Globalization and Pandemics*

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Abstract

This paper develops a model of human interaction that provides microfoundations for a gravity equation for international trade and a multi-country Susceptible-Infected-Recovered (SIR) model of disease dynamics. We study how decreases in technological and man-made barriers to trade and labor mobility affect the rate at which human beings interact at short and long distances, and the consequences of these interactions for trade flows across countries and for welfare. We examine the implications of these changes in cross-border human interactions for the spread, persistence and human toll of epidemics. We consider various versions of our model, including some in which the rate of human interactions responds to the outbreak of a pandemic, through general equilibrium effects from changes in relative labor supplies and through individual behavioral responses to the threat of infection. Global flows of goods and of human beings interact with the spread of a pandemic in a number of subtle ways.

KEYWORDS: Globalization, Pandemics, Gravity Equation, SIR Model
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"As to foreign trade, there needs little to be said. The trading nations of Europe were all afraid of us; no port of France, or Holland, or Spain, or Italy would admit our ships or correspond with us." (*A Journal of the Plague Year*, Daniel Defoe, 1665)

1 Introduction

Throughout human history, globalization and pandemics have been closely intertwined. The Black Death arrived in Europe in October 1347 when twelve ships from the Black Sea docked at the Sicilian port of Messina – the word quarantine originates from the Italian word for a forty-day period of isolation required of ships and their crews during the Black Death pandemic. Much more recently, on January 21, 2020, the first human-to-human infections of COVID-19 in Europe are presumed to have taken place in Starnberg, Germany, when a local car parts supplier (Webasto) organized a training session with a Chinese colleague from its operation in Wuhan, China. These examples are by no means unique; accounts of contagion through international business travel abound. In this paper we study the interplay between the human interactions motivated by an economically integrated world, and the likelihood and severity of pandemics.

We develop a conceptual framework to shed light on a number of central questions about the two-way interaction between trade and pandemics. Does a globalized world make societies more vulnerable to pandemics? To what extent are disease dynamics different in closed and open economies? What are the implications of pandemics for the volume and pattern of international trade? How do these changes in the volume and pattern of international trade in turn influence the spread of the disease? To what extent are there externalities between the health policies of different countries in the open economy equilibrium? Will an altered awareness of the threat of future pandemics have a permanent impact on the nature of globalization?

Our conceptual framework combines the canonical model of international trade from economics (the gravity equation) with the seminal model pandemics from epidemiology (the Susceptible-Infected-Recovered or SIR model). We provide joint microfoundations for these relationships in a single underlying theory in which both international trade and the spread of disease are driven by human interactions. Through jointly modelling these two phenomena, we highlight a number of interrelationships between them. On the one hand, the contact rate among individuals, which is a central parameter in benchmark epidemiology models, is endogenous in our framework, and responds to both economic forces (e.g., the gains from international trade) and to the dynamics of the pandemic (e.g., the perceived health risk associated with international travel). On the other hand, we study how the emergence of a pandemic and the perceived risk of future outbreaks shapes the dynamics of international trade, and the net gains from international trade once the death toll from the pandemic is taken into account.

We consider a setting in which agents in each country consume differentiated varieties and choose the measures of these varieties to source from home and abroad. We suppose that sourcing varieties is costly, both in terms of the fixed costs of meeting with other agents that sell varieties

– an activity that involves intranational or international travel – and the variable costs of shipping varieties. Within this environment, the measures of varieties sourced at home and abroad are endogenously determined by trade frictions, country sizes, and the state of a pandemic, thus determining the intensity with which agents meet one another. If a healthy (susceptible) agent meets an infected agent, the probability that the disease is transmitted between them depends on the local epidemiological environment where the meeting takes place. The contagion risk associated with that local epidemiological environment is in turn shaped by local climate, by local social and cultural norms, and also by local health policies. Therefore, since domestic agents meet with other agents at home and abroad, the rate at which they are infected by the disease depends not only on their home health policies but also on those abroad.

We show that human interactions and trade flows are characterized by gravity equations that feature origin characteristics, destination characteristics and measures of bilateral trade frictions. Using these gravity equations, we show that the welfare gains from trade can be written in terms of certain sufficient statistics, namely the domestic trade share, the change in a country's population (i.e., deaths) that can ascribed to trade integration, and model parameters. This is similar to the celebrated Arkolakis et al. (2012) formula for the gains from trade, but how trade shares map into welfare changes depends on a wider range of model parameters than the conventional elasticity of trade with respect to trade costs. These gravity equations also determine the dynamics of the pandemic, which take a similar form to those of multi-group SIR model, but one in which the intensity of interactions between the different groups is endogenously determined by international trade, and potentially evolves over the course of the disease outbreak. We find that these disease dynamics differ systematically between the open-economy case and the closed-economy case. In particular, in the open economy, the condition for a pandemic to be self-sustaining ($\mathcal{R}_0$) depends critically on the epidemiological environment in the country with the highest rates of domestic infection.

We show that globalization and pandemics interact in a number of subtle ways. To build intuition, we begin by studying a situation in which the pandemic does not affect the ability of agents to work and trade and has no effects on mortality. Although the pandemic has no aggregate real income implications in that case, the dynamics of the disease are significantly impacted by the degree of trade openness. More specifically, we show that a decline in any international trade or mobility friction reduces the rates at which agents from the same country meet one another and increases the rates at which agents from different countries meet one another. If countries are symmetric, a decline in any (symmetric) international trade friction also leads to an overall increase in the total number of human interactions (domestic plus foreign). As a result, whenever countries are symmetric, a decline in any (symmetric) international trade friction *increases* the likelihood of a global pandemic occurring. More precisely, even if a pandemic would not be self-sustaining in either of the two symmetric countries in the closed economy (because $\mathcal{R}_0^{Closed} < 1$), it can be self-sustaining in an open economy ($\mathcal{R}_0^{Open} > 1$), because of the enhanced rate of interactions between agents in the open economy.

In contrast, if countries are sufficiently different from one another in terms of some of their primitive epidemiological parameters (i.e., the exogenous component of the infection rate or the recovery rate from the disease), a decline in any international trade friction can have the opposite effect of *decreasing* the likelihood of a global pandemic occurring. This situation arises because the condition for the pandemic to be sustaining in the open economy depends critically on the domestic rate of infections in the country with the worst disease environment. As a result, when one country has a much worse disease environment than the other, trade liberalization can reduce the share of that country's interactions that occur in this bad disease environment, thereby taking the global economy below the threshold for a pandemic to be self-sustaining for the world as a whole. Hence, in this case, on top of the negative effect on income, tightening trade or mobility restrictions can worsen the spread of the disease in all countries, including the relatively healthy one.

More generally, when a pandemic occurs in the open economy, we show that its properties are influenced by the disease environments in all countries, and can display significantly richer dynamics than in the standard closed-economy SIR model. For instance, even without lock downs, multiple waves of infection can occur in the open economy, when there would only be a single wave in each country in the closed economy.

As mentioned above, all of these complex interactions between globalization and pandemics occur even when agents do not alter their individual behavior in response to the pandemic, and all economic variables – such as wages and trade flows – are unaffected by the disease outbreak. In such a case, globalization influences disease transmission only by affecting the time-invariant rate at which agents from different countries interact with one another. Once we allow the disease to affect mortality, globalization and pandemics interact through two further channels: (i) general-equilibrium effects from the reduction in labor supply as a result of deaths; and (ii) behavioral effects as individuals internalize the threat of costly infection and alter their decisions about where to travel to seek consumption goods.

To isolate the general-equilibrium effects, we first assume that the pandemic generates deaths and thus a decline in population, but that agents remain unaware of the source of infection and continue to work effectively independently of their health status. In this case, a country with a worse disease environment tends to experience a larger reduction in population and labor supply, which in turn leads to an increase in its relative wage. This wage increase reduces the share of interactions that occur in that country's bad disease environment, and increases the share that occur in better disease environments, which again can take the global economy below the threshold for a pandemic to be self-sustaining. Therefore, the general equilibrium effects of the pandemic on wages and trade patterns induces a form of "general-equilibrium social distancing" from bad disease environments that operates even in the absence purposeful social distancing motivated by health risks.

We next allow individuals to become aware of the source of infection and optimally adjust their behavior depending on the state of the pandemic. As in recent work (see Farboodi et al., 2020), it proves useful to assume that agents are uncertain about their health status, and simply infer their health risk from the aggregate share of their country's population that dies from the disease. Technically, this turns the problem faced by agents into a dynamic optimal control problem in which the number of varieties that agents source from each country responds directly to the relative severity of the disease in each country. As in recent closed-economy models of social distancing (such as Farboodi et al., 2020, or Toxvaerd et al., 2020), these behavioral responses reduce domestic and international interactions and thereby tend to flatten the curve of infections, but these behavioral responses interact in rich ways across countries. We also consider an extension of our dynamic framework in which there are adjustment costs of establishing the human interactions needed to sustain trade, and show that the expectation of future pandemics (which due to recency effects might be particularly high in the aftermath of a pandemic) can lead to a protracted recovery of international trade after a pandemic.

Throughout, we use as our core setup an economy with two countries where agents can interact across borders but are subject to trade an migration frictions. Most of our results can be easily extended to contexts with multiple regions or even a continuum of them. These extensions could flexibly be used to study interactions across regions within countries or neighborhoods in a city. Ultimately, the decision of which stores to patron in a city, and how these decisions affect local disease dynamics, is shaped by many of the same economic trade-offs that we study in this paper.

Our paper connects with several strands of existing research. Within the international trade literature, we build on the voluminous gravity equation literature, which includes, among many others, the work of Anderson (1979), Anderson and Van Wincoop (2003), Eaton and Kortum (2003), Chaney (2008), Helpman et al. (2008), Allen and Arkolakis (2014), and Allen et al. (2020). As in the work of Chaney (2008) and Helpman et al. (2008), international trade frictions affect both the extensive and intensive margin of trade, but our model features selection into importing rather than selection into exporting (as in Antràs et al., 2017) and, more importantly, it emphasizes human interactions among buyers and sellers. In that latter respect, we connect with the work on the diffusion of information in networks, which has been applied to a trade context by Chaney (2014). By endogenizing the interplay between globalization and pandemics, we study the nature and size of trade-induced welfare losses associated with disease transmission, thereby contributing to the very active recent literature on quantifying the gains from international trade (see, for instance, Eaton and Kortum, 2002, Arkolakis et al., 2012, Melitz and Redding, 2014, Costinot and Rodriguez-Clare, 2015, Ossa, 2015).

Although our model is admittedly abstract, we believe that it captures the role of international business travel in greasing the wheels of international trade. With this interpretation, our model connects with an empirical literature that has studied the role of international business travel in facilitating international trade (see Cristea, 2011, Blonigen and Cristea, 2015, and Startz, 2018), and more generally, in fostering economic development (see Hovhannisyan and Keller, 2015, Campante and Yanagizawa-Drott, 2018). Our simple microfounded model of trade through human interaction provides a natural rationalization for a gravity equation in international trade and shows how different types of trade frictions affect the extensive and intensive margins of trade.

Our paper also builds on the literature developing epidemiological models of disease spread, starting with the seminal work of Kermack and McKendrick (1927, 1932). More specifically, our multi-country SIR model shares many features with multigroup models of disease transmission, as in the work, among others, of Hethcote (1978), Hethcote and Thieme (1985) and van den Driessche and Watmough (2002).[1] A key difference is that the interaction between groups is endogenously determined by the gravity structure of international trade. The recent COVID-19 pandemic has triggered a remarkable explosion of work by economists studying the spread of the disease (see, for instance, Fernández-Villaverde and Jones, 2020) and exploring the implications of several types of policies (see, for instance, Alvarez et al., 2020, Acemoglu et al., 2020, Atkeson, 2020, or Jones et al., 2020). Within this literature, a few papers have explored the spatial dimension of the COVID-19 pandemic by simulating multi-group SIR models applied to various urban and regional contexts (see, among others, Argente et al., 2020, Bisin and Moro, 2020, Cuñat and Zymek, 2020, Birge et al., 2020, and Fajgelbaum et al., 2020). Our paper also connects with a subset of that literature, exemplified by the work of Alfaro et al. (2020), Farboodi et al. (2020), Fenichel et al. (2011), and Toxvaerd (2020) that has studied how the behavioral response of agents (e.g., social distancing) affects the spread and persistence of pandemics. Whereas most of this research is concerned with COVID-19 and adopts a simulation approach, our main goal is to develop a model of human interaction that jointly provides a microfoundation for a gravity equation and multi-group SIR dynamics, and can be used to analytically characterize the two-way relationship between globalization and pandemics.

Our work is also related to a literature in economic history that has emphasized the role of international trade in the transmission of disease. For the case of the Black Death, Christakos, Olea, Serre, Yu and Wang (2005), Boerner and Severgnini (2014), Ricci, Lee and Connor (2017), and Jedwab, Johnson and Koyama (2019) argue that trade routes are central to understanding the spread of the plague through medieval Europe. In a review of a broader range of infection diseases, Saker, Lee, Cannito, Gilmore and Campbell-Lendrum (2002) argue that globalization has often played a pivotal role in disease transmission. The recent COVID-19 pandemic has also provided a number of examples of spread of the virus through business travel.[2]

The rest of the paper is structured as follows. In Section 2, we present our baseline gravity-style model of international trade with endogenous intranational and international human interactions. In Section 3, we consider a first variant of the dynamics of disease spread in which the rate of

[1] See Hetchote (2000) and Brauer and Castillo-Chavez (2012) for very useful reviews of mathematical modelling in epidemiology, and Ellison (2020) for an economist's overview of SIR models with heterogeneity.

[2] A well-known example in the U.S. is the conference held by biotech company Biogen in Boston, Massachusetts on February 26 and 27, and attended by 175 executive managers, who spread the covid-19 virus to at least six states, the District of Columbia and three European countries, and caused close to 100 infections in Massachusetts alone http://www.nytimes.com/2020/04/12/us/coronavirus-biogen-boston-superspreader.html). Another example is Steve Walsh, the so-called British "super spreader," who is linked to at least 11 new infections of COVID-19, and who caught the disease in Singapore, while he attended a sales conference in late January of 2020 (see https://www.washingtonpost.com/world/europe/british-coronavirus-super-spreader-may-have-infected-at-least-11-people-in-three-countries/2020/02/10/016e9842-4c14-11ea-967b-e074d302c7d4_story.html). The initial spread of COVID-19 to Iran and Nigeria has also been tied to international business travel.

contact between agents (though endogenous) is time-invariant during the pandemic. In Section 4, we incorporate labor supply responses to the pandemic, which affect the path of relative wages (and thus the rate of contact of agents within and across countries) during the pandemic. In Section 5, we incorporate individual behavioral responses motivated by agents adjusting their desired number of human interactions in response to their fear of being infected by the disease. We offer some concluding remarks in Section 6.

2 Baseline Economic Model

We begin by developing a stylized model of the global economy in which international trade is sustained by human interactions. Our baseline model is a simple two-country world model, in which countries use labor to produce differentiated goods that are exchanged in competitive markets via human interactions. In section 2.3, we outline how our model can be easily extended to settings featuring (i) multiple countries, (ii) intermediate inputs, and (iii) scale economies and imperfect competition.

2.1 Environment

Consider a world with two locations: East and West, indexed by i or j. We denote by $\mathcal{J}$ the set of countries in the world, so for now $\mathcal{J} = \{East, West\}$. Location $i \in \mathcal{J}$ is inhabited by a continuum of measure L_i of households, and each household is endowed with the ability to produce a differentiated variety using labor as the only input in production. We denote by w_i the wage rate in country i.

Trade is costly. There are iceberg bilateral trade cost $\tau_{ij} = t_{ij} \times (d_{ij})^{\delta}$, when shipping from j back to i, where $d_{ij} \geq 1$ is the symmetric distance between i and j, and t_{ij} is a man-made additional trade friction imposed by i on imports from country j. We let these man-made trade costs be potentially asymmetric reflecting the fact that one country may impose higher restrictions to trade (e.g., tariffs, or delays in goods clearing customs) than the other country. For simplicity, there are no man-made frictions to internal shipments, so $t_{ii} = 1$ and $\tau_{ii} = (d_{ii})^{\delta}$, where $d_{ii} < d_{ij}$ can be interpreted as the average internal distance in country $i = East, West$.

Each household is formed by two individuals. One of these individuals – the *seller* – is in charge of producing and selling the household-specific differentiated variety from their home, while the other individual – the *buyer* – is in charge of procuring varieties for consumption from other households in each of the two locations. We let all households in country i be equally productive in manufacturing varieties, with one unit of labor delivering Z_i units of goods. Goods markets are competitive and sellers make their goods available at marginal cost. Households have CES preferences over differentiated varieties, with an elasticity of substitution $\sigma > 1$ regardless of the origin of these varieties, and they derive disutility from the buyer spending time away from home.

More specifically, a household in country i incurs a utility cost

$$c_{ij}\left(n_{ij}\right)=\frac{c}{\phi}\times\mu_{ij}\times\left(d_{ij}\right)^{\rho}\times\left(n_{ij}\right)^{\phi}, \tag{1}$$

whenever the household's buyer secures n_{ij} varieties from location j, at a distance $d_{ij}\geq 1$ from i. The parameter μ_{ij} captures (possibly asymmetric) travel restrictions imposed by j on visitors from i. The parameter c governs the cost of travel and we assume it is large enough to ensure an interior solution in which $n_{ij}\leq L_j$ for all i and $j\in\mathcal{J}$. We assume that whenever $n_{ij}<L_j$, the set of varieties procured from j are chosen at random, so if all households from i procure n_{ij} from j, each household's variety in j will be consumed by a fraction n_{ij}/L_j of households from i.[3]

Welfare of households in location i is then given by

$$W_i=\left(\sum\nolimits_{j\in\mathcal{J}}\int_0^{n_{ij}}q_{ij}\left(k\right)^{\frac{\sigma-1}{\sigma}}dk\right)^{\frac{\sigma}{\sigma-1}}-\frac{c}{\phi}\sum\nolimits_{j\in\mathcal{J}}\mu_{ij}\left(d_{ij}\right)^{\rho}\times\left(n_{ij}\right)^{\phi}, \tag{2}$$

where $q_{ij}\left(k\right)$ is the quantity consumed in location i of the variety produced in location j by household k.

2.2 Equilibrium

Let us first consider consumption choices in a given household for a given n_{ij}. Maximizing (2) subject to the households' budget constraint, we obtain:

$$q_{ij}=\frac{w_i}{\left(P_i\right)^{1-\sigma}}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{-\sigma}, \tag{3}$$

where w_i is household income, w_j/Z_j is the common free-on-board price of all varieties produced in location j, τ_{ij} are trade costs when shipping from j to i, and P_i is a price index given by

$$P_i=\left(\sum\nolimits_{j\in\mathcal{J}}n_{ij}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\right)^{1/(1-\sigma)}. \tag{4}$$

Multiplying equation (3) by $\left(q_{ij}\right)^{(\sigma-1)/\sigma}$, summing across locations, and rearranging, it is straightforward to show that

$$Q_i=\left(\sum\nolimits_{j\in\mathcal{J}}n_{ij}\left(q_{ij}\right)^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}=\frac{w_i}{P_i}, \tag{5}$$

so real consumption equals real income.

In order to characterize each household's choice of n_{ij}, we first plug (3) and (4) into (2) to

[3] It may seem arbitrary that it is buyers rather than sellers who are assumed to travel. In section 2.3, we offer an interpretation of the model in which trade is in intermediate inputs and the buyer travels in order to procure the parts of components necessary for the household to produce a final consumption good. In section 2.3, we also consider the case in which travel costs are in terms of labor, rather than a utility cost. Finally, in that same section 2.3, we also explore a variant of the model in which it is sellers rather than buyers who travel.

obtain

$$W_i = w_i \left(\sum\nolimits_{j \in \mathcal{J}} n_{ij} \left(\frac{\tau_{ij} w_j}{Z_j} \right)^{1-\sigma} \right)^{\frac{1}{(\sigma-1)}} - \frac{c}{\phi} \sum\nolimits_{j \in \mathcal{J}} \mu_{ij} \left(d_{ij} \right)^{\rho} \times \left(n_{ij} \right)^{\phi}. \tag{6}$$

The first order condition associated with the choice of n_{ij} delivers (after plugging in (5)):

$$n_{ij} = \left(c \left(\sigma - 1 \right) \mu_{ij} \right)^{-1/(\phi-1)} \left(d_{ij} \right)^{-\frac{\rho + (\sigma-1)\delta}{\phi-1}} \left(\frac{t_{ij} w_j}{Z_j P_i} \right)^{-\frac{\sigma-1}{(\phi-1)}} \left(\frac{w_i}{P_i} \right)^{1/(\phi-1)}. \tag{7}$$

Notice that bilateral human interactions follow a 'gravity-style' equation that is log-separable in origin and destination terms, and a composite of bilateral trade frictions. Evidently, natural and man-made barriers to trade (d_{ij}, t_{ij}) and to labor mobility (μ_{ij}) will tend to reduce the number of human interactions sought by agents from country i in country j. As we show in Appendix A.1, for the second-order conditions to be met for all values of μ_{ij}, d_{ij}, and t_{ij}, we need to impose $\phi > 1/\left(\sigma - 1\right)$ and $\sigma > 2$.

Bilateral import flows by country i from country j are in turn given by

$$X_{ij} = n_{ij} p_{ij} q_{ij} L_i = \left(c \left(\sigma - 1 \right) \mu_{ij} \right)^{-\frac{1}{\phi-1}} \left(d_{ij} \right)^{-\frac{\rho + \phi(\sigma-1)\delta}{\phi-1}} \left(\frac{t_{ij} w_j}{Z_j P_i} \right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \left(\frac{w_i}{P_i} \right)^{\frac{1}{\phi-1}} w_i L_i. \tag{8}$$

Notice that the trade share can be written as

$$\pi_{ij} = \frac{X_{ij}}{\sum_{\ell \in \mathcal{J}} X_{i\ell}} = \frac{\left(w_j / Z_j \right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times \left(\mu_{ij} \right)^{-\frac{1}{\phi-1}} \left(d_{ij} \right)^{-\frac{\rho + \phi(\sigma-1)\delta}{\phi-1}} \left(t_{ij} \right)^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{\ell \in \mathcal{J}} \left(\mu_{i\ell} \right)^{-\frac{1}{\phi-1}} \left(d_{i\ell} \right)^{-\frac{\rho + \phi(\sigma-1)\delta}{\phi-1}} \left(t_{i\ell} w_\ell / Z_\ell \right)^{-\frac{\phi(\sigma-1)}{\phi-1}}}, \tag{9}$$

and is thus log-separable in an origin-specific term S_j, a destination-specific term Θ_i, and a composite bilateral trade friction term given by:[4]

$$\left(\Gamma_{ij} \right)^{-\varepsilon} = \left(\mu_{ij} \right)^{-\frac{1}{\phi-1}} \left(d_{ij} \right)^{-\frac{\rho + \phi(\sigma-1)\delta}{\phi-1}} \left(t_{ij} \right)^{-\frac{\phi(\sigma-1)}{\phi-1}}. \tag{10}$$

Following Head and Mayer (2014), it then follows that bilateral trade flows in (8) also follow a standard gravity equation

$$X_{ij} = \frac{X_i}{\Phi_i} \frac{Y_j}{\Omega_j} \left(\Gamma_{ij} \right)^{-\varepsilon},$$

where X_i is total spending in country i, Y_j is country j's value of production, and

$$\Phi_i = \sum\nolimits_{j \in \mathcal{J}} \frac{Y_j}{\Omega_j} \left(\Gamma_{ij} \right)^{-\varepsilon}; \qquad \Omega_j = \sum\nolimits_{i \in \mathcal{J}} \frac{X_i}{\Phi_i} \left(\Gamma_{ji} \right)^{-\varepsilon}.$$

Notice that the distance elasticity is affected by the standard substitutability σ, but also by the traveling cost elasticity ρ, and by the convexity ϕ of the traveling costs. It is clear that both $\rho > 0$ and $\phi > 1$ increase the distance elasticity relative to a standard Armington model (in which

[4] More specifically, $S_j = \left(w_j / Z_j \right)^{-\frac{\phi(\sigma-1)}{\phi-1}}$ and $\Theta_i = \sum_{\ell \in \mathcal{J}} \left(\mu_{i\ell} \right)^{-\frac{1}{\phi-1}} \left(d_{i\ell} \right)^{-\frac{\rho + \phi(\sigma-1)\delta}{\phi-1}} \left(t_{i\ell} w_\ell / Z_\ell \right)^{-\frac{\phi(\sigma-1)}{\phi-1}}$.

the distance elasticity would be given by $\delta(\sigma-1)$). The other man-made bilateral frictions also naturally depress trade flows.[5]

We next solve for the price index and household welfare in each country. Invoking equation (5), plugging (3) and (7), and simplifying delivers

$$P_i = \left(\frac{w_i}{c(\sigma-1)}\right)^{-\frac{1}{\phi(\sigma-1)-1}} \left(\sum\nolimits_{j\in\mathcal{J}} (\Gamma_{ij})^{-\varepsilon} (w_j/Z_j)^{-\frac{(\sigma-1)\phi}{\phi-1}}\right)^{-\frac{(\phi-1)}{\phi(\sigma-1)-1}}. \tag{11}$$

Going back to the expression for welfare in (2), and plugging (5), (7) and (11), we then find

$$W_i = \frac{\phi(\sigma-1)-1}{\phi(\sigma-1)}\frac{w_i}{P_i}, \tag{12}$$

which combined with (9) implies that welfare at the household level is given by

$$W_i = \frac{\phi(\sigma-1)-1}{\phi(\sigma-1)} \times (\pi_{ii})^{-\frac{(\phi-1)}{\phi(\sigma-1)-1}} \times \left(\frac{(Z_i)^{\phi(\sigma-1)}}{c(\sigma-1)}(\Gamma_{ii})^{-\varepsilon(\phi-1)}\right)^{\frac{1}{\phi(\sigma-1)-1}}. \tag{13}$$

This formula is a variant of the Arkolakis et al. (2012) welfare formula indicating that, with estimates of ϕ and σ at hand, one could compute the change in welfare associated with a shift to autarky only with information of the domestic trade share π_{ii}. A key difference relative to their contribution, however, is that the combination of ϕ and σ relevant for welfare cannot easily be backed out from estimation of a 'trade elasticity' (see equation (10)). Later, when we allow trade to affect the transmission of disease and this disease to affect mortality, a further difference will be that the effect of trade on aggregate welfare (W_iL_i) will also depend on its effect on mortality (via a decline in L_i).

We conclude our description of the equilibrium of our model by discussing the determination of equilibrium wages. For that, it is simplest to just invoke the equality between income and spending in each country, that is $\pi_{ii}w_iL_i + \pi_{ji}w_jL_j = w_iL_i$, which plugging in (9), can be written as

$$\pi_{ii}(w_i,w_j) \times w_iL_i + \pi_{ji}(w_i,w_j) \times w_jL_j = w_iL_i, \tag{14}$$

where $\pi_{ii}(w_i,w_j)$ and $\pi_{ji}(w_i,w_j)$ are given in equation (9). These pair of equations (one for i and one for j) allow us to solve for w_i and w_j as a function of the unique distance d_{ij}, the pair of mobility restriction parameters μ_{ij} and μ_{ji}, the pair of man-made trade barriers t_{ij} and t_{ji}, and the parameters ϕ, σ, δ, and ρ. Setting one of the country's wages as the numéraire, the general equilibrium only requires solving one of these non-linear equations in (14). Once one has solved for this (relative) wage, it is straightforward to solve for trade flows and for the flow of buyers across

[5] It is also worth noting that when $\mu_{ij} = \mu_{ji}$ and $t_{ji} = t_{ij}$, this gravity eqauation is fully symmetric, and

$$\Phi_i = \Omega_i = \sum\nolimits_j S_j\phi_{ij} = \sum\nolimits_j (w_j/Z_j)^{-\frac{\phi(\sigma-1)}{\phi-1}} (\mu_{ij})^{-\frac{1}{\phi-1}} (d_{ij})^{-\frac{\rho+\phi(\sigma-1)\delta}{\phi-1}} (t_{ij})^{-\frac{\phi(\sigma-1)}{\phi-1}}.$$

locations, as well as for the implied welfare levels.

Note that the general-equilibrium condition in (14) is identical to that obtained in standard gravity models, so from the results in Alvarez and Lucas (2007), Allen and Arkolakis (2014), or Allen et al. (2020), we can conclude that:[6]

Proposition 1 *As long as trade frictions Γ_{ij} are bounded, there exists a unique vector of equilibrium wages $\mathbf{w}^* = (w_i, w_j) \in \mathbb{R}^2_{++}$ that solves the system of equations in (14).*

Using the implicit-function theorem, it is also straightforward to see that the relative wage w_j/w_i will be increasing in L_i, Γ_{ii}, Γ_{ji}, and Z_j, while it will be decreasing in L_j, Γ_{jj}, Γ_{ij}, and Z_i.

Given the vector of equilibrium wages $\mathbf{w} = (w_i, w_j)$, we are particularly interested in studying how changes in trade frictions (d_{ij}, t_{ij}, or μ_{ij}) affect the rate of human-to-human interactions at home, abroad and worldwide. Note that, combining equations (3), (8), and (9), we can express

$$n_{ij}(\mathbf{w}) = \left(\frac{t_{ij}(d_{ij})^{\delta} w_j}{P_i(\mathbf{w}) Z_j} \right)^{\sigma-1} \pi_{ij}(\mathbf{w}), \tag{15}$$

where $\pi_{ij}(\mathbf{w})$ is given in (9) and $P_i(\mathbf{w})$ in (11). Studying how $n_{ii}(\mathbf{w})$ and $n_{ij}(\mathbf{w})$ are shaped by the primitive parameters of the model is complicated by the general equilibrium nature of our model, but in Appendix A.3 we are able to show that (see Appendix A.2):

Proposition 2 *A decline in any international trade or mobility friction ($d_{ij}, t_{ij}, t_{ji}, \mu_{ij}, \mu_{ji}$) leads to: (a) a decline in the rates (n_{ii} and n_{jj}) at which individuals will meet individuals in their own country; and (b) an increase in the rates at which individuals will meet individuals from the other country (n_{ij} and n_{ji}).*

In words, despite the fact that changes in trade and mobility frictions obviously impact equilibrium relative wages, the more open are economies to the flow of goods and people across borders, the larger will be international interactions and the lower will be domestic interactions.

We can also study the effect of reductions in international trade and mobility frictions on the *overall* measure of varieties consumed by each household, which also corresponds to the number of human interactions experienced by each household's buyer (i.e., $n_{ii} + n_{ij}$). Similarly, we can also study the total number of human interactions carried out by each household's seller (i.e., $n_{ii} + n_{ji}$). General equilibrium forces complicate this comparative static, but we are able to show that (see Appendix A.3).

Proposition 3 *Suppose that countries are symmetric, in the sense that $L_i = L$, $Z_i = Z$, and $\Gamma_{ij} = \Gamma$ for all i. Then, a decline in any (symmetric) international trade frictions leads to an overall increase in human interactions ($n_{dom} + n_{for}$) experienced by both household buyers and household sellers.*

[6] In Alvarez and Lucas (2007), uniqueness requires some additional (mild) assumptions due to the existence of an intermediate-input sector. Because our model features no intermediate inputs, we just need to assume that trade frictions remain bounded.

The assumption of full symmetry is extreme, but the result of course continues to hold true if country asymmetries are small and trade frictions are not too asymmetric across countries. Furthermore, exhaustive numerical simulations suggest that the result continues to hold true for arbitrarily (asymmetric) declines in trade frictions, as long as countries are symmetric in size ($L_i = L$) and in technology ($Z_i = Z$).

Reverting back to our general equilibrium with arbitrary country asymmetries, we can also derive results for how changes in the labor force in either country affect the per-household measure of interactions at home and abroad. More specifically, from equation (14), it is straightforward to see that the relative wage w_j/w_i is monotonic in the ratio L_i/L_j. Furthermore, working with equations (7) and (11), we can establish (see Appendix A.4 for a proof):

Proposition 4 *A decrease in the relative size of country i's population leads to a decrease in the rates n_{ii} and n_{ji} at which individuals from all countries will meet individuals in country i, and to an increase in the rates n_{jj} and n_{ij} at which individuals from all countries will meet individuals in the other country j.*

This result will prove useful in section 4, where we study how general equilibrium forces partly shape the dynamics of an epidemic. For instance, if the epidemic affects labor supply disproportionately in one of the countries, then the implied increase in that country's relative wage will induce a form of general equilibrium induced social distancing, as it will incentivize home buyers to avoid that country, even without social distancing motivated by health risks.

2.3 Extensions

Our baseline economic model is special among many dimensions, so it is important to discuss the robustness of some of the key insights we take away from our economic model. Because the gravity equation of international trade can be derived under a variety of economic environments and market structures, it is perhaps not too surprising that many of the key features of our model carry over to alternative environments featuring multiple countries, intermediate input trade, scale economies and imperfect competition. We next briefly describe four extensions of our model, but we leave all mathematical details to the Appendix (see Appendix A.5).

Some readers might object to the fact that, in our baseline model, production uses labor, while the traveling cost is specified in terms of a utility cost. We make this assumption to identify international travel with specific members of the household, which facilitates a more transparent transition to a model of disease transmission driven by human-to-human interactions. Nevertheless, in terms of the mechanics of our economic model, this assumption is innocuous. More specifically, in the first extension studied in Appendix A.5, we show that Propositions 1 through 4 continue to hold whenever travel costs in equation (1) are specified in terms of labor rather than being modelled as a utility cost. In fact, this version of the model is isomorphic to our baseline model above, except for a slightly different expression for the equilibrium price index P_i.

The assumption that households travel internationally to procure consumption goods may seem unrealistic. Indeed, international business travel may be better thought as being a valuable input when firms need specialized inputs and seek potential providers of those inputs in various countries. Fortunately, it is straightforward to re-interpret our model along those lines by assuming that the differentiated varieties produced by households are intermediate inputs, which all households combine into a homogeneous final good, which in equilibrium is not traded. The details of this re-interpretation are worked out in the second extension studied in Appendix A.5.

Returning to our baseline economic model, in Appendix A.5 we next derive our key equilibrium conditions for a world economy with multiple countries. In fact, all the equations above, except for the labor-market clearing condition (14) apply to that multi-country environment once the set of countries $\mathcal{J}$ is re-defined to include multiple countries. The labor-market condition is in turn simply given by $\sum_{j\in\mathcal{J}} \pi_{ij}(\mathbf{w}) w_j L_j = w_i L_i$, where $\pi_{ij}(\mathbf{w})$ is defined in (9). Similarly, the model is also easily adaptable to the case in which there is a continuum of locations $i \in \Omega$, where Ω is a closed and bounded set of a finite dimensional Euclidean space. The equilibrium conditions are again unaltered, with integrals replacing summation operators throughout.

Finally, in Appendix A.5 we explore a variant of our model in which it is the household's seller rather than the buyer who travels to other locations. We model this via a framework featuring scale economies, monopolistic competition and fixed cost of exporting, as in the literature on selection into exporting emanating from the seminal work of Melitz (2003), except that the fixed costs of selling are defined at the buyer level rather than at the country level. This extension is still work in progress.

3 A Two-Country SIR Model with Time-Invariant Interactions

So far, we have just characterized a static (steady-state) model of international trade supported or fueled by international travel. Now let us consider the case in which the model above describes a standard "day" in the household. More specifically, in the morning the buyer in each household in i leaves the house and visits n_{ii} sellers in i and n_{ij} sellers in j, procuring goods from each of those households. For simplicity, assume that buyers do not travel together or otherwise meet each other. While the buyer visit other households and procures goods, the seller in each household sells its own goods to visitors to their household. There will be n_{ii} domestic visitors and n_{ji} foreign visitors. In the evening, the two members of the household reunite.

3.1 Preliminaries

With this background in mind, consider now the dynamics of contagion. As in the standard epidemiological model, we divide the population at each point in time into **S**usceptible households, **I**nfectious households, and **R**ecovered households (we will incorporate deaths in the next section). We think of the health status as being a household characteristic, implicitly assuming a perfect rate of transmission within the household (they enjoy a passionate marriage), and also that recovery is experienced contemporaneously by all household members.

In this section we seek to study the dynamics of a two-country SIR model in which the pandemic only generates cross-country externalities via contagion (and not via terms of trade effects), and in which households do not exert any pandemic-motivated social distancing. Hence, we simply assume that agents are unaware of their health status. As a result, agent's behavior is unaffected by the disease. Labor supply and aggregate income are constant in each country and over time because households exerts no social distancing and the pandemic is (for now) assumed to cause no deaths. We relax these assumptions in section 4 where, although agents remain unaware of their health status, some of the infected agents end up dying. The result is a model in which the dynamics of the pandemic affect the evolution of the labor supply and aggregate income in each country. In Section 5 we go further and assume that agents understand that if they become infected, they have a positive probability of dying (an event that they, of course, do notice!). The possibility of dying generates behavioral responses to prevent contagion by reducing interactions.

In sum, the goal of this section is to understand how cross-country interactions motivated by economic incentives affect the spread of a pandemic in a world in which these interactions are *time-invariant* during the pandemic. It is important to emphasize, however, that the fixed measure of interactions chosen by each household is still endogenously shaped by the primitive parameters of our model, as described in section 2. We will be particularly interested in studying the incidence and dynamics of the pandemic for different levels of trade integration, and different values of the primitive epidemiological parameters (the contagion rate conditional on a number of interactions and the recovery rate) in each country.

3.2 The Dynamic System

As argued above, the population, technology and relative wage will be time-invariant, so we can treat n_{ii}, n_{ij}, n_{ji} and n_{jj} as fixed parameters (though obviously their constant level is shaped by the primitives of the model).

The share of households of each type evolve according to the following laws of motion (we ignore time subscripts for now to keep the notation tidy):

$$\dot{S}_i = -2n_{ii} \times \alpha_i \times S_i \times I_i - n_{ij} \times \alpha_j \times S_i \times I_j - n_{ji} \times \alpha_i \times S_i \times I_j \quad (16)$$

$$\dot{I}_i = 2n_{ii} \times \alpha_i \times S_i \times I_i + n_{ij} \times \alpha_j \times S_i \times I_j + n_{ji} \times \alpha_i \times S_i \times I_j - \gamma_i I_i \quad (17)$$

$$\dot{R}_i = \gamma_i I_i \quad (18)$$

To better understand this system, focus first on how infections grow in equation (17). The first term $2n_{ii} \times S_i \times I_i$ in this equation captures newly infected households in country i. Sellers in i receive (in expectation) n_{ii} domestic buyers, while buyers meet up with n_{ii} domestic sellers. The household thus jointly has $2n_{ii}$ domestic contacts. In those encounters, a new infection occurs with probability α_i whenever one of the agents is susceptible (which occurs with probability S_i) and the other agent is infectious (which occurs with probability I_i). The second term of equation (17) reflects new infections of country i's households occurring in the foreign country when susceptible buyers from i (of which there are S_i) visit foreign households with infectious sellers. There are n_{ij} of those meetings, leading to an new infection with probability α_j whenever the foreign seller is infectious (which occurs with probability I_j). Finally, the third term in (17) reflects new infections associated with susceptible sellers in country i receiving infectious buyers from abroad (country j). Each susceptible domestic buyer (constituting a share S_i of i's population) has n_{ji} such meetings, which cause an infection with probability α_i whenever the foreign buyer is infectious (which occurs with probability I_j). The final term in equation (17) simply captures the rate at which infectious individuals recover (γ_i).

Once the equation determining the dynamics of new infections is determined, the one determining the change of susceptible agents in (16) is straightforward to understand, as it just reflects a decline in the susceptible population commensurate with new infections. Finally, equation (18) governs the transition from infectious households to recovered households.

In Section A.6 of the Appendix, we provide further details on the numerical simulations of the two-country SIR model that we use in the figures below to illustrate our results, including the parameter values we use.

3.3 The Closed-Economy Case

Our model reduces to a standard SIR model when there is no movement of people across countries, and thus no international trade. In such a case, the system in (16)-(18) reduces to

$$\begin{aligned}
\dot{S}_i &= -\beta_i \times S_i \times I_i \\
\dot{I}_i &= \beta_i \times S_i \times I_i - \gamma_i I_i \\
\dot{R}_i &= \gamma_i I_i
\end{aligned}$$

where $\beta_i = 2n_{ii}$ is the so-called contact rate. The dynamics of this system have been studied extensively since the pioneering work of Kermack and McKendrick (1927, 1932). Suppose that at some time t_0, there is an outbreak of a disease which leads to initial infections $I_i(t_0) = \varepsilon > 0$, where ε is small. Because ε is small, $S_i(t_0)$ is very close to 1, and from the second equation, we have that if the so-called *reproduction number* $\mathcal{R}_{0i} = \beta_i/\gamma_i$ is less than one, then, $\dot{I}_i(t) < 0$ for all $t > t_0$, and the infection quickly dies out. In other words, when $\mathcal{R}_{0i} = \beta_i/\gamma_i < 1$ a *pandemic-free* equilibrium is globally stable. If instead $\mathcal{R}_{0i} = \beta_i/\gamma_i > 1$, the number of new infections necessarily rises initially and the share of susceptible households declines until the system reaches a period t^* at which $S_i(t^*) = \gamma_i/\beta_i$, after which infections decline and eventually go to 0. The steady-state values of $S_i(\infty)$ in this *pandemic* equilibrium is determined by the solution to this simple non-linear equation:[7]

$$\ln S_i(\infty) = -\frac{\beta_i}{\gamma_i}\left(1 - S_i(\infty)\right). \tag{19}$$

This equation admits a unique solution with $1 > S_i(\infty) > 0$.[8] Because $S_i(\infty) < \gamma_i/\beta_i$ (since $S_i(t^*) = \gamma_i/\beta_i$ at the peak of infections), $S_i(\infty)$ is decreasing in $\mathcal{R}_{0i}$. In sum, in the closed-economy case, $S_i(\infty) = 1$ as long as $\mathcal{R}_{0i} \leq 1$, but when $\mathcal{R}_{0i} > 1$, the higher is $\mathcal{R}_{0i}$, the lower is $S_i(\infty)$, which means that more people get infected.

3.4 Pandemic-Free World Equilibrium

We can now return to the two-country system in (16)-(18). We first explore the conditions under which a pandemic-free equilibrium is stable, and infections quickly die out worldwide, regardless of where the disease originated. For that purpose, it suffices to focus on the laws of motion for (S_i, S_j, I_i, I_j) evaluated at the pandemic-free equilibrium, in which $S_i = S_j \simeq 1$ and $I_i = I_j \simeq 0$. The Jacobian of this system is given by

$$J = \begin{bmatrix} 0 & 0 & -2\alpha_i n_{ii} & -\left(\alpha_j n_{ij} + \alpha_i n_{ji}\right) \\ 0 & 0 & -\left(\alpha_j n_{ij} + \alpha_i n_{ji}\right) & -2\alpha_j n_{jj} \\ 0 & 0 & 2\alpha_i n_{ii} - \gamma_i & \alpha_j n_{ij} + \alpha_i n_{ji} \\ 0 & 0 & \alpha_j n_{ij} + \alpha_i n_{ji} & 2\alpha_j n_{jj} - \gamma_j \end{bmatrix},$$

and the largest positive eigenvalue of this matrix (see Appendix A.7) is given by

$$\lambda_{\max} = \frac{1}{2}\left(2\alpha_i n_{ii} - \gamma_i\right) + \frac{1}{2}\left(2\alpha_j n_{jj} - \gamma_j\right) + \frac{1}{2}\sqrt{4\left(\alpha_j n_{ij} + \alpha_i n_{ji}\right)^2 + \left(\left(2\alpha_i n_{ii} - \gamma_i\right) - \left(2\alpha_j n_{jj} - \gamma_j\right)\right)^2}.$$

Since we are interested in finding necessary conditions for stability of this equilibrium (i.e., $\lambda_{\max} < 0$), noting that $\lambda_{\max}$ is increasing in n_{ij} and n_{ji}, we have that

$$\lambda_{\max} \geq \lambda_{\max}\big|_{n_{ij}=n_{ji}=0} = \max\left\{2\alpha_i n_{ii} - \gamma_i, 2\alpha_j n_{jj} - \gamma_{jj}\right\}. \tag{20}$$

As a result, a pandemic-free equilibrium can only be stable whenever $2\alpha_i n_{ii}/\gamma_i \leq 1$ and $2\alpha_j n_{jj}/\gamma_{jj} \leq 1$. In words, if the reproduction number $\mathcal{R}_{0i}$ based only on domestic interactions (but evaluated at the world equilibrium value of n_{ii}) is higher than 1 in *any* country, the pandemic-free equilibrium is necessarily unstable, and both countries will experience at least one period of rising infections

[7] To see this, begin by writing

$$\frac{\dot{S}_i}{S_i} = -\beta_i I_i = -\frac{\beta_i}{\gamma_i}\dot{R}(i).$$

Now taking logs and integrating, and imposing $I_i(\infty) = 0$, delivers

$$\ln S_i(\infty) - \ln S_i(t_0) = -\frac{\beta_i}{\gamma_i}\left(1 - S_i(\infty) - R_i(0)\right).$$

Finally, imposing $\ln S_i(t_0) = R_i \simeq 0$, we obtain equation (19).

[8] Equation (19) is obviously also satisified when $S_i(\infty) = 1$, but this equilibrium is not stable when $\mathcal{R}_{0i} > 1$.

along the dynamics of the pandemic (see Appendix A.7). This result highlights the externalities that countries exert on other countries when the disease is not under control purely based on the domestic interactions of agents.

It is interesting to note that we achieve the exact same result when studying the global reproduction number $\mathcal{R}_0$ associated with the world equilibrium dynamics. Remember that $\mathcal{R}_0$ is defined as the expected number of secondary cases produced by a single (typical) infection starting from a completely susceptible population. Because our model maps directly to multigroup models of disease transmission, we can invoke (and verify) results from that literature to provide an alternative analysis of the stability of the pandemic-free equilibrium in our two-country dynamic system (c.f., Hethcote, 1978, Hethcote and Thieme, 1985, van den Driessche and Watmough, 2002). In particular, it is a well-known fact that the pandemic-free equilibrium is necessarily stable if $\mathcal{R}_0 < 1$. In order to compute $\mathcal{R}_0$, we following the approach in Diekmann et al. (1990), and write the two equations determining the dynamics of infections as

$$\begin{bmatrix} \dot{I}_i \\ \dot{I}_j \end{bmatrix} = \underbrace{\begin{bmatrix} 2\alpha_i n_{ii} S_i & (\alpha_j n_{ij} + \alpha_i n_{ji}) S_i \\ (\alpha_j n_{ij} + \alpha_i n_{ji}) S_j & 2\alpha_j n_{jj} S_j \end{bmatrix}}_{F} \begin{bmatrix} I_i \\ I_j \end{bmatrix} - \underbrace{\begin{bmatrix} \gamma_i & 0 \\ 0 & \gamma_j \end{bmatrix}}_{V} \begin{bmatrix} I_i \\ I_j \end{bmatrix}$$

The *next generation matrix* FV^{-1} (evaluated at $t = t_0$, and thus $S_i(t_0) = S_j(t_0) \simeq 1$) is given by

$$FV^{-1} = \begin{bmatrix} 2\alpha_i n_{ii}/\gamma_i & (\alpha_j n_{ij} + \alpha_i n_{ji})/\gamma_j \\ (\alpha_j n_{ij} + \alpha_i n_{ji})/\gamma_i & 2\alpha_j n_{jj}/\gamma_j \end{bmatrix}.$$

From the results in Diekmann et al. (1990), we thus have that

$$\mathcal{R}_0 = \rho\left(FV^{-1}\right),$$

where $\rho\left(FV^{-1}\right)$ is the spectral radius of the next generation matrix. In our case, this is given by

$$\mathcal{R}_0 = \frac{1}{2}\left(\frac{2\alpha_i n_{ii}}{\gamma_i} + \frac{2\alpha_j n_{jj}}{\gamma_j}\right) + \frac{1}{2}\sqrt{\left(\frac{2\alpha_i n_{ii}}{\gamma_i} - \frac{2\alpha_j n_{jj}}{\gamma_j}\right)^2 + 4\frac{(\alpha_j n_{ij} + \alpha_i n_{ji})^2}{\gamma_i \gamma_j}}. \tag{21}$$

As in the case of $\lambda_{\max}$ in equation (20), we have that $\mathcal{R}_0$ is nondecreasing in n_{ij} and n_{ji}, and thus

$$\mathcal{R}_0 \geq \mathcal{R}_0|_{n_{ij}=n_{ji}=0} = \max\left\{\frac{2\alpha_i n_{ii}}{\gamma_i}, \frac{2\alpha_j n_{jj}}{\gamma_j}\right\}. \tag{22}$$

This confirms again that the disease can only be contained (that is, the pandemic-free equilibrium is stable) only if both countries' disease reproduction rate based on their domestic interactions is less than one.[9] Therefore, even if a country has a strict disease environment that would prevent a pandemic under autarky, it may be drawn into a world pandemic in the open economy equilibrium,

[9] Although the expressions for $\lambda_{\max}$ and $\mathcal{R}_0$ appear different, it is straightforward to show that a necessary condition

if its trade partner has a lax disease environment, as measured by its open economy domestic reproduction rate.

Figure 1 presents the results of simulations of our model for different values of the exogenous infection rate in Country 2. The starting point are two identical countries with exogenous infection rate $\alpha_1 = \alpha_2 = 0.04$. The rest of the parameter values are described in Appendix A.6. For this exogenous infection rate the global reproduction number is $\mathcal{R}_0 = 0.75$, and the open economy domestic reproduction rates are $\mathcal{R}_{01} = \mathcal{R}_{02} = 0.46$. Thus, the initial infection quickly dies out and there is no global pandemic. The fraction of recovered agents in the long run, $R_i(\infty)$, which is equal to the cumulative number of infected agents since we have not introduced deaths yet, is close to zero in both countries. The left panel of Figure 1 plots $R_i(\infty)$ as a function of $\mathcal{R}_0$ as we progressively increase α_2 from 0.04 to 0.10. The value of $\mathcal{R}_0$ is monotone in α_2 and increases from 0.75 to 1.46. Hence as the exogenous infection rate of Country 2 increases, the global reproduction rate increases beyond the critical value of 1, and the world experiences a global pandemic. Note how the fraction of the cumulative number of recovered agents rises rapidly once $\mathcal{R}_0$ increases beyond 1 and both countries go through increasingly severe pandemics. Note also the importance of cross-country contagion in the open economy. Even though nothing is changing in the domestic characteristics of Country 1, it is dramatically affected by the worsened conditions in Country 2. The right panel shows the evolution of the pandemic in Country 1 for different levels of severity of the disease environment in Country 2.[10] The more severe and rapid pandemics are associated with the highest values of α_2 (the lightest curve in the graph). As α_2 declines and $\mathcal{R}_0$ lowers down and crosses the value of 1, the evolution of inflections flattens and becomes longer, until the pandemic eventually disappears.

The value of $\mathcal{R}_0$ is critical to determine the stability of a pandemic-free equilibrium. However, it is not critical to determine the existence of a pandemic cycle in *each* country. For values of $\mathcal{R}_0$ close enough to 1, an individual country can experience a pandemic, even if the world as a whole does not, if the declining number of cases in the other country is sufficiently large. Similarly, even if $\mathcal{R}_0 > 1$, some countries might not experience a pandemic when $\mathcal{R}_0$ is close enough to 1, even if the world economy as a whole is, since cases might be rising sufficiently fast in the other country. In Figure 1, in fact, cases rise slowly when the economy crosses the $\mathcal{R}_0 = 1$ threshold. At that point, pandemics are small and happen only in the sick country, while the number of cases in the healthy country remain essentially steady. The peak of infections in both countries is a smooth function of the value of α_2.

for *both* $\lambda_{\max} < 0$ and $\mathcal{R}_0 < 1$ is

$$\frac{2\alpha_i n_{ii}}{\gamma_i} + \frac{2\alpha_j n_{jj}}{\gamma_j} - \frac{2\alpha_i n_{ii}}{\gamma_i}\frac{2\alpha_j n_{jj}}{\gamma_j} + \frac{(\alpha_j n_{ij} + \alpha_i n_{ji})^2}{\gamma_i \gamma_j} < 1.$$

If either $2\alpha_i n_{ii}/\gamma_i > 1$ or $2\alpha_j n_{jj}/\gamma_j > 1$, this condition cannot possibly hold.

[10] The color of each curve, correspond to the colors of the points in the left panel.

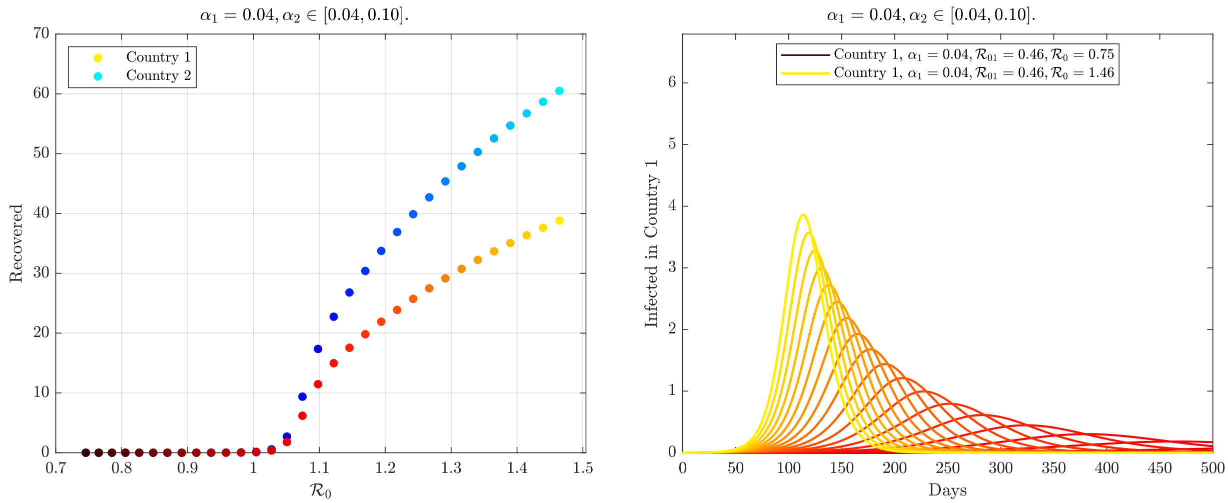

Figure 1: The Impact of Changes in the Exogenous Infection Rate in Country 2, α_2

3.5 Trade Integration and Global Pandemics

We now turn to the question of how globalization affects the likelihood that a pandemic-free equilibrium is achieved in all countries. Inspection of equations (20) and (22) might lead one to infer that avoiding a pandemic is always more difficult in a globalized world. One the one hand, it is obvious that, for given positive values of n_{ii} and n_{jj}, if the recovery rate is sufficiently low in *any* country in the world, a global pandemic affecting *all* countries cannot be avoided, even though the country with the higher recovery rate might well have avoided it under autarky. On the other hand, it would seem that even when $\gamma_i = \gamma_j$, the max operator in (20) and (22) implies that the pandemic-free equilibrium is less likely to be stable in the open economy. It is important to emphasize, however, that n_{ii} and n_{jj} are endogenous objects and will naturally be lower, the lower are trade frictions, as formalized in Proposition 2. Still, it seems intuitive that globalization will typically foster more human interactions, as these are necessary to materialize the gains associated with trade integration, and that this will generally lead to an increased likelihood of pandemics.

To explore this more formally, let us first consider a fully symmetric world in which all primitives of the model (population size, technology, trade barriers, recovery rates, etc.) are common in both countries, so that we have $n_{ii} = n_{jj} = n_{dom}$, $n_{ij} = n_{ji} = n_{for}$, $\alpha_i = \alpha_j = \alpha$, and $\gamma_i = \gamma_j = \gamma$. In such a case, we have

$$\lambda_{\max} = 2\alpha\left(n_{dom} + n_{for}\right) - \gamma; \qquad \mathcal{R}_0 = \frac{2\alpha\left(n_{dom} + n_{for}\right)}{\gamma}.$$

It then follows immediately from Proposition 3 that:

Proposition 5 *Suppose that countries are symmetric, in the sense that $L_i = L$, $Z_i = Z$, $\Gamma_{ij} = \Gamma$, $\alpha_i = \alpha$, and $\gamma_i = \gamma$ for all i. Then, a decline in any (symmetric) international trade friction increases $\mathcal{R}_0$ and decreases the range of parameters for which a pandemic-free equilibrium is stable.*

More generally, and as noted in footnote 9, a necessary condition for the pandemic-free equilibrium to be stable is

$$\frac{2\alpha_i n_{ii}}{\gamma_i} + \frac{2\alpha_j n_{jj}}{\gamma_j} - \frac{2\alpha_i n_{ii}}{\gamma_i}\frac{2\alpha_j n_{jj}}{\gamma_j} + \frac{(\alpha_j n_{ij} + \alpha_i n_{ji})^2}{\gamma_i \gamma_j} < 1, \tag{23}$$

and thus what is key for the effects of reductions of trade and mobility barriers on the likelihood of avoiding a pandemic is whether the left-hand-side of this expression increases or declines with those reductions in barriers.

Figure 2 investigates further the result in Proposition 5 for a case in which we introduce an asymmetry in the exogenous infection rate. We let $\alpha_1 = 0.04$ and $\alpha_2 = 0.07$ and study the cumulative number of recovered agents when we increase symmetric international trade (left panel) and mobility (right panel) frictions. The first point on both graphs, when $t_{12} = t_{21} = \mu_{12} = \mu_{21} = 1$, is one of the cases we studied in Figure 1. The large infection rate in Country 2 generates a pandemic in both countries. Globalization is essential to generate this pandemic. As both graphs illustrate, as we increase either tariffs or mobility restrictions, global interactions decline, and the total number of recovered agents decreases. Eventually, when the world is sufficiently isolated, the pandemic disappears and the pandemic-free equilibrium becomes stable. In both graphs, the value of $\mathcal{R}_0$ declines smoothly with frictions. The vertical line in the figure indicates the value of tariffs or mobility frictions, respectively, corresponding to $\mathcal{R}_0 = 1$.[11] Clearly, both types of barriers generate similar qualitative reductions in $R_i(\infty)$, although for this specific set of parameter values, the migration restrictions needed to eliminate the pandemic are larger than the corresponding trade frictions.

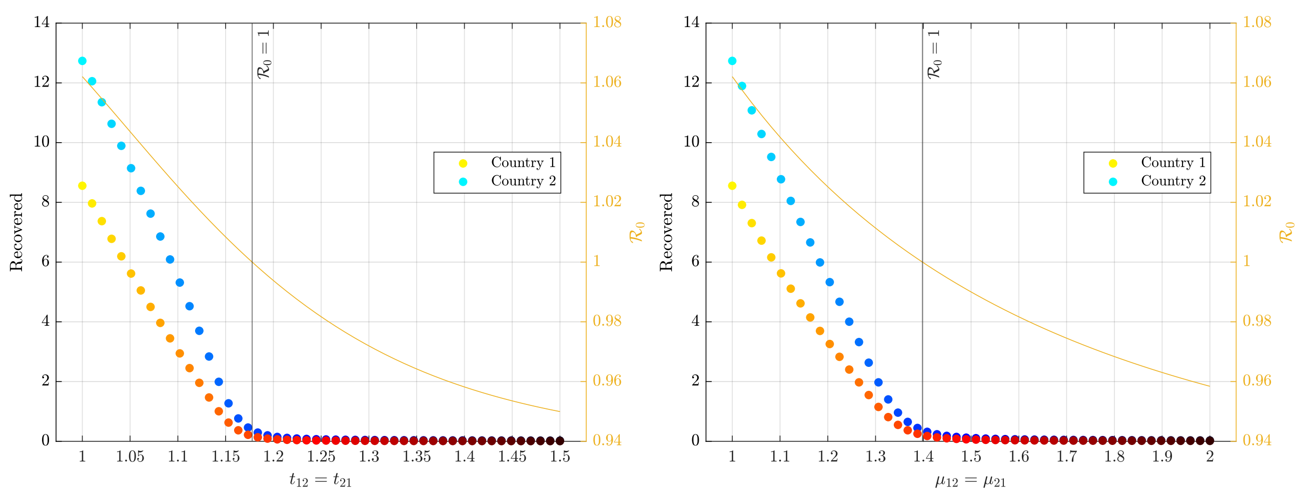

Figure 2: The Impact of Changes in Trade (left) and Mobility (right) Frictions

[11] Note that the value of $R_i(\infty)$, does not become zero for either country right at the point where tariffs or mobility frictions lead $\mathcal{R}_0$ to become greater than one. The reason is that even though one of the countries necessarily avoids a pandemic, it lingers close to its initial value of infections for a long time, which accumulates to a positive cumulative number of recovered agents.

Although in most cases condition (23) becomes tighter the lower are trade and mobility barriers, it is instructive to explore scenarios in which greater integration may actually reduce the risk of a pandemic. Suppose, in particular, that country j is a much lower risk environment, in the sense that α_j is very low – so infections are very rare – and γ_j is very high – so infected households quickly recover in that country. In the limiting case $\alpha_j \to 0$, condition (23) reduces to

$$\frac{2\alpha_i n_{ii}}{\gamma_i} + \frac{1}{\gamma_j}\frac{(\alpha_i n_{ji})^2}{\gamma_i \gamma_j} < 1.$$

For a high value of γ_j, it is then straightforward to see that the fall in country i's domestic interactions n_{ii} associated with a reduction in international barriers makes this constraint laxer, even if n_{ji} goes up with that liberalization. In those situations it is perfectly possible for a pandemic-free equilibrium worldwide to only be stable when barriers are low. The intuition for this result is straightforward. In such a scenario, globalization makes it economically appealing for agents from a high-risk country to increase their interactions with agents in a low-risk country, and despite the fact that overall interactions by these agents may increase, the reduction in domestic interactions in their own high-risk environment is sufficient to maintain the disease in check. We summarize this result as follows:

Proposition 6 *When the contagion rate α_i and the recovery rate γ_i vary sufficiently across countries, a decline in any international trade friction decreases $\mathcal{R}_0$ and increases the range of parameters for which a pandemic-free equilibrium is stable.*

An interesting implication of this result is that although it would seem intuitive that a healthy country should impose high restrictions to the inflow of individuals from a high-risk country where a disease has just broken out, in some cases such restrictions may in fact contribute to the spread of the disease in the high-risk country, which may then make a global pandemic inevitable unless mobility restrictions are set at prohibitive levels.

Figure 3 presents examples in which increases in trade and mobility barriers eliminate the possibility of a pandemic-free equilibrium. As we argued above, to generate these examples we need large differences in exogenous infection rates. The figure makes the exogenous infection rate in the healthy country, Country 1, extremely small at $\alpha_1 = 0.008$, and sets $\alpha_2 = 0.052$ (a standard value).[12] In both panels, increases in frictions now lead to increases in $\mathcal{R}_0$. Without frictions the pandemic-free equilibrium is stable. Agents in Country 2 interact sufficiently with the healthier Country 1, which helps them avoid the pandemic. As both economies impose more frictions, domestic interactions increase rapidly, while foreign interactions drop. This is bad new for Country 2, since its larger infection rate now leads to a pandemic. Perhaps surprisingly, it is also bad news for Country 1 since, although it interacts less with Country 2, it does so sufficiently to generate a pandemic there as well. Larger frictions, which decrease aggregate income in both countries

[12] Relative to the baseline parameters the example also lowers c to 0.1 and ϕ to 1.5. These additional changes increase the overall number of domestic and foreign interaction.

smoothly, also worsen the pandemic in both countries, at least when frictions are not too large; a clear case for free trade and mobility. Of course, as frictions increase further, eventually they isolate Country 1 sufficiently and so the severity of its local pandemic declines. In autarky, Country 1 avoids the pandemic completely, but at a large cost in the income of both countries. In contrast, higher frictions always worsen the pandemic in Country 2. Contacts with the healthy country are always beneficial, since they dilute interactions with locals, which are more risky.

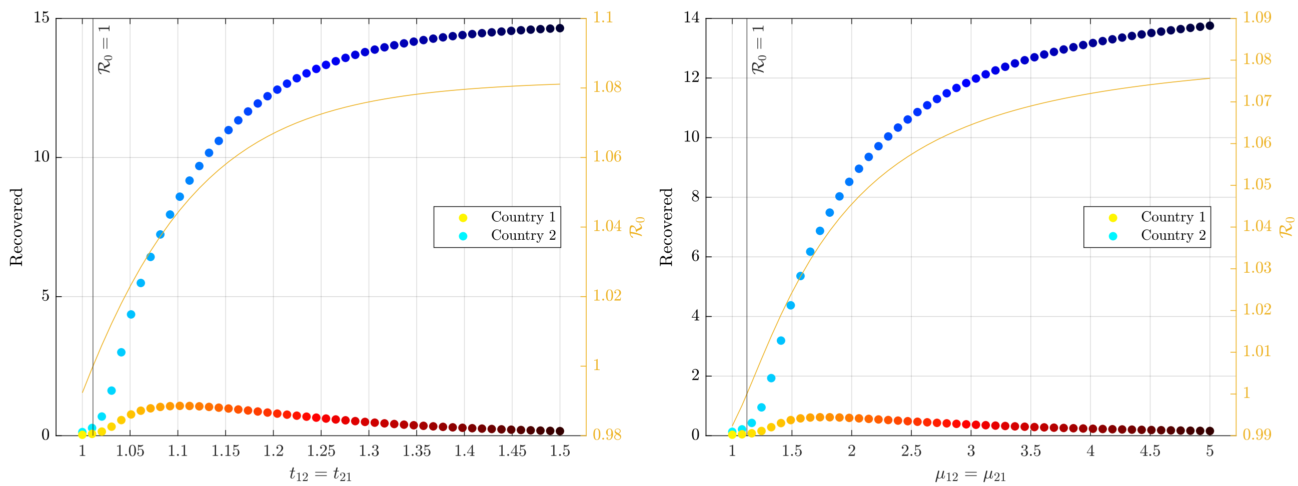

Figure 3: The Impact of Changes in Trade (left) and Mobility (right) Frictions with Large Differences in Infection Rates Across Countries ($\alpha_1 = 0.008$ and $\alpha_2 = 0.052$)

3.6 Pandemic Equilibrium

Having described in detail the existence and stability of a pandemic-free equilibrium, we next turn to a situation in which $\mathcal{R}_0 > 1$ and the resulting contagion dynamics lead to a pandemic. Building on the voluminous literature on multigroup models of disease transmission, it is well known that whenever the global reproduction rate satisfies $\mathcal{R}_0 > 1$, there exists a unique asymptotically globally stable 'pandemic' equilibrium in which the growth in the share of *worldwide* infected households necessarily increases for a period of time in every country, and then declines to a point at which infections vanish and the share of susceptible households in the population in each country $(S_i(\infty), S_j(\infty))$ takes a value strictly between 0 and 1 (see Hethcote, 1978).[13] Starting from equation (18), and going through analogous derivations as in the closed-economy case (see Appendix A.8), we obtain the following system of nonlinear equations pinning down the steady-state values $(S_i(\infty), S_j(\infty))$ of the share of susceptible households in each country in that pandemic

[13] Proving global stability of the endemic equilibrium is challenging for some variants of the SIR model, but for the simple one in (16)-(18), featuring permanent immunity and no vital dynamics, global stability of the endemic equilibrium is implied by the results in Hethcote (1978), particularly section 6.

equilibrium:

$$\ln S_i(\infty) = -\frac{2\alpha_i n_{ii}}{\gamma_i}(1 - S_i(\infty)) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}(1 - S_j(\infty)) \tag{24}$$

$$\ln S_j(\infty) = -\frac{2\alpha_j n_{jj}}{\gamma_j}(1 - S_j(\infty)) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i}(1 - S_i(\infty)) \tag{25}$$

Although we cannot solve the system in closed-form, we can easily derive some comparative statics. In particular, total differentiating we find (see Appendix A.8) that:

Proposition 7 *The steady-state values of S_i and S_j in a pandemic equilibrium are decreasing in n_{ii}, n_{jj}, n_{ij}, and n_{ji}, and are increasing in γ_i and γ_j.*

In our recurring fully symmetric case with $L_i = L$, $Z_i = Z$, and $\Gamma_{ij} = \Gamma$, $\alpha_i = \alpha_j = \alpha$, and $\gamma_j = \gamma$, the steady-state share of susceptible households in the population is identical in both countries and implicitly given by

$$\ln S_i(\infty) = -\frac{2\alpha(n_{dom} + n_{for})}{\gamma}(1 - S_i(\infty)),$$

Since from Proposition 3 $n_{dom} + n_{for}$ higher, the lower are trade frictions, we can conclude that:

Proposition 8 *Suppose that countries are symmetric, in the sense that $L_i = L$, $Z_i = Z$, $\Gamma_{ij} = \Gamma$, and $\gamma_i = \gamma$ for all i. Then, a decline in any (symmetric) international trade friction increases the share of each country's population that becomes infected during the pandemic.*

Figure 4 presents the evolution of the fraction of agents infected for different levels of trade frictions. It corresponds to the exercise on the left panel of Figure 2, so the lightest curves represent the evolution of the fraction of infected for the case with free trade ($t_{12} = t_{21} = 1$), and the darkest curves represent the case when $t_{12} = t_{21} = 1.5$. The figure illustrates nicely the results of Proposition 8 although for a case that does not impose full symmetry. In the figure, Country 2 has a higher exogenous infection rate than Country 1 ($\alpha_2 = 0.07 > 0.04 = \alpha_1$). Clearly, as we increase tariffs, the epidemic in both countries becomes less severe and prolonged. The peak of the infection curve declines monotonically, as does the total number of recovered agents. Eventually, although impossible to appreciate in the graph, high tariffs eliminate the pandemic altogether and infections decline monotonically from their initial value.

Although this result appears to hold even in the presence of significant asymmetries across countries, if countries differ enough in their epidemiological parameters, it may well be that a decline in international trade frictions actually ameliorates the pandemic by incentivizing agents in the high-risk country to shift more of their interactions to the low-risk country.

Proposition 9 *When the contagion rate α_i and the recovery rate γ_i vary sufficiently across countries, a decline in any international trade or mobility friction reduces the share of the population in the high-risk (high α_i, low γ_i) country that becomes infected during the pandemic. Such a decline*

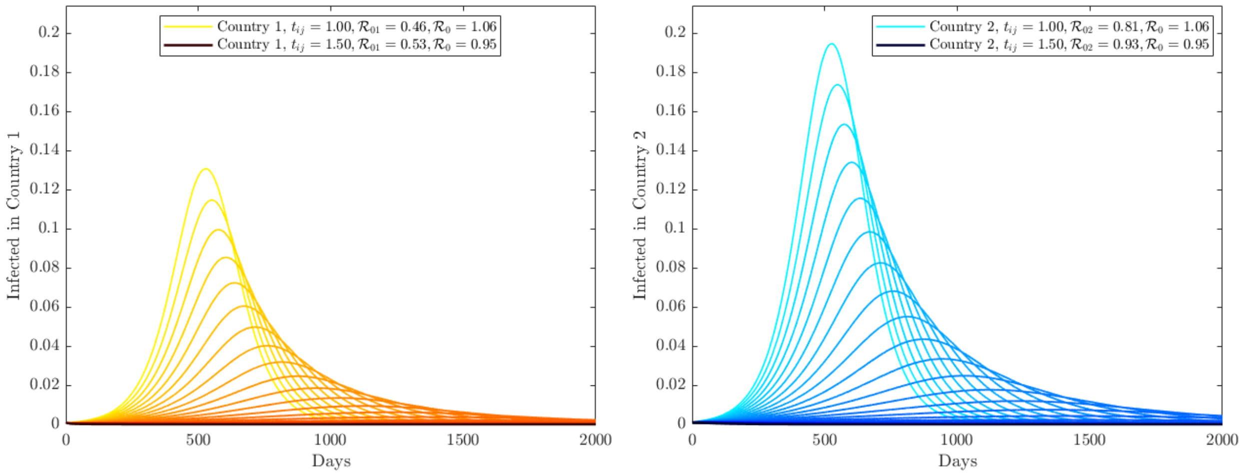

Figure 4: The Impact of Changes in Trade Frictions on the Evolution of Infections

in trade or mobility frictions may also reduce the share of the population in the low-risk (low α_i, high γ_i) country that become infected during the pandemic.

In Figure 5 we illustrate the result in Proposition 9 for the case with high differences in exogenous infection rates across countries that we presented in Figure 3. We focus on three specific exercises: A case with free trade where $t_{12} = t_{21} = 1$, another with intermediate tariffs where $t_{12} = t_{21} = 1.2$, and a third one where countries are in autarky. With free trade, there is no pandemic in either country. As we increase trade frictions, a pandemic develops in both countries, although it is much more severe in Country 2, the country with the higher exogenous infection rate. Still, the pandemic in Country 1 ends up infecting around 1% of the population. Moving to autarky eliminates the pandemic for Country 1, but makes it even more severe, faster, and with a higher peak, in Country 2. Closing borders helps the healthy country eliminate the pandemic only if trade is completely eliminated, and at the cost of a much more severe pandemic in Country 2 and larger income losses for everyone. Although Figure 5 uses countries of identical size and studies the case of changes in symmetric tariffs, we obtain very similar results when countries are asymmetric, or when Country 1, the healthy country, is the only country closing its borders. Similar examples can also be generated when considering mobility rather than trade frictions, as in the right panel of Figure 3. The essential ingredient for declines in international frictions to ameliorate the pandemic, on top of increasing incomes, is for countries to exhibit large asymmetries in epidemiological conditions.

3.7 Transitional Dynamics: A Second Wave

When $\mathcal{R}_0 > 1$ and the world economy converges to the pandemic steady-state equilibrium in equations (24) and (25), convergence to that steady-state may entail significantly richer than in the closed-economy SIR model. In particular, in the open economy, integrating the dynamics of infections in each country using the initial conditions $S_i(0) = S_j(0) = 1$ and $R_i(0) = R_j(0) = 0$,

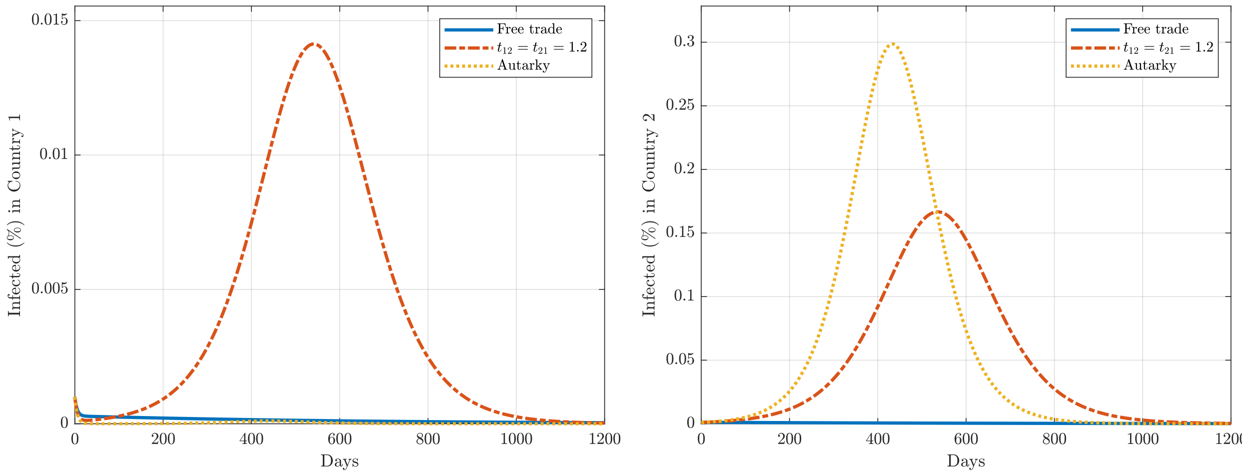

Figure 5: Evolution of Infections under Free Trade, Intermediate Trade Frictions, and Autarky with Large Differences in Infection Rates Across Countries ($\alpha_1 = 0.008$ and $\alpha_2 = 0.052$)

we have the following closed-form solutions for infections in each country at each point in time (I_{it}, I_{jt}) as a function of susceptibles in each country ($S_i(t)$, $S_j(t)$):

$$I_i(t) = 1 - S_i(t) + \frac{\log S_i(t) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{2\alpha_j n_{jj}} \log S_j(t)}{\frac{2\alpha_i n_{ii}}{\gamma_i} - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{2\alpha_j n_{jj}} \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{\gamma_i}}, \tag{26}$$

$$I_j(t) = 1 - S_j(t) + \frac{\log S_j(t) - \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{2\alpha_i n_{ii}} \log S_i(t)}{\frac{2\alpha_j n_{jj}}{\gamma_j} - \frac{\alpha_i n_{ji} + \alpha_j n_{ij}}{2\alpha_i n_{ii}} \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}}. \tag{27}$$

In the closed economy, there is necessarily a single wave of infections in the absence of a lock-down or other time-varying health policies. In contrast, in the open economy, it becomes possible for a country to experience multiple waves of infections, even in the absence of lock-downs or other time-varying health policies. From equations (26) and (27), the rate of growth of infections in each country is highest when $S_i(t) = S_j(t) = 1$, and declines as the number of susceptibles in each country falls, but the decline as $S_i(t)$ falls occurs at a different rate to the decline as $S_j(t)$ falls. It is this difference that creates the possibility of multiple waves. If one country has a wham-bam epidemic that is over very quickly in the closed economy, while the other country has an epidemic that builds slowly in the closed economy, this creates the possibility for the country with the quick epidemic in the closed economy to have multiple peaks of infections in the open economy. The first peak reflects the rapid explosion of infections in the country, which dissipates quickly. The second peak reflects which is in general smaller, reflects the evolution of the pandemic in its trading partner.

In Figure 6 we provide an example of such a case, in which Country 1 experiences two waves of infections in the open economy, whereas Country 2 experiences a single, more prolonged and severe, wave. Country 1 features a large value of α_1, but also a large value of γ_1. Thus, although the infection rate is large, people remain contagious only briefly (perhaps because of a good contact tracing program). The resulting domestic reproduction rate $\mathcal{R}_{01} = 1.08$ and the resulting first peak of the pandemic is relatively small and quick. Since Country 1 is assumed ten times smaller than Country 2, its small initial pandemic has no significant effect on Country 2. There, the infection rate is much smaller, but the disease remains contagious for much longer, leading to a larger $\mathcal{R}_{01} = 1.66$, which also results in a global reproduction number $\mathcal{R}_0 = 1.66$.[14] The result is a more protracted but also much longer singled-peaked pandemic in Country 2. This large pandemic does affect the smaller country through international economic interactions. The large country amounts for many of the interactions of the small country, which leads to the second wave of the pandemic in Country 2. Essential for this example is that countries have very different timings for their own pandemics in autarky, but that in the open economy the relationship is very asymmetric, with the small country having little effect on the large country but the large country influencing the small country significantly. If the interactions are large enough in both directions, both countries will end up with a synchronized pandemic with only one peak.

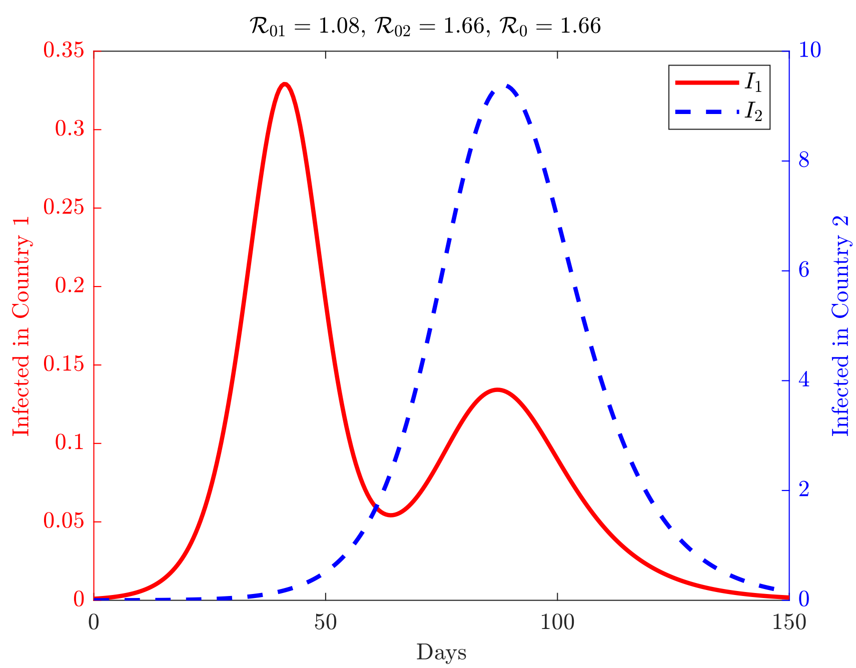

Figure 6: Multiple Waves of Infection in the Open Economy

[14] The parameter values used in the exercise are $\sigma = 4.5$, $L_1 = 2$, $L_2 = 20$, $d_{12} = d_{11}$, $c = 0.12$, $\alpha_1 = 0.69$, $\alpha_2 = 0.09$, $\gamma_1 = 2.1$ and $\gamma_2 = 0.18$. All other values are identical to the baseline case.

4 General-Equilibrium Induced Responses

In this section we introduce deaths into the system, although we continue to ignore direct behavioral responses at the household level, and we continue to assume that agents do not know their health status.[15] There will be two main implications of introducing deaths. First, the pandemic will now affect aggregate income (and thus welfare) in both countries, as households that die as a result of the pandemic will forego the net present discounted value of their future lifetime utility, which in our model is proportional to real income. Second, because deaths are not immediately replaced by new inflows into the labor force, the pandemic will affect labor supply and aggregate demand in each country, and this will impact equilibrium real wages.[16] In Section 5, we will introduce rich behavioral responses by agents as a result of the pandemic.

With this new assumption, the shares of households of each type evolve according to the following laws of motion (where we again ignore time subscripts to keep the notation tidy):

$$\dot{S}_i = -2n_{ii}(\mathbf{w}) \times \alpha_i \times S_i \times I_i - [n_{ij}(\mathbf{w}) \times \alpha_j + n_{ji}(\mathbf{w}) \times \alpha_i] \times S_i \times I_j \quad (28)$$

$$\dot{I}_i = 2n_{ii}(\mathbf{w}) \times \alpha_i \times S_i \times I_i + [n_{ij}(\mathbf{w}) \times \alpha_j + n_{ji}(\mathbf{w}) \times \alpha_i] \times S_i \times I_j - (\gamma_i + \eta_i) I_i \quad (29)$$

$$\dot{R}_i = \gamma_i I_i \quad (30)$$

$$\dot{D}_i = \eta_i I_i \quad (31)$$

There are two main differences between this dynamic system and the one above in (16)-(18). First, we now have four types of agents, as some infected agents transition to death rather than recovery. Second, we now need to make explicit the dependence of the contact rates $n_{ii}(\mathbf{w})$, $n_{ij}(\mathbf{w})$ and $n_{ij}(\mathbf{w})$ on the vector of equilibrium wages $\mathbf{w}$. Because the changes in each country's population induced by deaths affect wages, these contact rates are no longer time invariant, and evolve endogenously over the course of the pandemic. In particular, the equilibrium wage vector is determined by the following goods market clearing condition:

$$\sum\nolimits_{j \in \mathcal{J}} \pi_{ji}(\mathbf{w}) w_j (1 - D_j) L_j = w_i (1 - D_i) L_i,$$

where remember that $\pi_{ij}(\mathbf{w})$ and $n_{ij}(\mathbf{w})$ are given by (9) and (15), respectively.

We now show that this endogeneity of wages introduces a form of general equilibrium social distancing into the model. In particular, if the country with a worse disease environment experiences more deaths, its relative wage will rise. As this country's relative wage increases, its varieties become less attractive to agents in the country with the better disease environment compared to that country's domestic varieties. Therefore, purely from the general equilibrium force of changes in relative labor supplies, agents in the healthy country engage in a form of endogenous social distancing, in which they skew their interactions away from the country with a worse disease

[15] We implicitly assume that if one of the household members dies, the other one does too. So it is not only a passionate marriage, but also a *romantic* one (in the narrow sense of the word).

[16] We could easily introduce a set of agents that are symptomatic infected agents who also reduce their labor supply, but that would complicate the analysis and blur the comparison with the results in the previous section.

environment, as summarized in the following proposition (see Appendix A.9 for a proof):

Proposition 10 *If country j experiences more deaths than country i, the resulting change in relative wages (w_j/w_i) leads country i to reduce its interactions with country j and increase its interactions with itself (general equilibrium social distancing).*

Although we have illustrated this general equilibrium social distancing mechanism using death as the source of changes in relative labor supplies, if the disease also reduces the productivity of workers while they are infected, this additional source of labor supply movements introduces further subtle general equilibrium interactions between countries. For example, if the two countries' waves of infection are staggered in time, in the initial stages of the pandemic one country may experience a larger relative reduction in its labor supply (leading to endogenous social distancing in the other country), while in the later stages of the pandemic the other country experiences a larger relative reduction in its labor supply (leading to the opposite pattern of endogenous social distancing).

Another straightforward implication of explicitly modeling deaths is that they naturally affect aggregate income in both countries. More specifically, whenever changes in trade or mobility barriers affect population, aggregate real income $(w_i L_i/P_i)$ and aggregate welfare $(W_i L_i)$ are directly impacted by trade-induced changes in population. Because around $\mathcal{R}_0 = 1$ deaths are particularly responsive to changes in trade frictions, this effect is not necessarily negligible when evaluating the welfare implications of trade in a world with global pandemics.

5 Behavioral Responses

Up to this point, we have assumed that agents do not change their behavior during the pandemic, unless changes in relative wages induce them to do so. Implicitly, we were assuming that although households may observe that other households are dying, they do not understand the underlying cause of that death and go on with their lives.

In this section, we instead consider the more realistic (but also more complicated case) in which households realize that the deaths they observe are related to the outbreak of a pandemic. Following the approach in Farboodi et al. (2020), we continue to assume, however, that all infected individuals are asymptomatic, in the sense that household behavior is independent of their specific health status, though their actual behavior is shaped by their expectation of the probability with which they are susceptible, infected, or recovered households. How is that expectation formed? A natural assumption is that agents have rational expectations and their belief of the probability with which they have a specific health status is equal to the share of the population in their country with that particular health status.[17]

[17] This may raise the question among some readers as to how households are able to form this belief if, according to our assumptions, nobody observes their own health status. Fortunately, in order to form that belief, it is sufficient for individuals to have information on the number of deaths caused by the pandemic at each point in time, as well as common knowledge of the parameters η_i and γ_i. More specifically, notice from equation (31) that (i) I_{it} can be obtained from $I_{it} = (D_{it+1} - D_{it})\,\eta_i$; (ii) R_{it} can be obtained from equation (30) as $R_{it} = R_{it-1} + \gamma_i I_{it-1}$

We denote the individual beliefs of the probability of being infected, susceptible recovered or death with small cap letters, except for their belief of their death rate, which we denote by $k_i(t)$ to avoid a confusion with the notation we used for distance. The maximization problem of the individual is then given by

$$W_i^s(t) = \max_{n_{ij}(t)} \quad \int_0^\infty e^{-\xi t} \left[\left[Q_i\left(n_{ii}(t), n_{ij}(t)\right) - C_i\left(n_{ii}(t), n_{ij}(t)\right) \right] \left(1 - k_i(t)\right) \right] dt$$

$$\begin{aligned} \text{s.t.} \qquad \dot{s}_i(t) &= -s_i(t) \left[\left(\alpha_i n_{ii}(t) + \alpha_i n_{ii}^*(t)\right) I_i(t) + \left(\alpha_j n_{ij}(t) + \alpha_i n_{ji}^*(t)\right) I_j(t) \right], \\ \dot{i}_i(t) &= s_i(t) \left[\left(\alpha_i n_{ii}(t) + \alpha_i n_{ii}^*(t)\right) I_i(t) + \left(\alpha_j n_{ij}(t) + \alpha_i n_{ji}^*(t)\right) I_j(t) \right] \\ &\quad - \left(\gamma_i + \eta_i\right) i_i(t), \\ \dot{k}_i(t) &= \eta_i i_i(t), \end{aligned}$$

where ξ is the rate of time preference, and where from equation (6),

$$Q_i\left(n_{ii}(t), n_{ij}(t)\right) = w_i(t) \left(\sum\nolimits_{j \in \mathcal{J}} n_{ij}(t) \left(\frac{\tau_{ij} w_j(t)}{Z_j} \right)^{1-\sigma} \right)^{\frac{1}{(\sigma-1)}},$$

and

$$C_i\left(n_{ii}(t), n_{ij}(t)\right) = \frac{c}{\phi} \sum\nolimits_{j \in \mathcal{J}} \mu_{ij} \left(d_{ij}\right)^\rho \times \left(n_{ij}(t)\right)^\phi.$$

Notice that we denote with an asterisk variables chosen by *other* households that affects the dynamics of infection of a given household.[18] In equilibrium, aggregate consistency implies that $i_i(t) = I_i(t)$, $s_i(t) = S_i(t)$, and $k_i(t) = D_i(t)$.

The Hamiltonian of the problem faced by each household is given by

$$\begin{aligned} H(s, i, n_{ii}, n_{ij}, \theta^i, \theta^s, \theta^k) \quad = \quad & \left[Q_i\left(n_{ii}(t), n_{ij}(t)\right) - C_i\left(n_{ii}(t), n_{ij}(t)\right) \right] \left(1 - k_i(t)\right) e^{-\xi t} \\ & - \theta_i^s(t)\, s_i(t) \left[\left(\alpha_i n_{ii} + \alpha_i n_{ii}^*\right) I_i + \left(\alpha_j n_{ij} + \alpha_i n_{ji}^*\right) I_j \right] \\ & + \theta_i^i(t) \left[s_i(t) \left[\left(\alpha_i n_{ii} + \alpha_i n_{ii}^*\right) I_i + \left(\alpha_j n_{ij} + \alpha_i n_{ji}^*\right) I_j \right] - \left(\gamma_i + \eta_i\right) i_i(t) \right] \\ & + \theta_i^k(t)\, \eta_i i_i(t). \end{aligned}$$

Hence, the optimality condition with respect to the choice of n_{ij} is

$$\left[\frac{\partial Q_i\left(n_{ii}(t), n_{ij}(t)\right)}{\partial n_{ij}} - \frac{\partial C_i\left(n_{ii}(t), n_{ij}(t)\right)}{\partial n_{ij}} \right] \left(1 - k_i(t)\right) e^{-\xi t} = \left[\theta_i^s(t) - \theta_i^i(t) \right] s_i(t)\, \alpha_j I_j(t), \quad (32)$$

with $R_{i0} \simeq 0$; and S_{it} is then trivially $S_{it} = 1 - I_{it} - R_{it} - D_{it}$. Obviously, to update their expectations on their health status, agents need to have common knowledge on all remaining parameters of the model, so they can form expectations on the actions (n_{ii} and n_{ij}) taken by other agents.

[18] For instance, though the aggregate domestic rate of contact in i is $2\alpha_i n_{ii}$, a household has no control over how many buyers visit the household's seller, so the household only controls the rate $\alpha_i n_{ii}$ of contacts generated by the household's buyer.

while the optimality conditions associated with the co-state variables are given by:

$$-\dot{\theta}_i^s(t) = -\left[\theta_i^s(t) - \theta_i^i(t)\right]\left[\left(\alpha_i n_{ii} + \alpha_i n_{ii}^*\right) I_i + \left(\alpha_j n_{ij} + \alpha_i n_{ji}^*\right) I_j\right], \tag{33}$$

$$-\dot{\theta}_i^i(t) = \eta_i \theta_i^k(t) - \left(\gamma_i + \eta_i\right)\theta_i^i(t), \tag{34}$$

$$-\dot{\theta}_i^k(t) = -\left[Q_i\left(n_{ii}(t), n_{ij}(t)\right) - C_i\left(n_{ii}(t), n_{ij}(t)\right)\right] e^{-\xi t}. \tag{35}$$

Finally, the transversality conditions are

$$\begin{aligned} \lim_{t\to\infty} \theta_i^i(t)\, i_i(t) &= 0, \\ \lim_{t\to\infty} \theta_i^s(t)\, s_i(t) &= 0, \\ \lim_{t\to\infty} \theta_i^k(t)\, k_i(t) &= 0. \end{aligned}$$

This is obviously a rather complicated system characterized by several differential equations, and two (static) optimality conditions for the choices of n_{ii} and n_{ij} in each country. Nevertheless, in Appendix A.10, we are able to show the following Lemma.

Lemma 1 *Along the transition path, we must have* $\theta_i^s(t) - \theta_i^i(t) \geq 0$ *for all* t.

From this result, and from inspection of the static optimality condition in equation (32), it then follows that

$$\frac{\partial Q_i\left(n_{ii}(t), n_{ij}(t)\right)}{\partial n_{ij}} > \frac{\partial C_i\left(n_{ii}(t), n_{ij}(t)\right)}{\partial n_{ij}}$$

as long as $I_j(t) > 0$. In words, during a pandemic human interactions are depressed by social distancing practices by individuals.

To complete the description of the model, we need to specify the general equilibrium determination of wages. As in the version of our model with deaths in Section 4, we again have that wages are determined by the system of equations

$$\sum_{j\in\mathcal{J}} \pi_{ji}(\mathbf{w},t) \times w_j(t) \times \left(1 - D_j(t)\right) L_j = w_i(t) \times \left(1 - D_i(t)\right) \times L_i.$$

Importantly, however, the trade shares $\pi_{ji}(\mathbf{w},t)$ are now impacted by the fact that the level of interactions $n_{ij}(t)$ are directly affected by the dynamics of the pandemic. Still, computationally, it is straightforward to solve for a dynamic equilibrium in which $\pi_{ij}(\mathbf{w},t) = X_{ij}(t) / \sum_{\ell\in\mathcal{J}} X_{i\ell}(t)$, and $X_{ij}(t) = n_{ij}(t)\, p_{ij}(t)\, q_{ij}(t)\left(1 - D_i(t)\right) L_i$. More specifically, the dynamic model can be solved through a backward shooting algorithm. We are in the process of implementing this algorithm. The results in Farboodi et al. (2020) suggest that social distancing can have a marked impact on economic activity, which in our model would translate into a large collapse in international trade at the outset of a pandemic.

Adjustment Costs and the Risk of a Pandemic

Despite the potential for significant disruptions in international trade during a pandemic, a clear implication of the first-order condition (32) is that as long as $I_i(t) = I_j(t) = 0$, human interactions are at the same level as in a world without the potential for pandemics. In other words, although we have generated rich dynamics of international trade during a pandemic, as soon as a pandemic is overcome (via herd immunity or via the arrival of a vaccine), our model predicts that life goes back to normal instantly. We next explore an extension of our model that casts doubts on the notion of a quick V-shape recovery in economic activity and international trade flows after a global pandemic.

The main novel feature we introduce is adjustment costs associated with changes in the measures of human contacts $n_{ii}(t)$ and $n_{ij}(t)$. More specifically, we assume that whenever a household wants to change the measure of contacts $n_{ij}(t)$, it needs to pay a cost $\psi_1 \max[\dot{n}_{ij}(t), 0]^{\psi_2}$, where $\psi_2 > 1$. An analogous adjustment cost function applies to changes in domestic interactions n_{ii}. Notice that this formulation assumes that reducing the measure of contacts is costless. This leads to the following modified first-order condition for the choice of n_{ij} at any point in time t_0 (an analogous condition holds for n_{ii}):

$$\int_{t_0}^{\infty} e^{-\xi t} \left[\frac{\partial Q_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}} - \frac{\partial C_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}} \right] (1 - k_i(t)) dt$$
$$= \left[\theta_i^s(t_0) - \theta_i^i(t_0) \right] S_i(t_0) \beta_j I_j(t_0) + \psi_1 \psi_2 \dot{n}_{ij}(t_0)^{\psi_2 - 1}$$

if $\dot{n}_{ij}(t_0) \geq 0$, and

$$\int_{t_0}^{\infty} e^{-\xi t} \left[\frac{\partial Q_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}} - \frac{\partial C_i(n_{ii}(t), n_{ij}(t))}{\partial n_{ij}} \right] (1 - k_i(t)) dt = \left[\theta_i^s(t_0) - \theta_i^i(t_0) \right] S_i(t_0) \beta_j I_j(t_0)$$

if $\dot{n}_{ij}(t_0) < 0$. The rest of the system is as before with the added feature that the values of $n_{ii}(t)$ and $n_{ij}(t)$ are now state variables, with exogenous initial conditions $n_{ii}(0)$ and $n_{ij}(0)$.

As the first-order condition makes evident, the choice of $n_{ii}(t)$ and $n_{ij}(t)$ is now forward looking. This has two important implications. First, if the economy goes through a pandemic that destroys a lot of *contacts*, the recovery will not be immediate as households will optimally smooth the growth of contacts over a (potentially) long future. Second, if households anticipate that the probability of a future pandemic is $\lambda > 0$, the growth in the resurgence of human interactions will be slower than in the world in which the perceived probability of a future pandemic is 0, and the more so the larger is λ. As a result, if due to recency effects, households perceive a particularly high risk of future pandemics in the aftermath of a pandemic, this will again work to slow the recovery of international trade flows after a pandemic occurs.

6 Conclusions

Although globalization brings aggregate economic gains, it is often argued that it also makes societies more vulnerable to disease contagion. In this paper, we develop a new conceptual framework to study the interplay between trade integration (globalization) and the spread and persistence of pandemics. We jointly microfound both the canonical model of international trade from economics (the gravity equation) with the seminal model pandemics from epidemiology (the Susceptible-Infected-Recovered (SIR) model) using a theory of human interaction. Through jointly modelling these two phenomena, we highlight a number of subtle interactions between them. On the one hand, the contact rate among individuals, which is a central parameter in benchmark epidemiology models, is endogenous in our framework, and responds to both economic forces (e.g., the gains from international trade) and to the dynamics of the pandemic (e.g., the perceived health risk associated with business travel). On the other hand, we study how the emergence of a pandemic and the perceived risk of future outbreaks shapes the dynamics of international trade, and the net gains from international trade once the death toll from the pandemic is taken into account.

Even if the disease does not directly affect economic variables because it has no effects of the ability to work and trade or on mortality, globalization influences the dynamics of the disease by changing patterns of human interaction. If countries are symmetric, a decline in any (symmetric) international trade friction also leads to an overall increase in the total number of human interactions (domestic plus foreign), which *increases* the likelihood of a pandemic occurring. In this case, even if a pandemic would not be self-sustaining in a country in the closed economy, it can be self-sustaining in an open economy. In contrast, if countries are sufficiently different from one another in terms of their primitive epidemiological parameters (e.g., as a result of different health policies), a decline in any international trade friction can have the opposite effect of *decreasing* the likelihood of a pandemic occurring. When one country has a much worse disease environment than the other, trade liberalization can reduce the share of that country's interactions that occur in this bad disease environment, thereby taking the global economy below the threshold for a pandemic to be self-sustaining for the world as a whole. When a pandemic occurs in the open economy, we show that its properties are influenced by the disease environments in all countries, such that even without lock downs, multiple waves of infection can occur in the open economy, when there would only be a single wave in each country in the closed economy.

If the disease reduces the ability of agents to work and trade and affects mortality, globalization and pandemics interact through two further channels: general equilibrium effects from changes in relative labor supplies and behavioral effects as individual agents internalize the threat of costly infection and make different decisions about where to interact with other agents. To isolate these two channels, we first assume that agents remain unaware of the source of infection, such that only general equilibrium effects operate. In this case, a country with a worse disease environment experiences a larger reduction in labor supply, which in turn leads to an increase in its relative wage. This wage increase reduces the share of interactions that occur in that country's bad disease environment and increases the share that occur in better disease environments, which again can take the global economy below the threshold for a pandemic to be self-sustaining. Allowing individuals to become aware of the source of infection and optimally adjust their behavior in response introduces additional feedbacks, as agents in each country reduce the relative number of varieties that they source from countries with larger outbreaks of the disease. In the presence of costs of adjusting international trade relationships, changes in the perception of the likelihood of future pandemics occurring can have permanent effects on current patterns of international trade.

While our model is necessarily abstract in many dimensions, we capture the key idea that both international trade and disease transmission involve human interaction, which introduces a rich interdependence between them over the course of a pandemic.

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A Appendix

A.1 Second-Order Conditions for Choice of n_{ij}

From equation (6), we obtain, for all $j \in \mathcal{J}$,

$$
\begin{aligned}
\frac{\partial W(i)}{\partial n_{ij}} &= \frac{w_i}{(\sigma-1)}\left(\sum\nolimits_{j\in\mathcal{J}} n_{ij}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\right)^{\frac{1}{(\sigma-1)}-1}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma} - c\mu_{ij}\left(d_{ij}\right)^{\rho}\left(n_{ij}\right)^{\phi-1} \ ; \\
\frac{\partial W(i)}{\partial\left(n_{ij}\right)^2} &= \frac{w_i}{(\sigma-1)}\left(\frac{2-\sigma}{\sigma-1}\right)\left(\sum\nolimits_{j\in\mathcal{J}} n_{ij}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\right)^{\frac{1}{(\sigma-1)}-2}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma} \\
&\quad -(\phi-1)\,c\mu_{ij}\left(d_{ij}\right)^{\rho}\times\left(n_{ij}\right)^{\phi-2} \\
&= \left(\frac{2-\sigma}{\sigma-1}\right)\left(\sum\nolimits_{j\in\mathcal{J}} n_{ij}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\right)^{-1}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma} c\mu_{ij}\left(d_{ij}\right)^{\rho}\times\left(n_{ij}\right)^{\phi-1} \\
&\quad -(\phi-1)\,c\mu_{ij}\left(d_{ij}\right)^{\rho}\times\left(n_{ij}\right)^{\phi-2} \\
&= c\mu_{ij}\left(d_{ij}\right)^{\rho}\times\left(n_{ij}\right)^{\phi-2}\left[\left(\frac{1}{(\sigma-1)}-1\right)\left(\frac{n_{ij}\frac{\tau_{ij}w_j}{Z_j}}{\sum_{j\in\mathcal{J}} n_{ij}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}}\right)^{1-\sigma} - (\phi-1)\right]; \\
\frac{\partial^2 W(i)}{\partial n_{ij}\partial n_{ii}} &= \frac{w_i}{(\sigma-1)}\left(\frac{2-\sigma}{\sigma-1}\right)\left(\sum\nolimits_{j\in\mathcal{J}} n_{ij}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\right)^{\frac{1}{(\sigma-1)}-2}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\left(\frac{\tau_{ii}w_i}{Z_i}\right)^{1-\sigma}.
\end{aligned}
$$

Notice that $\frac{\partial W(i)}{\partial(n_{ij})^2} < 0$ if only if:

$$
\left(\frac{2-\sigma}{\sigma-1}\right)\left(\frac{n_{ij}\frac{\tau_{ij}w_j}{Z_j}}{\sum_{j\in\mathcal{J}} n_{ij}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}}\right)^{1-\sigma} < (\phi-1)\,,
$$

so this condition could be violated for large enough τ_{ij}, unless $\sigma > 2$, in which case the condition is surely satisfied as long as $\phi(\sigma-1) > 1$.

Next note that

$$
\left(\frac{\partial^2 W(i)}{\partial n_{ij}\partial n_{ii}}\right)^2 = \left(\frac{w_i}{\sigma-1}\frac{2-\sigma}{\sigma-1}\left(\sum\nolimits_{j\in\mathcal{J}} n_{ij}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\right)^{\frac{1}{(\sigma-1)}-2}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\left(\frac{\tau_{ii}w_i}{Z_i}\right)^{1-\sigma}\right)^2 = \Xi^2
$$

and

$$
\begin{aligned}
\frac{\partial W(i)}{\partial (n_{ii})^2}\frac{\partial W(i)}{\partial (n_{ij})^2} &= \left(\begin{array}{c} \frac{1}{(\sigma-1)}\frac{2-\sigma}{\sigma-1} w_i \left(\sum_{j\in\mathcal{J}} n_{ij}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\right)^{\frac{1}{(\sigma-1)}-2} \left(\frac{\tau_{ii}w_i}{Z_i}\right)^{1-\sigma}\left(\frac{\tau_{ii}w_i}{Z_i}\right)^{1-\sigma} \\ -(\phi-1)\, c\mu_{ii}(d_{ii})^{\rho}\times (n_{ii})^{\phi-2} \end{array}\right) \\
&\quad \times \left(\begin{array}{c} \frac{1}{(\sigma-1)}\frac{2-\sigma}{\sigma-1} w_i \left(\sum_{j\in\mathcal{J}} n_{ij}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\right)^{\frac{1}{(\sigma-1)}-2} \left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma}\left(\frac{\tau_{ij}w_j}{Z_j}\right)^{1-\sigma} \\ -(\phi-1)\, c\mu_{ij}(d_{ij})^{\rho}\times (n_{ij})^{\phi-2} \end{array}\right) \\
&= \Xi^2 - \varkappa_{ij}^{i} - \varkappa_{ij}^{j} + \varpi_{ij},
\end{aligned}
$$

where $\varkappa_{ij}^{i} < 0$ and $\varkappa_{ij}^{j} < 0$, and $\varpi_{ij} > 0$, whenever $\sigma > 2$ and $\phi > 1$.

In sum, when $\sigma > 2$ and $\phi(\sigma-1) > 0$, we have

$$
\frac{\partial W(i)}{\partial (n_{ii})^2}\frac{\partial W(i)}{\partial (n_{ij})^2} > \left(\frac{\partial^2 W(i)}{\partial n_{ij}\partial n_{ii}}\right)^2,
$$

and the second-order conditions are met.

A.2 Proof of Proposition 2

Proof of part a):

From equation (7), we can write

$$
n_{ii}(\mathbf{w}) = (c(\sigma-1)\mu_{ii})^{-1/(\phi-1)} (d_{ii})^{-\frac{\rho+(\sigma-1)\delta}{\phi-1}} \left(\frac{t_{ii}}{Z_i}\right)^{-\frac{\sigma-1}{(\phi-1)}} \left(\frac{w_i}{P_i}\right)^{-\frac{\sigma-2}{\phi-1}},
$$

but remember from (13) that

$$
\frac{w_i}{P_i} = (\pi_{ii})^{-\frac{(\phi-1)}{\phi(\sigma-1)-1}} \times \left(\frac{(Z_i)^{\phi(\sigma-1)}}{c(\sigma-1)} (\Gamma_{ii})^{-\varepsilon(\phi-1)}\right)^{\frac{1}{\phi(\sigma-1)-1}}.
$$

This implies that, in order to study the effect of international trade frictions on $n_{ii}(\mathbf{w})$, it suffices to study their effect on π_{ii}, with the dependence of n_{ii} on π_{ii} being monotonically positive. Now from

$$
\pi_{ii} = \frac{(w_i/Z_i)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times (\Gamma_{ii})^{-\varepsilon}}{\sum_{\ell\in\mathcal{J}} (w_\ell/Z_\ell)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times (\Gamma_{i\ell})^{-\varepsilon}},
$$

it is clear that the impact effect of a lower $\Gamma_{i\ell}$ is to decrease π_{ii} and thus to decrease n_{ii}. To take into account general-equilibrium forces, we can write equation (14) as

$$\frac{(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ii})^{-\varepsilon}}{(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ii})^{-\varepsilon}+(Z_j/\omega)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ij})^{-\varepsilon}}L_i+\frac{(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ji})^{-\varepsilon}}{(Z_j/\omega)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{jj})^{-\varepsilon}+(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ji})^{-\varepsilon}}\omega L_j=L_i, \tag{A.1}$$

where $\omega \equiv w_j/w_i$ is the relative wage in country j. From this equation, it is easy to see that if Γ_{ij} falls, ω cannot possibly decrease. If it did, both terms in the left-hand-side of (A.1) would fall. But if ω goes up, then π_{ii} goes up *by more* than as implied by the direct fall in Γ_{ij}. Similarly, if Γ_{ji} falls, π_{ij} falls on impact, so ω needs to increase to re-equilibriate the labor market, and again π_{ii} must decline.

Because the results above hold for Γ_{ij} and Γ_{ji}, they must hold for any of the constituents of those composite parameters.

Proof of part b):

Note from equations (2), (5), and (12) that

$$\frac{c}{\phi}\sum\nolimits_{j\in\mathcal{J}}\mu_{ij}\left(d_{ij}\right)^{\rho}\times\left(n_{ij}\right)^{\phi}=\frac{1}{\phi\left(\sigma-1\right)}\frac{w_i}{P_i}.$$

In part a) of the proof, we have established that when any international trade friction decreases, π_{ii} down, and from (13), w_i/P_i goes up. Thus, $\mu_{ii}\left(d_{ii}\right)^{\rho}\times\left(n_{ii}\right)^{\phi}+\mu_{ij}\left(d_{ij}\right)^{\rho}\times\left(n_{ij}\right)^{\phi}$ goes up when any international trade friction decreases. But because n_{ii} goes down and μ_{ij} and d_{ij} (weakly) go down, it must be the case that n_{ij} increases.

A.3 Proof of Proposition 3

We begin by considering the case with general country asymmetries. Consider the sum

$$\mu_{ii}\left(d_{ii}\right)^{\rho}\times\left(n_{ii}\right)^{\phi}+\mu_{ij}\left(d_{ij}\right)^{\rho}\times\left(n_{ij}\right)^{\phi}.$$

Differentiating:

$$\phi\left[\mu_{ii}\left(d_{ii}\right)^{\rho}\times\left(n_{ii}\right)^{\phi-1}\underbrace{dn_{ii}}_{<0}+\mu_{ij}\left(d_{ij}\right)^{\rho}\times\left(n_{ij}\right)^{\phi-1}dn_{ij}\right]+\underbrace{d\left(\mu_{ij}\left(d_{ij}\right)^{\rho}\right)}_{\leq 0}\times\left(n_{ij}\right)^{\phi}>0. \tag{A.2}$$

Clearly, we must have

$$\mu_{ii}\left(d_{ii}\right)^{\rho}\times\left(n_{ii}\right)^{\phi-1}dn_{ii}+\mu_{ij}\left(d_{ij}\right)^{\rho}\times\left(n_{ij}\right)^{\phi-1}dn_{ij}>0.$$

So if

$$\mu_{ii}\left(d_{ii}\right)^{\rho}\left(n_{ii}\right)^{\phi-1}>\mu_{ij}\left(d_{ij}\right)^{\rho}\times\left(n_{ij}\right)^{\phi-1},$$

we must have

$$dn_{ij} > -dn_{ii},$$

which would prove the Proposition.

Now, from the FOC for the choice of n's, that is equation (7),

$$\begin{aligned}
\mu_{ii}\left(d_{ii}\right)^{\rho}\left(n_{ii}\right)^{\phi-1} &= \left(\frac{w_i}{P_i}\right)^{1/(\phi-1)} \frac{\left(P_i\right)^{\frac{\sigma-1}{(\phi-1)}}}{(\sigma-1)\,c} \times \left(\frac{\left(d_{ii}\right)^{\delta} t_{ii} w_i}{Z_i}\right)^{-\frac{\sigma-1}{(\phi-1)}} \\
\mu_{ij}\left(d_{ij}\right)^{\rho}\left(n_{ij}\right)^{\phi-1} &= \left(\frac{w_i}{P_i}\right)^{1/(\phi-1)} \frac{\left(P_i\right)^{\frac{\sigma-1}{(\phi-1)}}}{(\sigma-1)\,c} \times \left(\frac{\left(d_{ij}\right)^{\delta} t_{ij} w_j}{Z_j}\right)^{-\frac{\sigma-1}{(\phi-1)}},
\end{aligned}$$

so a sufficient condition for the result is

$$\frac{\left(d_{ii}\right)^{\delta} t_{ii} w_i}{Z_i} < \frac{\left(d_{ij}\right)^{\delta} t_{ij} w_j}{Z_j}.$$

This amounts to prices for domestic varieties being lower than prices for foreign varieties. This makes sense, in such a case, desired quantities of domestic varieties will be higher, and the marginal benefit of getting more of them will be higher.

Note finally that with full symmetry, we must have $w_i = w_j$ and $Z_j = Z_i$, and the condition above trivially holds since $t_{ij} > t_{ii}$ and $d_{ij} > d_{ii}$.

A.4 Proof of Proposition 4

Note from equation (11), that we can write

$$\begin{aligned}
\frac{w_i}{P_i} &= const \times \left(\left(\frac{1}{Z_i}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \left(\Gamma_{ii}\right)^{-\varepsilon} + \left(\frac{\omega}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \left(\Gamma_{ij}\right)^{-\varepsilon}\right)^{\frac{(\phi-1)}{\phi(\sigma-1)-1}} \\
\frac{w_j}{P_i} &= const \times \omega \left(\left(\frac{1}{Z_i}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \left(\Gamma_{ii}\right)^{-\varepsilon} + \left(\frac{\omega}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \left(\Gamma_{ij}\right)^{-\varepsilon}\right)^{\frac{(\phi-1)}{\phi(\sigma-1)-1}}
\end{aligned}$$

where $\omega = w_j/w_i$. Plugging in (7), we have

$$n_{ii} = const \times \left(\frac{w_i}{P_i}\right)^{-\frac{\sigma-2}{\phi-1}} \times \left(\left(\frac{1}{Z_i}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \left(\Gamma_{ii}\right)^{-\varepsilon} + \left(\frac{\omega}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \left(\Gamma_{ij}\right)^{-\varepsilon}\right)^{-\frac{\sigma-2}{\phi(\sigma-1)-1}},$$

and thus n_{ii} increases in ω. Next, note

$$
\begin{aligned}
n_{ij} &= const \times \left(\frac{w_j}{P_i}\right)^{-\frac{\sigma-1}{(\phi-1)}} \left(\frac{w_i}{P_i}\right)^{1/(\phi-1)} \\
&= const \times \omega^{-\frac{\sigma-1}{(\phi-1)}} \left(\left(\frac{1}{Z_i}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} (\Gamma_{ii})^{-\varepsilon} + \left(\frac{\omega}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} (\Gamma_{ij})^{-\varepsilon}\right)^{-\frac{\sigma-2}{\phi(\sigma-1)-1}}
\end{aligned}
$$

The effect of ω may look ambiguous, but in fact we have that n_{ij} decreases if ω goes up. To see this, note that

$$
\frac{\partial \omega^{-a}\left(b+c\omega^{-d}\right)^{-g}}{\partial \omega} = -\frac{(a-dg)\,c+ab\omega^{d}}{\left(\frac{1}{\omega^{d}}\left(c+b\omega^{d}\right)\right)^{g}\omega^{a}\omega\left(c+b\omega^{d}\right)},
$$

which is negative if $a-dg>0$. But here we have

$$
a-dg=\frac{\sigma-1}{(\phi-1)}-\frac{\phi(\sigma-1)}{\phi-1}\frac{\sigma-2}{\phi(\sigma-1)-1}=\frac{\sigma-1}{\phi(\sigma-1)-1}>0.
$$

In sum, n_{ij} decreases in ω. Because an increase in L_i/L_j increases in ω (from straightforward use of the implicit function theorem to (14)), the Proposition follows.

Notice also that

$$
\begin{aligned}
n_{ji} &= const \times \left(\frac{w_i}{P_j}\right)^{-\frac{\sigma-1}{(\phi-1)}} \left(\frac{w_j}{P_j}\right)^{1/(\phi-1)} \\
&= const \times \omega^{\frac{\sigma-1}{(\phi-1)}} \left(\left(\frac{\omega}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} (\Gamma_{jj})^{-\varepsilon} + \left(\frac{1}{Z_i}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} (\Gamma_{ji})^{-\varepsilon}\right)^{-\frac{\sigma-2}{\phi(\sigma-1)-1}},
\end{aligned}
$$

and by an analogous argument above, we have that n_{ji} increases in ω, and thus an increase in population in i leads to an increase n_{ji} (while also decreasing n_{jj}).

A.5 Extensions of Economic Model

In this Appendix, we flesh out some of the details of the four extensions of our framework mentioned in section 2.3 of the main text.

A. Traveling Costs in Terms of Labor

If traveling costs are specified in terms of labor (rather than utility), welfare at the household level depends only on consumption

$$
W_i = \left(\sum_{j\in\mathcal{J}} \int_0^{n_{ij}} q_{ij}(k)^{\frac{\sigma-1}{\sigma}} dk\right)^{\frac{\sigma}{\sigma-1}},
$$

and the implied demand (for a given n_{ii} and n_{ij}) is given by

$$q_{ij}(k) = \left(\frac{p_{ij}}{P_i}\right)^{-\sigma} \frac{\mathcal{I}_i}{P_i},$$

where $\mathcal{I}_i$ is household income, which is given by

$$\mathcal{I}_i = w_i \left(1 - \frac{c}{\phi} \sum_{j \in \mathcal{J}} \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi}\right),$$

since the household now needs to hire labor to be able to secure final-good differentiated varieties, and where

$$P_i = \left(\sum_{j \in \mathcal{J}} n_{ij} p_{ij}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

Welfare can therefore be rewritten as

$$W_i = \frac{\mathcal{I}_i}{P_i} = w_i \left(1 - \frac{c}{\phi} \sum_{j \in \mathcal{J}} \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi}\right) \left(\sum_{j \in \mathcal{J}} n_{ij} p_{ij}^{1-\sigma}\right)^{\frac{1}{\sigma-1}}$$

The first-order condition for the choice of n_{ij} delivers:

$$n_{ij} = (c(\sigma - 1))^{-\frac{1}{\phi-1}} \left(\frac{\mathcal{I}_i}{w_i}\right)^{\frac{1}{\phi-1}} \left(\frac{t_{ij} w_j}{Z_j P_i}\right)^{-\frac{\sigma-1}{\phi-1}} \mu_{ij}^{-\frac{1}{\phi-1}} d_{ij}^{-\frac{\rho+\delta(\sigma-1)}{\phi-1}}$$

Bilateral import flows by country i from country j are given by

$$X_{ij} = n_{ij} p_{ij} q_{ij} L_i = (c(\sigma - 1))^{-\frac{1}{\phi-1}} \left(\frac{\mathcal{I}_i}{w_i}\right)^{\frac{1}{\phi-1}} \left(\frac{t_{ij} w_j}{Z_j P_i}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \mu_{ij}^{-\frac{1}{\phi-1}} d_{ij}^{-\frac{\rho+\phi\delta(\sigma-1)}{\phi-1}} \mathcal{I}_i L_i,$$

and the trade share can be written as

$$\pi_{ij} = \frac{X_{ij}}{\sum_{l \in \mathcal{J}} X_{il}} = \frac{\left(\frac{w_j}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times \mu_{ij}^{-\frac{1}{\phi-1}} d_{ij}^{-\frac{\rho+\phi\delta(\sigma-1)}{\phi-1}} t_{ij}^{-\frac{\phi(\sigma-1)}{\phi-1}}}{\sum_{l \in \mathcal{J}} \left(\frac{w_l}{Z_l}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \times \mu_{il}^{-\frac{1}{\phi-1}} d_{il}^{-\frac{\rho+\phi\delta(\sigma-1)}{\phi-1}} t_{il}^{-\frac{\phi(\sigma-1)}{\phi-1}}} = \frac{S_j}{\Phi_i} \times \Gamma_{ij}^{-\varepsilon},$$

where

$$\Gamma_{ij}^{-\varepsilon} = \mu_{ij}^{-\frac{1}{\phi-1}} d_{ij}^{-\frac{\rho+\phi\delta(\sigma-1)}{\phi-1}} t_i^{-\frac{\phi(\sigma-1)}{\phi-1}},$$

which is identical to equation (9) applying to our baseline model with traveling costs in terms of labor.

The price index is in turn given by

$$P_i = (c(\sigma-1))^{\frac{1}{\phi(\sigma-1)}} \left(\frac{\mathcal{I}_i}{w_i}\right)^{-\frac{1}{\phi(\sigma-1)}} \left(\sum_{j\in\mathcal{J}} \left(\frac{w_j}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}\right)^{-\frac{\phi-1}{\phi(\sigma-1)}},$$

and using this expression together for the one for π_{ij}, one can verify that we can write

$$n_{ij} = \left(\frac{t_{ij}d_{ij}^{\delta}w_j}{Z_jP_i}\right)^{1-\sigma} \pi_{ij},$$

just as in equation (15) of the main text.

Plugging this expression back into the budget constraint yields

$$\mathcal{I}_i = \frac{\phi(\sigma-1)}{\phi(\sigma-1)+1}w_i,$$

and a resulting price index equal to

$$P_i = \left(\frac{c\phi}{\phi(\sigma-1)+1}\right)^{\frac{1}{\phi(\sigma-1)}} \left(\sum_{j\in\mathcal{J}} \left(\frac{w_j}{Z_j}\right)^{-\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}\right)^{-\frac{\phi-1}{\phi(\sigma-1)}},$$

which is only slightly different than expression (11) in the main text,

The labor-market conditions are given by

$$\pi_{ii}\mathcal{I}_iL_i + \pi_{ji}\mathcal{I}_jL_j = \mathcal{I}_iL_i$$

or, equivalently,

$$\pi_{ii}w_iL_i + \pi_{ji}w_jL_j = w_iL_i,$$

just as in the main text, and remember that the expressions for π_{ii} and π_{ji} are also left unchanged.

We next turn to verifying that Propositions 1 through 4 in the main text continue to hold whenever travel costs in equation (1) are specified in terms of labor rather than being modelled as a utility cost.

Proposition 1: *As long as trade frictions (Γ_{ij}) are bounded, there exists a unique vector of equilibrium wages $w^* = (w_i, w_j) \in R^2_{++}$ that solves the system of equations above.*

Proof. By results in standard gravity models in Alvarez and Lucas (2007), Allen and Arkolakis (2014), and Allen et al. (2020). ■

Proposition 2: *A decline in any international trade or mobility friction ($d_{ij}, t_{ij}, t_{ji}, \mu_{ij}, \mu_{ji}$) leads to: (a) a decline in the rates (n_{ii} and n_{jj}) at which individuals will meet individuals in their own country; and (b) an increase in the rates at which individuals will meet individuals from the other country (n_{ij} and n_{ji}).*

Proof. **(a)** Given that $\mathcal{I}_i = \frac{\phi(\sigma-1)}{\phi(\sigma-1)+1} w_i$,

$$n_{ii} = (c(\sigma-1))^{-\frac{1}{\phi-1}} \left(\frac{\mathcal{I}_i}{w_i}\right)^{\frac{1}{\phi-1}} \left(\frac{t_{ii} w_i}{Z_i P_i}\right)^{-\frac{\sigma-1}{\phi-1}} \mu_{ii}^{-\frac{1}{\phi-1}} d_{ii}^{-\frac{\rho+\delta(\sigma-1)}{\phi-1}} = const \times \left(\frac{P_i}{w_i}\right)^{\frac{\sigma-1}{\phi-1}}$$

Then

$$\frac{P_i}{w_i} = \left(\frac{c\phi}{\phi(\sigma-1)+1}\right)^{\frac{1}{\phi(\sigma-1)}} \left(Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}\right)^{-\frac{\phi-1}{\phi(\sigma-1)}},$$

where $\omega = w_j / w_i$ is the relative wage in country j .

Note that the labor constraint can be rewritten as

$$\frac{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon}}{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ii}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ij}^{-\varepsilon}} L_i + \frac{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon}}{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{ji}^{-\varepsilon} + \left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}} \Gamma_{jj}^{-\varepsilon}} \omega L_j = L_i$$

Consider a case when Γ_{ij} decreases, while other Γ_{kl} remain constant. That means that the first term in the sum goes down, while the second term is constant. For the equality to hold, ω should increase. After re-equilibration, the second term in the sum increased, which means that the first term decreased. This means that P_i/w_i decreased, and n_{ii} as well.

Consider now a case when Γ_{ji} decreases, while other Γ_{kl} remain constant. The second term increases, so ω needs to go down to equilibrate the model. That means that the first term decreases, and P_i/w_i and n_{ii} decrease by extension.

Therefore, whenever one decreases any international friction $(d_{ij}, t_{ij}, t_{ji}, \mu_{ij}, \mu_{ji})$, Γ_{ij} or Γ_{ji} goes down, and, hence, n_{ii} and n_{jj} go down.

(b) Note that

$$\frac{\mathcal{I}_i}{w_i} = 1 - \frac{c}{\phi} \sum_{j \in \mathcal{J}} \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi}$$

Since $\mathcal{I}_i = \frac{\phi(\sigma-1)}{\phi(\sigma-1)+1} w_i$, the left-hand side is constant. Since n_{ii} and n_{jj} decrease, n_{ij} and n_{ji} must increase. ■

Proposition 3: *Suppose that countries are symmetric, in the sense that $L_i = L$, $Z_i = Z$, and $\Gamma_{ij} = \Gamma$ for all i. Then a decline in any (symmetric) international trade frictions leads to an overall increase in human interactions ($n_{dom} + n_{for}$) experienced by both household buyers and household sellers.*

Proof. We begin by considering the case with general country asymmetries. Consider the sum

$$\mu_{ii} d_{ii}^{\rho} n_{ii}^{\phi} + \mu_{ij} d_{ij}^{\rho} n_{ij}^{\phi} = \frac{1}{\phi(\sigma-1)+1}$$

Differentiating yields

$$\phi\mu_{ii}d_{ii}^{\rho}n_{ii}^{\phi-1}dn_{ii}+\phi\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi-1}dn_{ij}+\phi n_{ij}^{\phi}\underbrace{d\left(\mu_{ij}d_{ij}^{\rho}\right)}_{\leq 0}=0$$

Hence,

$$\phi\mu_{ii}d_{ii}^{\rho}n_{ii}^{\phi-1}dn_{ii}+\phi\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi-1}dn_{ij}\geq 0,$$

and if $\mu_{ii}d_{ii}^{\rho}n_{ii}^{\phi-1}>\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi-1}$, then $dn_{ij}>-dn_{ii}$.

From the FOC for the choice of n_{ii} and n_{ij},

$$\mu_{ii}d_{ii}^{\rho}n_{ii}^{\phi-1}=\frac{1}{c(\sigma-1)}\frac{\mathcal{I}_i}{w_i}\left(\frac{p_{ii}}{P_i}\right)^{1-\sigma}$$

$$\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi-1}=\frac{1}{c(\sigma-1)}\frac{\mathcal{I}_i}{w_i}\left(\frac{p_{ij}}{P_i}\right)^{1-\sigma}$$

Therefore, $\mu_{ii}d_{ii}^{\rho}n_{ii}^{\phi-1}>\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi-1}$ is satisfied if and only if $p_{ii}<p_{ij}$.

When countries are symmetric, this holds trivially because of international trade costs $t_{ij}>t_{ii}$ and $d_{ij}>d_{ii}$. Hence, $dn_{ij}>-dn_{ii}$, and $n_{dom}+n_{for}$ increases. ■

Proposition 4: *An increase in the relative size of country i's population leads to an increase in the rate n_{ii} at which individuals from i will meet individuals in their own country, and to a decrease in the rate n_{ij} at which individuals will meet individuals abroad.*

Proof. Consider again

$$\frac{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}}\Gamma_{ii}^{-\varepsilon}}{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}}\Gamma_{ii}^{-\varepsilon}+\left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}}\Gamma_{ij}^{-\varepsilon}}L_i+\frac{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}}\Gamma_{ji}^{-\varepsilon}}{Z_i^{\frac{\phi(\sigma-1)}{\phi-1}}\Gamma_{ji}^{-\varepsilon}+\left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}}\Gamma_{jj}^{-\varepsilon}}\omega L_j=L_i$$

An increase in L_i makes the left-hand side smaller then the right-hand side. Therefore, ω grows to re-equilibrate. Then

$$\frac{P_i}{w_i}=\left(\frac{c\phi}{\phi(\sigma-1)+1}\right)^{\frac{1}{\phi(\sigma-1)}}\left(Z_i^{\frac{\phi(\sigma-1)}{\phi-1}}\Gamma_{ii}^{-\varepsilon}+\left(\frac{Z_j}{\omega}\right)^{\frac{\phi(\sigma-1)}{\phi-1}}\Gamma_{ij}^{-\varepsilon}\right)^{-\frac{\phi-1}{\phi(\sigma-1)}},$$

increases, and $n_{ii}=const\times\left(\frac{P_i}{w_i}\right)^{\frac{\sigma-1}{\phi-1}}$ increases with it.

Since

$$\mu_{ii}d_{ii}^{\rho}n_{ii}^{\phi}+\mu_{ij}d_{ij}^{\rho}n_{ij}^{\phi}=\frac{1}{\phi(\sigma-1)+1},$$

n_{ij} decreases.

Therefore, following a growth in population L_i, n_{ii} increases while n_{ij} decreases. ■

B. International Sourcing of Inputs

The assumption that households travel internationally to procure final goods may seem unrealistic. Perhaps international travel is better thought as being a valuable input when firms need specialized inputs and seek potential providers of those inputs in various countries. It is straightforward to re-interpret our model along those lines. In particular, suppose now that all households in country i produce a homogeneous final good but also produce differentiated intermediate input varieties. The household's final good is produced combining a bundle of the intermediate inputs produced by other households. Technology for producing the final good is given by

$$Q_i = \left(\sum\nolimits_{j \in \mathcal{J}} \int_0^{n_{ij}^I} q_{ij}^I (k)^{\frac{\sigma-1}{\sigma}} \, dk \right)^{\frac{\sigma}{\sigma-1}}$$

and this final good is not traded (this is without loss of generality if households are homogeneous in each country and trade costs for final goods are large enough). Household welfare is linear in consumption of the final good and is reduced by the disutility cost of a household's member having to travel to secure intermediate inputs. In particular, we have

$$W_i = \left(\sum\nolimits_{j \in \mathcal{J}} \int_0^{n_{ij}^I} q_{ij}^I (k)^{\frac{\sigma-1}{\sigma}} \, dk \right)^{\frac{\sigma}{\sigma-1}} - \frac{c}{\phi} \sum\nolimits_{j \in \mathcal{J}} \mu_{ij} \left(d_{ij} \right)^{\rho} \times \left(n_{ij}^I \right)^{\phi}.$$

Under this model is isomorphic to the one above, except that trade will be in intermediate inputs rather than in final goods.

C. Multi-Country Model

We next consider a version of our model with a world economy featuring multiple countries. It should be clear that all our equilibrium conditions, except for the labor-market clearing condition (14) apply to that multi-country environment once the set of countries $\mathcal{J}$ is redefined to include multiple countries. The labor-market condition is in turn simply given by

$$\sum\nolimits_{j \in \mathcal{J}} \pi_{ij} \left(\mathbf{w} \right) w_j L_j = w_i L_i,$$

where $\pi_{ij} \left(\mathbf{w} \right)$ is defined in (9) for an arbitrary set of countries $\mathcal{J}$. Similarly, the model is also easily adaptable to the case in which there is a continuum of locations $i \in \Omega$, where Ω is a closed and bounded set of a finite dimensional Euclidean space. The equilibrium conditions are again unaltered, with integrals replacing summation operators throughout.

From the results in Alvarez and Lucas (2007), Allen and Arkolakis (2014), and Allen et al. (2020), it is clear that Proposition 1 in the main text on existence and uniqueness will continue to hold. In the presence of arbitrary asymmetries across countries, it is hard however to derive crisp comparative static results of the type in Propositions 2 and 4. Nevertheless, our result in Proposition 3 regarding the positive effect of declines of trade and mobility barriers on the overall level of human interactions between symmetric countries is easily generalizable to the case of many countries (details available upon request - future versions of the paper will include an Online Appendix with the details).

D. Traveling Salesman Model

Finally, we explore a variant of our model in which it is the household's seller rather than the buyer who travels to other locations. We model this via a framework featuring scale economies, monopolistic competition and fixed cost of exporting, as in the literature on selection into exporting emanating from the seminal work of Melitz (2003), except that the fixed costs of selling are defined at the buyer level rather than at the country level. This extension is still work in progress.

A.6 Simulation Appendix

In this section of the Appendix, we discuss the simulation of the two-country SIR model. The share of households of each type evolve according to the following laws of motion (we ignore time subscripts to keep the notation tidy):

$$\begin{aligned}
\dot{S}_i &= -2n_{ii} \times \alpha_i \times S_i \times I_i - n_{ij} \times \alpha_j \times S_i \times I_j - n_{ji} \times \alpha_i \times S_i \times I_j \\
\dot{I}_i &= 2n_{ii} \times \alpha_i \times S_i \times I_i + n_{ij} \times \alpha_j \times S_i \times I_j + n_{ji} \times \alpha_i \times S_i \times I_j - \gamma_i I_i \\
\dot{R}_i &= \gamma_i I_i.
\end{aligned}$$

The values of n_{ij} for all i, j are the outcome of the equilibrium described in Section 2. We initiate the simulation with $I_i(0) = 0.1 \times 10^{-4}$ in both countries. The simulation presented in the main text are supposed to be illustrative rather than a detailed calibration for a specific circumstance. Nevertheless, the baseline calibration adopts the central values of the epidemiology parameters in Fernández-Villaverde and Jones (2020). For example, in Figure 1 we set the value of the exogenous component of the infection rate in the healthy country, $\alpha_1 = 0.04$, and we vary the value for the sick country between $\alpha_2 \in [0.04, 0.1]$. Using the equilibrium values of interactions, this leads to a value of $2n_{ii}\alpha_i + n_{ij}\alpha_j + n_{ji}\alpha_i$ (the actual infection rate in Country i if $I_i = I_j$) in the range $[0.15, 0.20]$ in Country 1 and $[0.15, 0.33]$ in Country 2, well in the range of values estimated in Fernández-Villaverde and Jones (2020). We also set $\gamma_i = 0.2$, which implies an infectious period of 5 days.

The economic model also involves a number of parameters. We set the elasticity of substitution $\sigma = 5$, a central value in the trade literature (Costinot and Rodríguez-Clare, 2015), and normalize productivity $Z_i = 1$ for all i. We also set Country size $L_i = 3$ when countries are symmetric. We choose values so that the choice of trading partners n_{ij} is never constrained. We choose a baseline value for the elasticity of the cost of consuming more varieties in a region of $\phi = 2$. Hence, the second order conditions discussed in the text are satisfied since $\phi > 1/(\sigma - 1)$. Note that we also require $\phi > 1$. We eliminate all man-made frictions in the baseline, so $t_{ij} = \mu_{ij} = 1$ for all i, j, and let $d_{ij} = 1.1$ for $i \neq j$ and 1 otherwise. We set to one the elasticity of trade costs with respect to distance, so $\delta = 1$. Finally we set the level of the cost of creating contacts, $c = 0.15$, which guarantees that equilibrium contacts are always in an interior solution. Of course, in the main text we show a number of exercises in which we change these parameter values, and in particular introduce trade and mobility frictions. Whenever we vary from the parameter values mentioned above we state that in the discussion of the relevant graph.

A.7 Pandemic-Free versus Pandemic Equilibrium

Letting $\tilde{\beta}_{ii} = 2n_{ii} \times \alpha_i$, $\tilde{\beta}_{jj} = -2n_{jj} \times \alpha_j$, $\tilde{\beta}_{ij} = n_{ij} \times \alpha_j$, and $\tilde{\beta}_{ji} = n_{ji} \times \alpha_i$, we can write the full system as:

$$\begin{bmatrix} \dot{S}_{it} \\ \dot{S}_{jt} \\ \dot{I}_{it} \\ \dot{I}_{jt} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & -\tilde{\beta}_{ii}S_{it} & -\left(\tilde{\beta}_{ij}S_{it} + \tilde{\beta}_{ji}S_{jt}\right) \\ 0 & 0 & -\left(\tilde{\beta}_{ji}S_{jt} + \tilde{\beta}_{ij}S_{it}\right) & -\tilde{\beta}_{jj}S_{jt} \\ 0 & 0 & \left(\tilde{\beta}_{ii}S_{it} - \gamma_i\right) & \left(\tilde{\beta}_{ij}S_{it} + \tilde{\beta}_{ji}S_{jt}\right) \\ 0 & 0 & \left(\tilde{\beta}_{ji}S_{jt} + \tilde{\beta}_{ij}S_{it}\right) & \left(\tilde{\beta}_{jj}S_{jt} - \gamma_{jt}\right) \end{bmatrix}}_{J} \begin{bmatrix} S_{it} \\ S_{jt} \\ I_{it} \\ I_{jt} \end{bmatrix}.$$

Denote the spectral radius of the matrix J as $\rho(J)$.

Proposition 11 ***(A)*** *There exists a unique stable no-pandemic equilibrium if and only if:*

$$\rho(J) = \frac{1}{2}\left(\tilde{\beta}_{ii} - \gamma_i\right) + \frac{1}{2}\left(\tilde{\beta}_{jj} - \gamma_j\right) + \frac{1}{2}\sqrt{4\left[\left(\tilde{\beta}_{ij} + \tilde{\beta}_{ji}\right)^2\right] + \left(\left(\tilde{\beta}_{ii} - \gamma_i\right) - \left(\tilde{\beta}_{jj} - \gamma_j\right)\right)^2} < 0.$$

(B) *The no-pandemic equilibrium is unstable and there exists a unique pandemic equilibrium with $S_{i\infty} \in [0,1)$ and $S_{j\infty} \in [0,1)$ if and only if:*

$$\rho(J) = \frac{1}{2}\left(\tilde{\beta}_{ii} - \gamma_i\right) + \frac{1}{2}\left(\tilde{\beta}_{jj} - \gamma_j\right) + \frac{1}{2}\sqrt{4\left[\left(\tilde{\beta}_{ij} + \tilde{\beta}_{ji}\right)^2\right] + \left(\left(\tilde{\beta}_{ii} - \gamma_i\right) - \left(\tilde{\beta}_{jj} - \gamma_j\right)\right)^2} > 0.$$

Proof. (A) In a no-pandemic equilibrium in which $S_i = S_j = 1$, we have:

$$J = \begin{bmatrix} 0 & 0 & -\tilde{\beta}_{ii} & -\left(\tilde{\beta}_{ij} + \tilde{\beta}_{ji}\right) \\ 0 & 0 & -\left(\tilde{\beta}_{ji} + \tilde{\beta}_{ij}\right) & -\tilde{\beta}_{jj} \\ 0 & 0 & \left(\tilde{\beta}_{ii} - \gamma_i\right) & \left(\tilde{\beta}_{ij} + \tilde{\beta}_{ji}\right) \\ 0 & 0 & \left(\tilde{\beta}_{ji} + \tilde{\beta}_{ij}\right) & \left(\tilde{\beta}_{jj} - \gamma_j\right) \end{bmatrix}.$$

This no-pandemic equilibrium is stable if $\rho(J) < 0$. To find $\rho(J)$, we solve for the eigenvalues of the matrix J (denoted by λ) using the following characteristic equation:

$$|J - \lambda I| = \left| \begin{bmatrix} (0-\lambda) & 0 & -\tilde{\beta}_{ii} & -\left(\tilde{\beta}_{ij} + \tilde{\beta}_{ji}\right) \\ 0 & (0-\lambda) & -\left(\tilde{\beta}_{ji} + \tilde{\beta}_{ij}\right) & -\tilde{\beta}_{jj} \\ 0 & 0 & \left(\tilde{\beta}_{ii} - \gamma_i - \lambda\right) & \left(\tilde{\beta}_{ij} + \tilde{\beta}_{ji}\right) \\ 0 & 0 & \left(\tilde{\beta}_{ji} + \tilde{\beta}_{ij}\right) & \left(\tilde{\beta}_{jj} - \gamma_j - \lambda\right) \end{bmatrix} \right| = 0,$$

which has the following eigenvalues:

$$\lambda_1 = 0,$$

$$\lambda_2 = 0,$$

$$\lambda_3 = \frac{1}{2}\left(\tilde{\beta}_{ii} - \gamma_i\right) + \frac{1}{2}\left(\tilde{\beta}_{jj} - \gamma_j\right) - \frac{1}{2}\sqrt{4\left[\left(\tilde{\beta}_{ij} + \tilde{\beta}_{ji}\right)^2\right] + \left(\left(\tilde{\beta}_{ii} - \gamma_i\right) - \left(\tilde{\beta}_{jj} - \gamma_j\right)\right)^2},$$

,

$$\lambda_4 = \frac{1}{2}\left(\tilde{\beta}_{ii} - \gamma_i\right) + \frac{1}{2}\left(\tilde{\beta}_{jj} - \gamma_j\right) + \frac{1}{2}\sqrt{4\left[\left(\tilde{\beta}_{ij} + \tilde{\beta}_{ji}\right)^2\right] + \left(\left(\tilde{\beta}_{ii} - \gamma_i\right) - \left(\tilde{\beta}_{jj} - \gamma_j\right)\right)^2}.$$

We therefore have:

$$\begin{aligned} \rho(J) &= \max\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} = \frac{1}{2}\left(\tilde{\beta}_{ii} - \gamma_i\right) + \frac{1}{2}\left(\tilde{\beta}_{jj} - \gamma_j\right) \\ &\quad + \frac{1}{2}\sqrt{4\left[\left(\tilde{\beta}_{ij} + \tilde{\beta}_{ji}\right)^2\right] + \left(\left(\tilde{\beta}_{ii} - \gamma_i\right) - \left(\tilde{\beta}_{jj} - \gamma_j\right)\right)^2}. \end{aligned}$$

(B) If $\rho(J) > 0$, the no-pandemic equilibrium exists but is unstable. To establish that there exists another equilibrium with $S_{i\infty} \in [0,1)$ and $S_{j\infty} \in [0,1)$, note that for sufficiently small values of S_{it} and S_{jt}, the change in the number of infections is necessarily negative:

$$\lim_{S_{it}, S_{jt} \to 0} \left\{\dot{I}_{it}\right\} = \lim_{S_{it}, S_{jt} \to 0} \left\{\left[\tilde{\beta}_{ii} S_{it} + \tilde{\beta}_{ij} S_{it} + \tilde{\beta}_{ji} S_{jt} - \gamma_i\right] I_{it}\right\} < 0,$$

$$\lim_{S_{it}, S_{jt} \to 0} \left\{\dot{I}_{jt}\right\} = \lim_{S_{it}, S_{jt} \to 0} \left\{\left[\tilde{\beta}_{jj} S_{jt} + \tilde{\beta}_{ji} S_{jt} + \tilde{\beta}_{ij} S_{it} - \gamma_j\right] I_{jt}\right\} < 0.$$

Therefore, steady-state infections are necessarily zero ($I_{i\infty} = I_{j\infty} = 0$), which implies that the steady-state rate of change of susceptibles is zero:

$$\dot{S}_{i\infty} = -\left[\tilde{\beta}_{ii} S_{i\infty} + \tilde{\beta}_{ij} S_{i\infty} + \tilde{\beta}_{ji} S_{i\infty}\right] I_{j\infty} = 0,$$

$$\dot{S}_{j\infty} = -\left[\tilde{\beta}_{jj} S_{j\infty} + \tilde{\beta}_{ji} S_{j\infty} + \tilde{\beta}_{ij} S_{j\infty}\right] I_{i\infty} = 0,$$

which in turn implies that there exists a pandemic steady-state with $S_{i\infty} \in [0,1)$ and $S_{j\infty} \in [0,1)$. To establish the uniqueness of this pandemic steady-state with $S_{i\infty} \in [0,1)$ and $S_{j\infty} \in [0,1)$, note that it satisfies the following system of equations:

$$\log S_{i\infty} = -\frac{\tilde{\beta}_{ii}}{\tilde{\gamma}_i}\left(1 - S_{i\infty}\right) - \frac{\tilde{\beta}_{ij} + \tilde{\beta}_{ji}}{\tilde{\gamma}_j}\left(1 - S_{j\infty}\right) \tag{A.3}$$

$$\log S_{j\infty} = -\frac{\tilde{\beta}_{jj}}{\tilde{\gamma}_j}\left(1 - S_{j\infty}\right) - \frac{\tilde{\beta}_{ji} + \tilde{\beta}_{ij}}{\tilde{\gamma}_i}\left(1 - S_{i\infty}\right). \tag{A.4}$$

We can re-write this system of equations as:

$$S_{i\infty} = F_i\left(S_{i\infty}, S_{j\infty}\right) = \exp\left(-\frac{\tilde{\beta}_{ii}}{\tilde{\gamma}_i}\left(1 - S_{i\infty}\right) - \frac{\tilde{\beta}_{ij} + \tilde{\beta}_{ji}}{\tilde{\gamma}_j}\left(1 - S_{j\infty}\right)\right) \tag{A.5}$$

$$S_{j\infty} = F_j\left(S_{i\infty}, S_{j\infty}\right) = \exp\left(-\frac{\tilde{\beta}_{jj}}{\tilde{\gamma}_j}\left(1 - S_{j\infty}\right) - \frac{\tilde{\beta}_{ji} + \tilde{\beta}_{ij}}{\tilde{\gamma}_i}\left(1 - S_{i\infty}\right)\right). \tag{A.6}$$

or more compactly as:

$$\mathbf{S}_\infty = F\left(\mathbf{S}_\infty\right)$$

This system of equations satisfies the following properties:
(i) $F\left(\mathbf{S}_\infty\right)$ is continuous
(ii) $F\left(\mathbf{S}_\infty\right)$ satisfies monotonicity, since:

$$\frac{dS_{i\infty}}{S_{i\infty}} = \frac{\tilde{\beta}_{ii}}{\tilde{\gamma}_i} dS_{i\infty} + \frac{\tilde{\beta}_{ij} + \tilde{\beta}_{ji}}{\tilde{\gamma}_j} dS_{j\infty} > 0, \quad \text{for} \quad dS_{i\infty}, dS_{j\infty} > 0,$$

$$\frac{dS_{j\infty}}{S_{j\infty}} = \frac{\tilde{\beta}_{jj}}{\tilde{\gamma}_j} dS_{j\infty} + \frac{\tilde{\beta}_{ji} + \tilde{\beta}_{ij}}{\tilde{\gamma}_j} dS_{i\infty} > 0, \quad \text{for} \quad dS_{i\infty}, dS_{j\infty} > 0,$$

(iii) $F\left(\mathbf{S}_\infty\right)$ is bounded, such that $F_i \in \left[\underline{F}_i, \bar{F}_i\right] \subseteq [0,1]$ and $F_j \in \left[\underline{F}_j, \bar{F}_j\right] \subseteq [0,1]$, with $S_{i\infty} \in [0,1]$ and $S_{j\infty} \in [0,1]$:

$$0 < \underline{F}_i = \lim_{S_{i\infty}, S_{j\infty} \to 0} \left(F_i\left(S_{i\infty}, S_{j\infty}\right)\right) = \exp\left(-\frac{\tilde{\beta}_{ii}}{\tilde{\gamma}_i} - \frac{\tilde{\beta}_{ij} + \tilde{\beta}_{ji}}{\tilde{\gamma}_j}\right) < 1,$$

$$\overline{F}_i = \lim_{S_{i\infty}, S_{j\infty} \to 1} \left(F_i\left(S_{i\infty}, S_{j\infty}\right)\right) = 1,$$

$$0 < \underline{F}_j = \lim_{S_{i\infty}, S_{j\infty} \to 0} \left(F_j\left(S_{i\infty}, S_{j\infty}\right)\right) = \exp\left(-\frac{\tilde{\beta}_{jj}}{\tilde{\gamma}_j} - \frac{\tilde{\beta}_{ji} + \tilde{\beta}_{ij}}{\tilde{\gamma}_i}\right) < 1,$$

$$\overline{F}_j = \lim_{S_{i\infty}, S_{j\infty} \to 1} \left(F_j\left(S_{i\infty}, S_{j\infty}\right)\right) = 1.$$

Together these properties imply that the solution $\mathbf{S}_\infty = F\left(\mathbf{S}_\infty\right)$ for $S_{i\infty} \in [0,1)$ and $S_{j\infty} \in [0,1)$ is unique. ■

A.8 Comparative Statics of Pandemic Equilibrium

We begin with the law of motion for susceptible agents in each country in equation (16):

$$\begin{aligned}\dot{S}_i &= -2\alpha_i n_{ii} \times S_i \times I_i - \alpha_j n_{ij} \times S_i \times I_j - \alpha_i n_{ji} \times S_i \times I_j \\ \dot{S}_j &= -2\alpha_j n_{jj} \times S_j \times I_j - \alpha_j n_{ij} \times S_j \times I_i - \alpha_i n_{ji} \times S_j \times I_i\end{aligned}$$

Dividing by the own share of susceptibles, and plugging the expression for $\dot{R}_i$ and $\dot{R}_j$ in (18), we obtain

$$\begin{aligned}\frac{\dot{S}_i}{S_i} &= -\frac{2\alpha_i n_{ii}}{\gamma_i}\dot{R}_i - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}\dot{R}_j \\ \frac{\dot{S}_j}{S_j} &= -\frac{2\alpha_j n_{jj}}{\gamma_j}\dot{R}_j - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i}\dot{R}_i.\end{aligned}$$

Turning the growth rate in the left-hand-side to a log-difference, and integrating we get

$$\begin{aligned}\ln S_i(t) - \ln S_i(0) &= -\frac{2\alpha_i n_{ii}}{\gamma_i}(R_i(t) - R_i(0)) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}(R_j(t) - R_j(0)) \\ \ln S_j(t) - \ln S_j(0) &= -\frac{2\alpha_j n_{jj}}{\gamma_j}(R_j(t) - R_j(0)) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i}(R_i(t) - R_j(0))\end{aligned}$$

Finally, noting $S_i(0) \simeq 1$ and $R_i(0) \simeq 1$, and $R_i(\infty) = 1 - S_i(\infty)$ (since $I_i(\infty) = 0$), we obtain the system in (24)-(25), that is:

$$\begin{aligned}\ln S_i(\infty) &= -\frac{2\alpha_i n_{ii}}{\gamma_i}(1 - S_i(\infty)) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}(1 - S_j(\infty)) \\ \ln S_j(\infty) &= -\frac{2\alpha_j n_{jj}}{\gamma_j}(1 - S_j(\infty)) - \frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i}(1 - S_i(\infty))\end{aligned}$$

Although we cannot solve the system in closed-form, we can derive some comparative statics. In particular, total differentiating we find

$$\begin{aligned}& \frac{1}{S_i(\infty)}dS_i(\infty) - \frac{2\alpha_i n_{ii}}{\gamma_i}dS_i(\infty) + (1 - S_i(\infty))\, d\left(\frac{2\alpha_i n_{ii}}{\gamma_i}\right) \\ =\; & \left(\frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}\right)dS_j(\infty) - d\left(\frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_j}\right) \\ & \frac{1}{S_j(\infty)}dS_j(\infty) - \frac{2\alpha_j n_{jj}}{\gamma_j}dS_j(\infty) + (1 - S_j(\infty))\, d\left(\frac{2\alpha_j n_{jj}}{\gamma_j}\right) \\ =\; & \left(\frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i}\right)dS_i(\infty) - d\left(\frac{\alpha_j n_{ij} + \alpha_i n_{ji}}{\gamma_i}\right)\end{aligned}$$

Solving

$$dS_i(\infty) = -\frac{\left[\begin{array}{c}\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_j}\left(d\left(\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_i}\right)+(1-S_j(\infty))\, d\left(\frac{2\alpha_j n_{jj}}{\gamma_j}\right)\right)\\ +\left(\frac{1}{S_j(\infty)}-\frac{2\alpha_j n_{jj}}{\gamma_j}\right)\left(d\left(\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_j}\right)+(1-S_i(\infty))\, d\left(\frac{2\alpha_i n_{ii}}{\gamma_i}\right)\right)\end{array}\right]}{\left(\frac{1}{S_i(\infty)}-\frac{2\alpha_i n_{ii}}{\gamma_i}\right)\left(\frac{1}{S_j(\infty)}-\frac{2\alpha_j n_{jj}}{\gamma_j}\right)-\frac{(\alpha_j n_{ij}+\alpha_i n_{ji})^2}{\gamma_i\gamma_j}}$$

$$dS_j(\infty) = -\frac{\left[\begin{array}{c}\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_i}\left(d\left(\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_j}\right)+(1-S_i(\infty))\, d\left(\frac{2\alpha_i n_{ii}}{\gamma_i}\right)\right)\\ +\left(\frac{1}{S_i(\infty)}-\frac{2\alpha_i n_{ii}}{\gamma_i}\right)\left(d\left(\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_i}\right)+(1-S_j(\infty))\, d\left(\frac{2\alpha_j n_{jj}}{\gamma_j}\right)\right)\end{array}\right]}{\left(\frac{1}{S_j(\infty)}-\frac{2\alpha_j n_{jj}}{\gamma_j}\right)\left(\frac{1}{S_i(\infty)}-\frac{2\alpha_i n_{ii}}{\gamma_i}\right)-\frac{(\alpha_j n_{ij}+\alpha_i n_{ji})^2}{\gamma_i\gamma_j}}$$

Next, note that at the peak infection of infection (t^*) we have ($\dot{I}(t^*)=0$), and $S_i(t^*)$ and $S_j(t^*)$ satisfy:

$$\begin{aligned}\frac{2\alpha_i n_{ii}}{\gamma_i}S_i(t^*)+\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_j}S_i(t^*)\times\frac{I_j}{I_i} &= 1\\ \frac{2\alpha_j n_{jj}}{\gamma_j}S_j(t^*)+\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_i}S_j(t^*)\times\frac{I_i}{I_j} &= 1\end{aligned}$$

Because the steady-state values of S_i and S_j must be lower than those values, it follows that

$$\begin{aligned}\frac{2\alpha_i n_{ii}}{\gamma_i}S_i(\infty)+\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_j}S_i(\infty)\times\frac{I_j}{I_i} &\leq 1\\ \frac{2\alpha_j n_{jj}}{\gamma_j}S_j(\infty)+\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_i}S_j(\infty)\times\frac{I_i}{I_j} &\leq 1\end{aligned}$$

So it is clear that $\frac{2\alpha_i n_{ii}}{\gamma_i}S_i(\infty)\leq 1$ and $\frac{2\alpha_j n_{jj}}{\gamma_j}S_j(\infty)\leq 1$. In addition,

$$\left(\frac{1}{S_i(\infty)}-\frac{2\alpha_i n_{ii}}{\gamma_i}\right)\left(\frac{1}{S_j(\infty)}-\frac{2\alpha_j n_{jj}}{\gamma_j}\right)\geq\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_j}\times\frac{I_j}{I_i}\frac{\alpha_j n_{ij}+\alpha_i n_{ji}}{\gamma_i}\times\frac{I_i}{I_j}=\frac{(\alpha_j n_{ij}+\alpha_i n_{ji})^2}{\gamma_i\gamma_j}.$$

Going back to the system, this means that an increase in any n or a decrease in any γ will decrease the steady-state values for $S_i(\infty)$ and $S_j(\infty)$, and thus increase infections everywhere.

A.9 Proof of Proposition 10

The goods market clearing condition with deaths defines the following implicit function:

$$\Lambda_i=\left[\begin{array}{c}\frac{(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ii})^{-\varepsilon}}{(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ii})^{-\varepsilon}+(Z_j/\omega)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ij})^{-\varepsilon}}(1-D_i)L_i\\ +\frac{(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ji})^{-\varepsilon}}{(Z_j/\omega)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{jj})^{-\varepsilon}+(Z_i)^{\frac{\phi(\sigma-1)}{\phi-1}}(\Gamma_{ji})^{-\varepsilon}}\omega(1-D_j)L_j-(1-D_i)L_i\end{array}\right]=0.$$

Taking partial derivatives of this implicit function, we have:

$$\frac{\partial \Lambda_i}{\partial D_i} > 0, \qquad \frac{\partial \Lambda_i}{\partial D_j} < 0, \qquad \frac{\partial \Lambda_i}{\partial \omega} > 0.$$

Therefore, from the implicit function theorem, we have the following comparative statics of the relative wage with respect to deaths in the two countries:

$$\frac{d\omega}{dD_i} = -\frac{\partial \Lambda_i / \partial D_i}{\partial \Lambda_i / \partial \omega} < 0, \qquad \frac{d\omega}{dD_j} = -\frac{\partial \Lambda_i / \partial D_j}{\partial \Lambda_i / \partial \omega} > 0. \tag{A.7}$$

We now combine these results above with the comparative statics of bilateral interactions with respect to the relative wage (ω) from Proposition 4. In particular, from the proof of that proposition, we have the following results:

$$\frac{dn_{ii}}{d\omega} > 0, \qquad \frac{dn_{ij}}{d\omega} < 0. \tag{A.8}$$

Combining these two sets of relationships (A.7) and (A.8), we have the following results stated in the proposition:

$$\frac{dn_{ii}}{dD_i} = \underbrace{\frac{dn_{ii}}{d\omega}}_{>0}\underbrace{\frac{d\omega}{dD_i}}_{<0} < 0, \qquad \frac{dn_{ii}}{dD_j} = \underbrace{\frac{dn_{ii}}{d\omega}}_{>0}\underbrace{\frac{d\omega}{dD_j}}_{>0} > 0.$$

$$\frac{dn_{ij}}{dD_i} = \underbrace{\frac{dn_{ij}}{d\omega}}_{<0}\underbrace{\frac{d\omega}{dD_i}}_{<0} > 0, \qquad \frac{dn_{ij}}{dD_j} = \underbrace{\frac{dn_{ij}}{d\omega}}_{<0}\underbrace{\frac{d\omega}{dD_j}}_{>0} < 0.$$

A.10 Proof of Lemma 1

Because $Q_i\left(n_{ii}\left(t\right), n_{ij}\left(t\right)\right) \geq C_i\left(n_{ii}\left(t\right), n_{ij}\left(t\right)\right)$, from equation (35), we must have $\dot{\theta}_i^k\left(t\right) \geq 0$ at all t. This in turn implies that we must have $\theta_i^k\left(t\right) \leq 0$ at all t for the transversality condition to be met (i.e., convergence to 0 from below).

We next show that $\dot{\theta}_i^i\left(t\right) \geq 0$ and $\theta_i^i\left(t\right) \leq 0$ for all t. First note that we must have

$$\eta_i \theta_i^k\left(t\right) < \left(\gamma_i + \eta_i\right) \theta_i^i\left(t\right)$$

and thus (from equation (34)) $\dot{\theta}_i^i\left(t\right) > 0$ for all t. To see this, note that if instead we had

$$\eta_i \theta_i^k\left(t_0\right) > \left(\gamma_i + \eta_i\right) \theta_i^i\left(t_0\right),$$

at any time t_0, then $\dot{\theta}_i^i\left(t_0\right) < 0 < \dot{\theta}_k^i\left(t_0\right)$ so this inequality would continue to hold for all $t_0 > t$. But then we would have $\dot{\theta}_i^i\left(t\right) < 0$ for all $t > t_0$, and for $\theta_i^i\left(t\right)$ to meet its transversality condition, we would need to have $\theta_i^i\left(t\right) > 0$ at all $t > t_0$. But if $\theta_i^i\left(t\right) > 0$ and $\theta_i^k\left(t\right) \leq 0$ for $t > t_0$, it is clear from equation (34) that $\dot{\theta}_i^i\left(t\right) > 0$ for $t > t_0$, which is a contradiction. In sum, $\dot{\theta}_i^i\left(t\right) > 0$ for all t. But then for $\theta_i^i\left(t\right)$ to meet its transversality condition (from below), we need $\theta_i^i\left(t\right) \leq 0$ for all t.

Finally, to show that show that $\theta_i^s\left(t\right) > \theta_i^i\left(t\right)$ for all t, suppose that $\theta_i^s\left(t_0\right) < \theta_i^i\left(t_0\right)$ for some t_0. From equation (33), this would imply $\dot{\theta}_i^s\left(t_0\right) < 0$. But because $\dot{\theta}_i^i\left(t\right) > 0$ for all t, we would continue to have $\theta_i^s(t) < \theta_i^i(t)$ for all $t > t_0$, and thus $\dot{\theta}_i^s(t) < 0$ for all $t > t_0$. This would imply that, for $t > t_0$, $\theta_i^s(t)$ would converge to its steady-state value of 0 from above, i.e., $\theta_i^s(t) > 0$ for $t > t_0$. But because $\theta_i^i(t) \leq 0$ for all t, from equation (33), we would have $\dot{\theta}_i^s(t) > 0$ for $t > t_0$, which is a contradiction. In sum, we must have $\theta_i^s(t) > \theta_i^i(t)$ for all t.