 5-digit product code } g. \text{ All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Column (2) also controls for lagged <math>h_{it}$ levels, while column (3) adds changes in world input tariffs, and column (4) adds controls for world output tariffs, respectively. World input and output tariffs measured as $\sum_{g \in \Lambda^g_{i,t-1}} \alpha^g_{i,t-1} \Delta \tau^g_t$, where τ^g_t is the average value of input or output tariffs, respectively, by 5-digit ASICC code m. Standard errors treat demand and production function estimates as data.

8.8 Appendix H: The Transformation Function and Optimal Input Allocations

In this section, I compare the approach for estimating productivity in multi-product firms developed in this paper, to the transformation function approach, as in Diewert (1973) and more recently Dhyne et al. (2017). In particular, I show that that Assumptions 1 through 6 imply a simple functional form for the transformation function. However, this function depends on $J_{it} = |\mathbb{Y}_{it}|$ unobservable plant-product specific TFP terms. Therefore, without more restrictions, the transformation function cannot feasibly be estimated. I then show that the approach used in this paper can be characterized as a way to solve to "too many unobservables" problem inherent to the structure of the transformation function.

As per Appendix A, one may define the transformation function as the maximal level of output that can be obtained for some reference good $r \in \mathbb{Y}_{it}$, given aggregate input vector \vec{X}_{it} , a vector of firm-product specific TFP terms, $\vec{\omega}_{it}$, and some vector or other outputs \vec{Y}_{it}^{-r} , i.e. $Y_{it}^r = \tilde{T}(\vec{Y}_{it}^{-r}, \vec{X}_{it}, \vec{\omega}_{it})$. It turns out that it is very simple to solve for an equivalent representation of the technology, the symmetric for the transformation function (Diewert 1973), given by $Y_{it}^r - \tilde{T}(\vec{Y}_{it}^{-r}, \vec{X}_{it}, \vec{\omega}_{it}) \equiv T(\vec{Y}_{it}, \vec{X}_{it}, \vec{\omega}_{it}) = 0$. To obtain the symmetric transformation function, note that by Lemma 1, as long as the input allocation problem has a unique solution, then input ratios are constant across production lines, and therefore $X_{it}^j = S_{it}^j X_{it}$ for all $X \in (\mathbb{K}, \mathbb{M})$. Substituting this equation into the production function, and using the fact that the production technology is homogenous of degree $\phi > 0$ yields:

$$Y_{it}^j = \exp(\omega_{it}^j) (S_{it}^j)^{\phi} F(\vec{X}_{it}).$$

Rearranging the above yields:

$$\left(\frac{Y_{it}^j}{\exp(\omega_{it}^j)}\right)^{\frac{1}{\phi}} = S_{it}^j \left(F(\vec{X}_{it})\right)^{\frac{1}{\phi}}.$$

Finally, summing over all $j \in \mathbb{Y}_{it}$ and then taking this expression to the power of ϕ yields:

$$\left(\sum_{j\in\mathbb{Y}_{it}} \left(\frac{Y_{it}^j}{\exp(\omega_{it}^j)}\right)^{\frac{1}{\phi}}\right)^{\phi} = F(\vec{X}_{it}). \tag{75}$$

Note that (75) is a simple representation of the symmetric transformation function, since we can also use (75) to write $T(\vec{Y}_{it}, \vec{X}_{it}, \vec{\omega}_{it}) = \left(\sum_{j \in \mathbb{Y}_{it}} \left(\frac{Y_{it}^j}{\exp(\omega_{it}^j)}\right)^{\frac{1}{\phi}}\right)^{\phi} - F(\vec{X}_{it}) = 0.$

Note that the symmetric transformation function here is seperable, in the sense defined by Hall (1973), i.e. we are able to write the transformation function as the sum of two functions, one of which depends on outputs, and another that depends on inputs, $T(\vec{Y}_{it}, \vec{X}_{it}, \vec{\omega}_{it}) = A_{it}(\vec{Y}_{it}) - B(\vec{X}_{it})$. It is worth noting in passing that one of the key findings in Hall (1973), is that that separable transformation functions are almost always the result joint technologies, which are multi-product production processes that cannot be written as a series of independent production technologies. Since the production process considered in this paper is inherently non-joint, as per Hall (1973), the fact that I obtain a seperable transformation

function may appear surprising. Note, however, that the only case where a a non-joint production process leads to a separable transformation function is if the production technology across production lines within a firm only differs by a scalar multiple. This is exactly equivalent to my Assumption 2, which is why the transformation function is separable.

The key challenge for identifying the shape of transformation function empirically is that (75) has $J_{it} \equiv |\mathbb{Y}_{it}|$ unobservables. As a result, estimation via GMM is immediately unavailable, since it is impossible to simply use (75) to solve for each value of ω_{it}^j . There are, of course, restrictions that one could place on the variation of TFP that could lead to identification. The easiest of these is to, for example, follow De Loecker et al. (2016) and assume $\omega_{it}^j = \omega_{it} \ \forall j \in \mathbb{Y}_{it}$. This leads to the simple transformation function:

$$\left(\sum_{j\in\mathbb{Y}_{it}} \left(Y_{it}^{j}\right)^{\frac{1}{\phi}}\right)^{\phi} = \exp(\omega_{it})F(\vec{X}_{it}). \tag{76}$$

The above simplified transformation function could immediately be used to identify firm-level TFP using standard non-linear GMM techniques, and would have the added advantage of allowing one use to multi-product firms directly in estimation, rather than relying on a selected sample of single product firms to estimate the production technology. Another approach would be to put some parametric structure on the distribution of the unobservable ω_{it}^j terms, and then using maximum likelihood for estimation. This would have the cost of making endogeneity difficult to deal with, as one would also have to model the correlation between ω_{it}^j and \vec{Y}_{it} .

Another approach, which is equivalent to the approach described in this paper, is to use price variation within the firm to identify within-firm TFP dispersion, and then combine that with (75). Specifically, rewrite (75) as:

$$\left(\sum_{j \in \mathbb{Y}_{it}} \left(Y_{it}^{j}\right)^{\frac{1}{\phi}} \left(\frac{\exp(\omega_{it}^{r})}{\exp(\omega_{it}^{j})}\right)^{\frac{1}{\phi}}\right)^{\phi} = \exp(\omega_{it}^{r}) F(\vec{X}_{it}), \tag{77}$$

where ω_{it}^r is productivity of some reference good $r \in \mathbb{Y}_{it}$. To make estimation of (77) feasible, we need to identify relative TFP within the firm, $\frac{\exp(\omega_{it}^r)}{\exp(\omega_{it}^r)}$. To do this, first consider the firm's conditional cost minimization problem after having solved for the optimal input allocations given \vec{X}_{it} , as described in Appendix A. Rewriting their conditional cost minimization problem in terms of the symmetric transformation function yields:¹⁰³

$$\underset{\vec{M}_{it}}{\text{Min}} \sum_{M \in \mathbb{M}} \mathbf{W}^{M} \left(M_{it}, A_{it}^{M} \right) M_{it}$$
subject to
$$\left(\sum_{j \in \mathbb{Y}_{it}} \left(\frac{Y_{it}^{j}}{\exp(\omega_{it}^{j})} \right)^{\frac{1}{\phi}} \right)^{\phi} = F(\vec{X}_{it}), \tag{78}$$

¹⁰³Recall that the conditional cost minimization problem takes the level of dynamic inputs, such as capital, as given, and as such the level of dynamic input costs are taken as given, and are therefore suppressed in the formulation of costs.

with associated Lagrangian:

$$L = \sum_{M \in \mathbb{M}} \mathbf{W}^{M} \left(M_{it}, A_{it}^{M} \right) M_{it} + \lambda \left(\left(\sum_{j \in \mathbb{Y}_{it}} \left(\frac{Y_{it}^{j}}{\exp(\omega_{it}^{j})} \right)^{\frac{1}{\phi}} \right)^{\phi} - F(\vec{X}_{it}) \right).$$
 (79)

Letting $C\left(\vec{K}_{it}, \vec{Y}_{it}, \vec{\omega}_{it}, \vec{A}_{it}\right)$ denote the conditional cost function for static inputs, as before, then then by the envelope theorem:

$$MC_{it}^{j} = \frac{\partial C}{\partial Y_{it}^{j}} = \lambda \left(\sum_{j \in \mathbb{Y}_{it}} \left(\frac{Y_{it}^{j}}{\exp(\omega_{it}^{j})} \right)^{\frac{1}{\phi}} \right)^{\phi - 1} \left(\frac{Y_{it}^{j}}{\exp(\omega_{it}^{j})} \right)^{\frac{1}{\phi}} \frac{1}{Y_{it}^{j}}. \tag{80}$$

Using the above expression to examine marginal cost ratios between an arbitrary good j and the reference good r yields $\frac{MC_{it}^{j}}{MC_{it}^{r}}$ then yields:

$$\frac{MC_{it}^{j}}{MC_{it}^{r}} = \left(\frac{\frac{Y_{it}^{j}}{\exp(\omega_{it}^{j})}}{\frac{Y_{it}^{r}}{\exp(\omega_{it}^{r})}}\right)^{\frac{1}{\phi}} \frac{Y_{it}^{r}}{Y_{it}^{j}}.$$
(81)

Solving for $\left(\frac{\exp(\omega_{it}^r)}{\exp(\omega_{it}^j)}\right)^{\frac{1}{\phi}}$ in this expression yields:

$$\left(\frac{\exp(\omega_{it}^r)}{\exp(\omega_{it}^j)}\right)^{\frac{1}{\phi}} = \frac{Y_{it}^j M C_{it}^j}{Y_{it}^r M C_{it}^r} \left(\frac{Y_{it}^r}{Y_{it}^j}\right)^{\frac{1}{\phi}}.$$
(82)

Substituting (82) back into the transformation function (77) yields:

$$\frac{Y_{it}^r}{(Y_{it}^r M C_{it}^r)^{\phi}} \left(\sum_{j \in \mathbb{Y}_{it}} Y_{it}^j M C_{it}^j \right)^{\phi} = \exp(\omega_{it}^r) F(\vec{X}_{it}), \tag{83}$$

or, after re-arranging and using the fact that the production function is homogeneous of degree $\phi > 0$

$$Y_{it}^{r} = \left(\frac{Y_{it}^{r}MC_{it}^{r}}{\sum_{j \in \mathbb{Y}_{it}} Y_{it}^{j}MC_{it}^{j}}\right)^{\phi} \exp(\omega_{it}^{r})F(\vec{X}_{it}) = \exp(\omega_{it}^{r})F\left(\frac{Y_{it}^{r}MC_{it}^{r}}{\sum_{j \in \mathbb{Y}_{it}} Y_{it}^{j}MC_{it}^{j}}\vec{X}_{it}\right). \tag{84}$$

Since marginal costs can be estimated from price and quantity data, there is only one unobservable in (84), the firm-product level TFP term, and therefore standard GMM techniques can be used. More important, note that (84) equivalent to the firm-product level production function equation after using the input allocation rule (9) to estimate input shares. As a result, the approach used in this paper to estimate productivity in multiproduct firms can also be characterized as using pricing (and therefore marginal cost information) to pin down relative TFP within a firm, to make estimation of the transformation function feasible.

8.9 Appendix I: Alternative Measures of Plant-level TFPR

Consider the following definition of plant-level log TFPR:

$$h_{it}^{ALT} \equiv \ln \left(\sum_{j \in \mathbb{Y}_{it}} P_{it}^{jg} Q_{it}^{jg} \right) - \frac{\frac{\alpha}{\sigma}}{1 + \frac{\alpha}{\sigma}} \ln \left(F(X_{it}) \right)$$
 (85)

Exponentiate (32) and then substitute this expression for revenues into the above, yielding:

$$h_{it}^{ALT} = \ln \left(\sum_{j \in \mathbb{Y}_{it}} \left(F(\vec{X}_{it}^{jg}) \right)^{\frac{\alpha}{1+\frac{\alpha}{\sigma}}} \exp\left(h_{it}^{jg} \right) \right) - \frac{\frac{\alpha}{\sigma}}{1+\frac{\alpha}{\sigma}} \ln\left(F(X_{it}) \right)$$

Substituting $X_{it}^{jg} = S_{it}^{jg} X_{it} \quad \forall X$ into the above, and then using the fact that F(.) is homogenous of degree $\phi > 0$, yields:

$$h_{it}^{ALT} = \ln \left(\sum_{j \in \mathbb{Y}_{it}} \left(S_{it}^{jg} \right)^{\frac{\phi \frac{\alpha}{\sigma}}{1 + \frac{\alpha}{\sigma}}} \left(F(X_{it}) \right)^{\frac{\alpha}{1 + \frac{\alpha}{\sigma}}} \exp \left(h_{it}^{jg} \right) \right) - \frac{\frac{\alpha}{\sigma}}{1 + \frac{\alpha}{\sigma}} \ln \left(F(X_{it}) \right)$$

$$= \ln \left(\sum_{j \in \mathbb{Y}_{it}} \left(S_{it}^{jg} \right)^{\frac{\phi \frac{\alpha}{\sigma}}{1 + \frac{\alpha}{\sigma}}} \exp \left(h_{it}^{jg} \right) \right) + \frac{\frac{\alpha}{\sigma}}{1 + \frac{\alpha}{\sigma}} \ln \left(F(X_{it}) \right) - \frac{\frac{\alpha}{\sigma}}{1 + \frac{\alpha}{\sigma}} \ln \left(F(X_{it}) \right)$$

$$= \ln \left(\sum_{j \in \mathbb{Y}_{it}} \left(S_{it}^{jg} \right)^{\frac{\phi \frac{\alpha}{\sigma}}{1 + \frac{\alpha}{\sigma}}} \exp \left(h_{it}^{jg} \right) \right) \right)$$

$$(86)$$

Note that (86) has a very similar structure to the notion of log plant-level TFPR, (36), defined in the main text. In particular, (86) is also a input-share weighted sum of variety specific TFPR, although in this expression input shares are weighted by the revenue production function returns to scale $\frac{\phi_{\sigma}^{\alpha}}{1+\frac{\alpha}{\sigma}}$, and the variety specific TFPR terms that are multiplied by the shares are in levels, rather than in logs.

In practice, the core difference between (86) and (36) is the way in which they weight variation in within-plant specialization. To see this, decompose each h_{it}^{jg} term into the sum of their plant-level average, $\overline{h}_{it} \equiv \frac{1}{J_{it}} \sum_{j \in \mathbb{Y}_{it}} h_{it}^{jg}$ and the deviation from that average, $\widehat{h}_{it}^{jg} \equiv h_{it}^{jg} - \overline{h}_{it}$ i.e.:

$$h_{it}^{ALT} = \ln \left(\sum_{j \in \mathbb{Y}_{it}} \left(S_{it}^{jg} \right)^{\frac{\phi \frac{\alpha}{\sigma}}{1 + \frac{\alpha}{\sigma}}} \exp \left(\overline{h}_{it} + \widehat{h}_{it}^{jg} \right) \right)$$

$$= \underbrace{\overline{h}_{it}}_{\text{Mean TFPR}} + \ln \left(\sum_{j \in \mathbb{Y}_{it}} \left(S_{it}^{jg} \right)^{\frac{\phi \frac{\alpha}{\sigma}}{\sigma}} \exp \left(\widehat{h}_{it}^{jg} \right) \right)$$
Alternative Specialization (87)

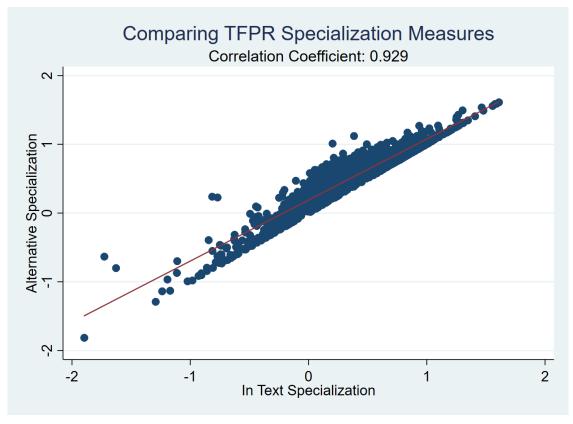
One can then clearly see that h_{it}^{ALT} and h_{it} as defined in the main text only differ in their treatment of within-plant specialization by taking the difference between (86) and (37), which yields after some simplification:

$$h_{it}^{ALT} - h_{it} = \underbrace{\ln\left(\sum_{j \in \mathbb{Y}_{it}} \left(S_{it}^{jg}\right)^{\frac{\phi \frac{\alpha}{\sigma}}{1 + \frac{\alpha}{\sigma}}} \exp\left(\widehat{h}_{it}^{jg}\right)\right)}_{\text{Alternative Specialization}} - \underbrace{\sum_{j \in \mathbb{Y}_{it}} S_{it}^{jg} \widehat{h}_{it}^{jg}}_{\text{In Text Specialization}}$$
(88)

since the mean TFPR terms are common to both measures of log TFPR.

Figure 4, below, shows the scatter plot and correlation between between the in text specialization measure and the appendix measure. Clearly these two measure of specialization are very strongly correlated, taking a correlation coefficient of 0.929. As a result, in practice the in text measure of plant-level TFPR appears to vary very little from the more "standard" approach to measuring TFPR described in this appendix. Tables 48 to 52, below, further underscore this point, where I repeat much of the same analysis as in the main text for this alternative measure of TFPR. Specifically, in Table 48, I decompose the variation in h_{it}^{ALT} into that explained by mean TFPR variation, and specialization, as in the main text. Note, however, that since this version of TFPR is no longer log-linear in the specialization term, I can no longer further decompose this term into the specialization in cost and demand comparative advantage discussed in the main text. On the other hand, note that this measure of plant-level TFPR also appears to primarily be driven by average TFPR variation, rather than specialization effects. 49 through 52 repeat the regression analysis in the main text where I examine Chinese import competitions effect on plant-level TFPR growth, replacing Δh_{it} with Δh_{it}^{ALT} . The results here vary little with those in the main text- in particular, Chinese import competition appears to increase plant-level TFPR, but primarily through mean TFPR increases, rather than within-plant specialization effects.

Figure 4: Correlation Between In Text (Eq. 37) and Alternative (Eq. 86) Specialization Terms



Notes:

Table 48: Plant-Level TFPR Variance Decompositions

	\overline{h}_{it}	$\ln\left(\sum_{j\in\mathbb{Y}_{it}} \left(S_{it}^{jg}\right)^{\frac{\phi\frac{\alpha}{\sigma}}{1+\frac{\alpha}{\sigma}}} \exp\left(\widehat{h}_{it}^{jg}\right)\right)$			
A: All Plants					
h_{it}^{ALT}	0.996	0.004			
	(0.001)	(0.001)			
B: Multi-product Plants	;				
h_{it}^{ALT}	1.020	-0.020			
	(0.003)	(0.003)			
Notes: The above reports OLS regressions of \overline{h}_{it} and $\ln \left(\sum_{j \in \mathbb{Y}_{it}} \left(S_{it}^{jg} \right)^{\frac{\phi \frac{\alpha}{\sigma}}{1 + \frac{\alpha}{\sigma}}} \exp \left(\widehat{h}_{it}^{jg} \right) \right)$ on h_{it}^{ALT} , as					
defined in (86). Panel A has 19,261 plant-year observations. Panel B has plant-year 3,852 observations. Standard errors, which treat demand and production function estimates as data and are clustered by plant,					

reported in parentheses.

Table 49: Effect of Chinese Import Competition on TFPR: OLS

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	TFPR	TFPR	TFPR	TFPR
Change in Chinese Imports	0.253	0.322	0.301	0.298
	(0.0807)	(0.0811)	(0.0843)	(0.0849)
Lagged TFPR level		-0.298	-0.298	-0.298
		(0.0164)	(0.0164)	(0.0164)
Change in World Input Tariffs			-0.0313	-0.0332
			(0.0258)	(0.0269)
Change in World Output Tariffs				0.00834
				(0.0158)
Observations	5,236	5,236	5,236	5,236

Notes: The above reports OLS regressions Δh_{it}^{ALT} , given by (86), on the described variables. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in Chinese Imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\mathrm{IM}_{\mathrm{IND,CHN},t}^m \right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \tau_t^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Table 50: Effect of Chinese Import Competition on TFPR: IV

	(1)	(2)	(3)	(4)
	Change in	Change in	Change in	Change in
VARIABLES	TFPR	TFPR	TFPR	TFPR
Change in Chinese Imports	1.272	1.460	1.331	1.312
	(0.986)	(0.979)	(0.732)	(0.703)
Lagged TFPR level		-0.304	-0.303	-0.303
		(0.0171)	(0.0168)	(0.0168)
Change in World Input Tariffs			0.0247	0.0254
			(0.0499)	(0.0508)
Change in World Output Tariffs				-0.00800
				(0.0196)
Observations	5,236	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports IV regressions of Δh_{it}^{ALT} , given by (86), on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t}^m \right) \right)$ instrumenting the change in Chinese imports. All regressions also include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year. Change in Chinese imports measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \ln \left(\text{IM}_{\text{IND,CHN},t}^m \right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda_{i,t-1}^m} \alpha_{i,t-1}^m \Delta \tau_t^m$, where τ_t^m is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. First-Stage F-statistics are 16.80, 16.74, 27.73, and 30.57, for columns (1) through (4), respectively. Standard errors treat demand and production function estimates as data.

Table 51: Effect of Chinese Import Competition on TFPR: OLS

	(1)	(2)	(3)
	Full Plant	Average	Alternative
	TFPR	TFPR	Specialization
VARIABLES	Growth	Growth	Term
Change in Chinese Imports	0.300	0.298	0.00207
	(0.0847)	(0.0852)	(0.00570)
Change in World Input Tariffs	-0.0328	-0.0333	0.000560
	(0.0268)	(0.0273)	(0.00214)
Change in World Output Tariffs	0.00845	0.00981	-0.00137
	(0.0157)	(0.0159)	(0.00104)
Observations	5,236	5,236	5,236

Notes: The above reports OLS regressions of the various elements of (87) on the described variables. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership

structure, and year, while also controling for
$$\overline{h}_{i,t-1}$$
 and $\ln \left(\sum_{j \in \mathbb{Y}_{i,t-1}} \left(S_{i,t-1}^{jg} \right)^{\frac{\phi \frac{\alpha}{\alpha}}{1+\frac{\alpha}{\sigma}}} \exp \left(\widehat{h}_{i,t-1}^{jg} \right) \right)$.

Change in Chinese Imports measured as $\sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \ln \left(\mathrm{IM}^m_{\mathrm{IND,CHN},t} \right)$, as defined in the main text. Change in world input and output tariffs measured as $\sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \tau^m_t$, where τ^m_t is the average value of input or output tariffs, respectively, by 3-digit ASICC code m. Standard errors treat demand and production function estimates as data.

Table 52: Effect of Chinese Import Competition on TFPR: IV

	(1)	(2)	(3)
	Full Plant	Average	Alternative
	TFPR	TFPR	Specialization
VARIABLES	Growth	Growth	Term
Change in Chinese Imports	1.318	1.336	-0.0185
	(0.704)	(0.709)	(0.0422)
Change in World Input Tariffs	0.0254	0.0260	-0.000618
	(0.0508)	(0.0514)	(0.00336)
Change in World Output Tariffs	-0.00789	-0.00686	-0.00103
	(0.0196)	(0.0198)	(0.00130)
Observations	5,236	5,236	5,236

Standard errors clustered by plant

Notes: The above reports IV regressions of the various elements of (41) on the described variables, with $Z_{it}^{\text{CHN}} = \sum_{m \in \Lambda_{i,t-1}^m} \left(\alpha_{i,t-1}^m \Delta \ln \left(\sum_{k \in \mathbb{C}^{\text{No India}}} \text{IM}_{k,\text{CHN},t}^m \right) \right)$ instrumenting the change in Chinese imports. All regressions include a series of indicator variables that control for current and lagged number of products, state, plant age, census status, rural versus urban, type of organization, ownership structure, and year, while also controling for

sus urban, type of organization, ownership structure, and year, while also controling for
$$\overline{h}_{i,t-1}$$
 and $\ln\left(\sum_{j\in\mathbb{Y}_{i,t-1}}\left(S_{i,t-1}^{jg}\right)^{\frac{\phi}{1+\frac{\phi}{\sigma}}}\exp\left(\widehat{h}_{i,t-1}^{jg}\right)\right)$. Change in Chinese imports measured as

 $\sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \ln \left(\mathrm{IM}^m_{\mathrm{IND,CHN},t} \right), \text{ as defined in the main text. Change in world input and output tariffs measured as } \sum_{m \in \Lambda^m_{i,t-1}} \alpha^m_{i,t-1} \Delta \tau^m_t, \text{ where } \tau^m_t \text{ is the average value of input or output tariffs, respectively, by 3-digit ASICC code } m. First-Stage F-statistic is 30.65. Standard errors treat demand and production function estimates as data.}$