Financial Multinationals∗

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March 2019—Version 1.6

Abstract

We develop a quantitative-oriented model that integrates the production and financing decisions of multinational corporations (MNCs). Firms differ by their productivity and net worth. Productive firms expand by accumulating net worth over time and by using funding from external investors. Firms can deploy their technology for production in affiliates overseas and become MNCs. However, the affiliates’ scale of operation is partially dependent on funding from the parent, giving rise to foreign direct investment (FDI). The model links FDI, affiliate production, and financial market conditions of host and home countries in ways consistent with the data, and remains tractable with heterogeneous firms and many asymmetric countries. We use the model to explore various issues such as the drivers of the global FDI growth over 2001-2007, the role of credit crunch for its growth slowdown since the Great Recession, and the welfare gains from FDI.

∗For helpful comments we thank Yan Bai, Saki Bigio, Stefania Garetto, Ezra Oberfield, Kim Ruhl, Stephen Yeaple, as well as workshop and conference participants at Hong Kong University, Tsinghua University, Aarhus University, SJTU, SHUFE, NYU Junior Trade Jamboree, the SAET (Taipei), the SED (Mexico City), and the Chinese University of Hong Kong.
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1 Introduction

Multinational corporations (MNCs) tend to be more productive and have abundant financial capital at disposal. By mobilizing both technology and capital across borders, they exert a significant impact on the world economy. Recent studies have made great progress in developing quantitative models to explain the patterns of MNC activities and examine their impacts on the aggregate economy. Most existing work, however, treats MNCs merely as a vehicle for cross-border transfer of technology, overlooking the accompanying movement of capital. As a result, a number of questions on the activities of MNCs and the welfare implications of these activities remain unanswered. For example, it is natural to conjecture that factors affecting the access to capital, such as credit market tightness, should affect MNC activities, but how important are these factors quantitatively, and whether the effects differ by country? On the normative side, policy makers from many developing countries hold the view that MNCs could be a source of scarce capital for the country, yet we still know very little about the importance of this channel.

We answer these questions by developing a quantitative-oriented model of firm dynamics that integrates the movements of both technology and capital within MNCs. One key ingredient of the model is that, due to imperfections in the financial market, the operations of an affiliate partially rely on the internal capital market of the MNC. This generates an incentive for MNCs to invest in destination countries using internal funds, so foreign direct investment (FDI) arises as within-firm capital flow from the parent to the affiliate. We parameterize the model to match the transitional dynamics of FDI flows among 36 countries over 2001-2012, a period that witnessed rapid growth in the global FDI (2001-2007) and its slowdown (2008-2012). This overall change coincided with the credit boom among our sample countries over 2001-2007, and the credit crunch during the Great Recession. Through counterfactual experiments, we show that the changes in credit market conditions explain a substantial part of the aggregate FDI dynamics. We also show that our model generates different predictions on the welfare effects of MNCs than existing studies.

In Section 2, we start by documenting three facts on the financing and production decisions of MNCs. Our analysis encompasses two related measures of MNCs that has been treated largely in isolation in the literature: FDI—which captures the within-firm capital flow from the parent to the affiliate—and MP, which is defined as the production by the affiliates of MNCs. Using bilateral information on MP and FDI among a large number of countries, we show that, first, MP and FDI are highly correlated. This correlation exists even after controlling for extensive margin of MNC activities at the bilateral level (the number of affiliates). Second, host countries with a more developed financial market attract more FDI; conditional on the level of FDI, the affiliates in these countries also produce more. Third, home countries with a more developed financial market send out more FDI. All these results are robust to the inclusion of a number of host and home country characteristics, such as income, market size, tax rate, etc. Together, they suggest that: 1) operations of affiliates at least partially on the internal capital market of the MNCs; 2) access to external finance in both the home country and the host country matter for the allocation of FDI and the distribution of MP across countries.
Motivated by the above facts, in Section 3, we develop a model of firm dynamics that highlights the role of internal capital markets and external finance. The goal of the model is to illustrate channels and provide quantitative answers to the questions posed earlier. In the model, firms are heterogeneous along three dimensions: their (pre-determined) home country, their productivity efficiency, which follows an exogenous Markov process, and their net worth, which is endogenous and can be accumulated over time. Firms decide whether and which country to deploy their productive technology. When a firm moves production abroad, an MNC emerges. MNCs (and domestic firms) combine their technology with physical capital and labor for the production of a homogeneous good. In addition to their net worth, firms have access to two sources of finance for physical capital: they can borrow against their net worth in the home country, and/or partner up with investors in the host country. Financial market conditions affect the availability of funding through both channels. To capture this effect most parsimoniously, we assume that both types of external finance are impeded by exogenous collateral constraints, with the collateral for the former being a firm’s net worth and the collateral for the latter being the amount of capital a firm brings into the host country, namely FDI. We further allow for the collateral requirement for the parent to be different from that for the affiliate, so a change in the financial market in a country can have differential effects on the collateral constraints faced by parent firms and foreign affiliates in the country.\footnote{While in the model, financial frictions in both stages take the form of a collateral constraint, their interpretations could be different. The former captures the extent to which parent firms could borrow externally from the credit market, therefore should be best interpreted as credit market conditions. Studies of financial frictions in macroeconomics often also interpret this as a measure of financial development (see, for example, Buera et al., 2011; Midrigan and Xu, 2014; Moll, 2014). The latter, the leverage constraint at the affiliate level captures the extent to which a parent company can rely on local partners. One interpretation for this friction is that with imperfect contract enforcement, stakes in the affiliates provide an incentive for the parents to exert effort, which makes local investors willing to invest—in other words, the market for management technology is imperfect. (See Antras et al. (2009) for a theory that links institution quality to the financing of affiliates endogenously and the empirical evidence based on US MNCs.) Despite the difference, frictions in both stages are clearly influenced by the overall financial institution and credit market conditions in a country. In the quantitative exercise, we discipline these two objects using different data and find they are indeed correlated.}

The model generates empirically consistent relationships between FDI, MP, and the financial market condition of a country. Because of the financial frictions, the total production of an affiliate depends on how much funding the parent brings in. As a host country becomes financially more developed, foreign affiliates can more easily scale up by partnering with local investors, so the internal rate of return to investing in that affiliate increases, drawing more FDI into the country. On the other hand, as a home country becomes financially more developed, productive firms will gain market share. In the short run, this drives up the labor cost, so more firms will invest abroad; in the long run, a better financial market enables faster accumulation of net worth, especially for the most productive firms, who are more likely to become MNCs. Both short- and long-run effects increase the level of FDI outflow.

In Section 4, we take the model to the data to quantify the importance of financial markets on the dynamics of FDI for individual countries and the world as a whole. We assemble a new dataset of bilateral FDI information for a sample of 36 major developed and developing countries over the period of 2001-2012. We supplement this dataset with country-specific time series on domestic
credit market conditions (captured by credit/GDP ratio), investment rate, parents’ position in affiliates’ balance sheet, and aggregate TFP. We fully saturate the model with country-specific wedge for investment and bilateral wedge for FDI, and calibrate these wedges, as well as the ‘structural parameters’ of the model that characterize financial conditions and firms’ productivity dynamics, so that the transitional dynamics of the model match the data in all above dimensions.\footnote{As in Eaton et al. (2016), once the time series of all structural parameters and wedges are fed in, the model will produce the pattern in the data.}

We validate the model by showing that a few non-targeted moments are consistent with the data. The calibration reveals a gradual improvement in credit market conditions until 2007 in the form of continuously relaxing collateral constraints for both parents and affiliates; this trend was reversed since the great financial crisis. Our first set of quantitative exercises focus on how changes of individual countries affect their FDI in each of the two periods, holding fundamentals of other countries at the benchmark, mimicking empirical studies that use a diff-in-diff design (see, e.g., Klein et al., 2002). We find that the credit boom, measured through the lens of our model, can explain up to 50-80\% of the cumulative FDI outflow during 2002-2007 in most of our sample countries. On the other hand, if the financial markets have stayed at the level of 2007, then the cumulative FDI outflows during 2008-2012 would have been 30-100\% higher than the factual values among the sample countries.

The above analysis overlooks domestic general equilibrium effect and the third country effect—for example, as financial market condition in the U.S. improves, the outward investment by American MNCs increases; the increasing competition created by these MNCs in their destination countries drive out investment to the same destination from third countries; changes in domestic wage also affect inward FDI into the U.S. We find that these equilibrium responses are large—the cumulative aggregate FDI flow between 2002-2007, across all source and destination countries, would decrease by around a third, if the collateral constraints stay at the value of 2001; the cumulative FDI during 2008-2012 would have been 40\% higher, if the credit conditions stay at the peak value of 2007. While these numbers are still quantitatively significant, the difference in magnitude compared to country-specific exercises shows the importance of using a multi-country general equilibrium model.

Finally, in Section 5, we study the normative implications of our model and compare them to those predicted by existing models. Following a tradition in international economics (see, e.g., Arkolakis et al., 2012; Ramondo and Rodríguez-Clare, 2013), the focus of our comparison will be on ex-post effects, defined as the percentage change in real income of workers as a country moves from autarky to the observed level of MNC activities. Because of the dynamic incomplete market setting, the full welfare effects of MP cannot be characterized analytically. To clarify the channels, we first derive the static impacts of inward MNC activities on workers’ real income. Our formula shows that, unlike in papers with Ricardian models of MNC (Ramondo and Rodríguez-Clare, 2013; Alviarez, 2016; Cravino and Levchenko, 2017), in which the MP share is the sufficient statistic for the welfare effect—and the studies that modeled FDI as movement of physical capital only (Mundell, 1957), in which FDI share in domestic capital is the sufficient statistic, in our setting,
even the static income change depends on both MP and FDI. MP captures the relative importance of foreign affiliates in local production. Conditioning on the MP share, a higher fraction of investment by foreign parents reduces affiliates’ dependence on credits from the host country, thus reducing the crowd-out effect on domestic firms.

When the world credit market is fully integrated, the credit crowd out channel is minimal. In that case, the MP share is a sufficient statistic. Yet it still encompasses two forces. First, a ‘technological’ channel—because foreign affiliates are on average more productive, their entry improve the productivity distribution of firms in an economy. Second, the ‘capital deepening’ channel—the capital brought in by foreign firms total amount of capital used in domestic production. This latter channel corresponds to what one would obtain, if they were to use a model that treats FDI as merely capital for the welfare evaluation.

We evaluate the importance of the two static channels for the sample countries, focusing on the year 2001 as an example. The median country has around 8% static welfare gains. On average, about half of the gains are through the technological channel, but this ratio differs significantly across countries—the technology channel matters more for countries with low domestic productivity, whereas the capital channel matters more for countries that are relatively scarce in capital. This result echoes policy makers’ emphasis that for developing countries with severe financial constraints, FDI is not only a source of advanced technologies, but also a source of financial capital (UNCTAD, 2011). It also underscores the importance of jointly modeling both FDI and MP in studying the aggregate effects of MNC activities.

The above static analysis takes as given the evolution of the joint distribution between productivity and capital. The financial frictions, coupled with the incomplete market assumption, lead to additional dynamic implications because the joint distribution will change as a result of MNC activities. To make the point, we calculate the full effects of MNC activities by shutting down the activities for the entire period starting 2001, and computing the equilibrium objects over this alternative transitional dynamics. We find that all countries experience a decrease in wage in 2002, the first period of moving to autarky. Over time, however, domestic firms are able to accumulate net worth. As a result, the economy gradually catches up with the benchmark economy. Interestingly, countries with a higher short-run TFP growth rate, such as China, might even have a higher income than in the economy with MNCs—that is, shutting down inward FDI lead to long-term gains at the expense of short-term losses. The intuition is as follows: because of financial frictions, the growth of domestic firms is constrained by their size. The presence of MNCs depresses the market share of domestic firms and thus slow down their growth. For countries in which domestic firms have a higher TFP growth rate compared to the rest of the world, having more MNCs therefore might result in lower long-run income.

Our paper is most closely related to a recent literature that studies quantitatively the patterns of MNC activities and their impacts on the aggregate economy (Burstein and Monge-Naranjo, 2009; McGrattan and Prescott, 2009; Ramondo and Rappoport, 2010; Ramondo and Rodríguez-

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3Although the world credit market is integrated, the amount of capital used in domestic production might still be constrained by domestic firms’ net worth.
Clare, 2013; Alviarez, 2016; Cravino and Levchenko, 2017; Tintelnot, 2017; Arkolakis et al., 2018; Anderson et al., 2017; Ghironi and Wolfe, 2018, among others). Most existing studies only consider the movement of technology (either rival or non-rival) and abstract from modeling financial capital. As a result, they are silent on FDI, a widely collected and used statistics, and do not speak to how financial market conditions affect the activities of MNCs, which has been documented extensively (Antras et al., 2009; Alfaro and Chen, 2012; Desai et al., 2008; Klein et al., 2002; Bilir et al., 2014; Manova et al., 2015). Our contribution is to develop a tractable heterogeneous-firm model, which combines the technology channel emphasized in existing studies with the movement of capital inside MNCs. This model gives different questions to old questions, such as how large are the welfare gains from MNC activities and, moreover, allow us to answer new questions.

To be able to account for capital accumulation and reallocation, our model is naturally dynamic, which is related to a small but growing literature in the intersection between firm dynamics and FDI (Fillat and Garetto, 2015; Garetto et al., 2017).

As an important form of cross-border capital flow, FDI has also received considerable attention from the international finance literature. Within this literature, our paper is related to both the empirical studies on the determinants and aggregate implications of FDI, as well as the theoretical and quantitative work on the welfare effects of international capital flows (Mundell, 1957; Feldstein, 1995; Gourinchas and Jeanne, 2006a). Relative to the empirical work, our contribution is to document the joint relationship between MP, FDI, and financial market conditions; relative to the theoretical and quantitative work, which usually treat FDI in the same way as other types of capital flows, our contribution is to formally model FDI as within-firm capital that enables MNCs to transfer their technology to affiliates, thus connecting this literature to the studies of MNCs in international trade.

More broadly, this paper is related to the literature on the theory of firm, in particular the recent studies focusing on how firms reallocate activities across production units in response to plant-level policy and economic shocks and the implications of this reallocation for the transmission of business cycle across regions (see, e.g., Giroud and Mueller, 2015; Giroud and Rauh, forthcoming). While focusing mostly in the domestic setting, the patterns document in this literature are supportive of the mechanisms in our model. Our paper complements this literature by developing a framework suitable for quantitative analysis.

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4Indeed, in their survey of MNCs studies, Antràs and Yeaple (2014) concludes by highlighting the lack of research that incorporates the allocation of capital within firms.

5The literature has documented that FDI flows and financial development (see, for example, Desbordes and Wei, 2017; Bilir et al., 2014), and that host country financial development is important for a country to benefit from FDI (Alfaro et al., 2004).

6A separate but related literature studies exporting and innovation in models of firm dynamics (see, for example, Atkeson and Burstein, 2010; Alessandria et al., 2014).

7The literature is too voluminous to discuss here. Most related to our focus on financial market conditions, we refer the readers to Desbordes and Wei (2017) on empirical studies of financial market conditions on FDI; and to Alfaro et al. (2004) and the reference thereto on the effects of FDI on growth.

8Also relatedly, Caballero et al. (2008), Ju and Wei (2010), and Wang et al. (2016) examine how financial frictions might explain the patterns of global imbalances. Our model is also suitable to for jointly analyzing the determinants and welfare implications of financial of portfolio capital and FDI flows in a multi-country setting, but we abstract from this as global imbalances are not the focus of this paper.
In terms of methodology, our accounting procedure in matching the dynamics in the data is related to Eaton et al. (2016) and Kehoe et al. (2018), but different in that our model allows for dynamic decisions with firm heterogeneity, incomplete markets, and multinational corporations, while still retaining tractability. This allows us, for the first time in the literature, to perform a dynamic analysis of FDI in a setting that is amenable to the multi-country data. As discussed in the conclusion section, the model and the calibration algorithm developed here can also easily extended and used to answer other questions on FDI policies.

2 Empirical Relationship Between MP, FDI, and Financial Markets

This section documents relationships between the production of MNC affiliates (MP), FDI, and the financial markets. Our measures of FDI and MP are both based on Ramondo et al. (2015), which documents the average MP and FDI over 1996-2001 at the country-pair level. MP is defined as the total sales of affiliates from a given home country. FDI is defined as the stock of capital the parents invest in the affiliate in the form of equity and intra-company loans. We supplement this data set with a number of country characteristics, such as their income, business tax rate, restriction on FDI, and indexes for the quality of financial institution. The appendix provides detailed descriptions of the data sources and results from additional robustness exercises. Using the data, we document three facts.

2.1 Correlation Between MP and FDI

Fact 1: MP is systematically correlated with FDI at country-pair level, controlling for extensive margin variation.

Our first fact speaks to the relationship between the production and financing of affiliates. Table 1 documents the corresponding results. The dependent variable is log of MP. We use the Poisson Pseudo Maximum Likelihood estimator to avoid the biases, as recommended by Silva and Tenreyro (2006). The first column reports the univariate regression of MP on FDI, which has an R-squared of over 0.7. That MP and FDI are strongly correlated should not be surprising—in fact, the literature has often used these two as interchangeable measures of MNC activities. We interpret this correlation as reflecting that affiliate production relies at least partially on financing from the parent, for either fixed investment or working capital. Of course, since a plant will only be defined as a foreign affiliate if the foreign ownership of its equity exceeds a certain threshold, the correlation could be entirely mechanical. In the second column, we control for the number of affiliates in a host country owned by parents from the home country. The coefficients for both independent variables are economically significant. Conditional on the number of affiliates, the more funding the affiliates receive from their parent, the more they produce. To further control for unobserved heterogeneity, in the third column, we include home and host country fixed effects;
in the fourth column we include various measures for bilateral distance. The coefficients barely change.

The strong correlation between MP and FDI after controlling for the extensive margin variation is inconsistent with models featuring a perfect financial market—in the absence of frictions, affiliates can borrow locally, so their real activity should be independent of how much financial investment they receive from their parents. Instead, it supports a model in which affiliates have limited capability to raise funding externally. This is in line with findings from firm-level empirical studies that affiliates rely on access to parents’ internal capital markets.10 Using information at country-pair level, we show that the patterns hold for the aggregate outcomes, in a large number of countries.

### 2.2 Host Financial Institutions and MNC Activities

**Fact 2: More FDI are made in countries with a better financial institution; conditional on the level of FDI, MP in these countries are even higher.**

The second fact relates financial market conditions in the host country to financial and production activities of MNCs. A large empirical literature finds financial development to be among the most robust predictors of FDI (see, Desbordes and Wei (2017) and the reference therein). The first part of Fact 2 confirms this result in our data set.

Following Desbordes and Wei (2017), we use the log of financial development index developed by the World Bank as our primary measure of financial institution.11 The dependent variable in

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10For example, Manova (2013) looks at Chinese exporters and find that MNC affiliates less affected by financial constraint; Alfaro and Chen (2012) shows that during the global financial crisis, multinational affiliates benefit from their access to the internal capital market.

11The index is the sum of two sub components, with the first measuring the depth of credit history info in the economy and the second measuring the legal protection of creditors’ rights.
Table 2: Host Financial Institution and MNC Activities

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (financial development index)</td>
<td>3.189***</td>
<td>0.589*</td>
<td>0.420*</td>
<td>0.257**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (FDI)</td>
<td></td>
<td></td>
<td>0.584***</td>
<td>0.600***</td>
<td>0.586***</td>
<td>0.370***</td>
</tr>
<tr>
<td>Log (credit info depth)</td>
<td></td>
<td></td>
<td></td>
<td>0.246</td>
<td></td>
<td>(0.274)</td>
</tr>
<tr>
<td>Log (creditors’ right)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.274*</td>
<td>(0.156)</td>
</tr>
<tr>
<td>Log (number of affiliates)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.543***</td>
</tr>
<tr>
<td>Observations</td>
<td>2840</td>
<td>1833</td>
<td>982</td>
<td>915</td>
<td>982</td>
<td>793</td>
</tr>
<tr>
<td>R²</td>
<td>0.250</td>
<td>0.850</td>
<td>0.962</td>
<td>0.960</td>
<td>0.963</td>
<td>0.979</td>
</tr>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Additional host characteristics</td>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Note: Bilateral distance measures include: distance between host and home countries; whether there is a colonial tie between the two; whether two countries share a border; whether they speak the same language. Additional host characteristics include: size (log gdp), average productivity, business tax information, and an index for policy restrictions on inward FDI.

The estimation specification is PPML. Standard errors (clustered at host-country level) in parentheses. * p < 0.10, ** p < 0.05, *** p < 0.01.

The first two columns of Table 2 is log FDI. The first column includes only financial development index and home country fixed effects in the regression. The coefficient is economically large and statistically significant, indicating a strong positive correlation between inward FDI and financial development. To rule out alternative explanations, in the second column we include host country characteristics, including size (log gdp), aggregate TFP, business tax rate, whether it is viewed as low-tax country, index for policy restrictions on inward FDI, and bilateral distance measures. Not surprisingly, the inclusion of these variables shrink the coefficient, but it is still economically significant. According to the coefficient, a 10% increase in the financial development index is associated with a 6% increase in inward FDI.

Columns 3 to 6 of Table 2 report the results on the second part of Fact 2. The dependent variable in these regressions is MP and the primary variable of interest is financial development. In Column 3, we include home country fixed effects, bilateral distance measures, and additional host country characteristics. The coefficient associated with financial development indicates that a 10% increase in the index is associated with a 4% increase in the production of affiliates, controlling for total investment from parents. This is consistent with the notion that a better financial institution allows parent companies to more easily raise capital from local investors—in the form of either equity or debt—to expand the scale of their affiliates.

The above correlation has several explanations—for example, financially more developed countries might have more investors abundant in resources, who are willing to lend with relatively less collateral. Alternatively, Antras et al. (2009) demonstrates theoretically and empirically, countries with better protection of creditors’ right might reassure local investors that the parent company
Table 3: The Effects of Home Financial Development on Outward FDI

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
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<tr>
<td>Log (financial development index)</td>
<td>4.102***</td>
<td>0.968***</td>
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<td>0.290</td>
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<td></td>
<td>(1.586)</td>
<td>(0.285)</td>
<td>(0.348)</td>
<td>(0.348)</td>
<td>(0.348)</td>
</tr>
<tr>
<td>Log (credit info depth)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.578)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (creditors’ right)</td>
<td>0.642***</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.208)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (FDI)</td>
<td>0.752***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>2947</td>
<td>1717</td>
<td>1616</td>
<td>1717</td>
<td>998</td>
</tr>
<tr>
<td>R²</td>
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<td>0.882</td>
<td>0.873</td>
<td>0.885</td>
<td>0.965</td>
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<td>Additional home characteristics</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Bilateral distance measure</td>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Note: See Table 2 for definition of variables. Columns 1 through 4 show that financial development in a home country is correlated with outward FDI. Column 4 shows home country financial development has no effect on MP, once FDI is controlled for.

The estimation specification is PML. Standard errors (clustered at home-country level) in parentheses. * p < 0.10, ** p < 0.05, *** p < 0.01.

will make effort in transferring the technology, increasing local partners’ willingness to invest. While we will not take a stand on the exact channel of operation, consistently with the latter explanation, when we include the two subcomponents of the financial development index separately in Columns 4 and 5, the effects are statistically stronger for protection of creditors’ right than for the depth of credit history information.

Finally, in Column 6, we include log number of affiliates in the regression. The coefficient associated with financial development decreases by around a third, but remains statistically and economically significant, so it seems the channel operates through both extensive and intensive margin.

In summary, Fact 2 shows that host country financial conditions will affect MP in two ways both directly by attracting more FDI, and indirectly by allowing these affiliates to scale up. By showing that the correlations hold on aggregate data across a large number of countries, our analysis complements that of Antras et al. (2009), which focuses on U.S. MNCs and exploits exogenous variation in financial development to show a causal effect.

2.3 Home Financial Institutions and MNC Activities

Fact 3: Countries with a better financial institution have a higher level of outward FDI but—conditional on FDI—not necessarily a higher level of outward MP.

The third fact relates outward FDI to home country financial institutions. Desbordes and Wei (2017) also looks at home country financial development on outward FDI by exploiting the interaction between country-level financial development and sectoral variation in dependence on external finance. Using bank-firm linked data, Klein et al. (2002) and Biermann and Huber (2018) provides causal evidence that the collapse in banking sector reduces outward FDI from Japanese and German firms, respectively.
4 in Table 3 reports the results. The dependent variable is log FDI, and the coefficient of interest is home financial development. We start with a regression in which the only control variables are host country fixed effects. The coefficient is economically large but of course, it may simply be picking up other effects, such as the size of a host country. In the second column, we control for other host country characteristics including their size, productivity, business tax information, and an index for policy restriction of FDI. The coefficient diminishes to 0.97 but remains statistically significant. As before, we further estimate the same specification separately for the two subcomponents of financial development index, and find that legal protection of creditors’ rights seem to be more important.

Finally, Column 5 can be viewed as a placebo test. It shows that home country financial institution has no effect on MP, once FDI is controlled for, so home financial institution increase MP primarily by increasing parent investment.

To summarize, using bilateral data, we document a strong relationship between MP and FDI. Our results further suggest that both home and host country financial development are correlated with FDI; but conditional on FDI, only host country financial development are correlated with MP. In the next section, we develop a quantitative model that are consistent with these facts. In the Appendix, we show these results are robust to different measures of financial development and alternative specifications using OLS, and are not driven by a number of small countries.

3 Model

3.1 Endowments, Preferences, and Technology

Time $t$ is discrete and goes from 0 to infinity. There are $N$ countries, indexed by $i \in I$. Each country is endowed with an exogenous number of workers, denoted by $L_{i,t}$. Workers are immobile, each supplying one unit of labor inelastically and consuming all their labor income.

Each country also has a continuum of firms. Following a large literature in firm dynamics with financial constraint (Buera et al., 2011; Midrigan and Xu, 2014; Moll, 2014), we interpret firms as owned by entrepreneurs and further assume that entrepreneurs have the following preference:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $c_t$ is the entrepreneurs’ private consumption at period $t$. Entrepreneurs make operational decisions to maximize their personal utility. Of course, most MNCs are large corporations which are owned by not individual entrepreneurs but rather shareholders. In that case, we can think of the entrepreneur as the CEO of the company, and interpret $c$ as the compensation of the CEO.\footnote{In modern corporations, CEO are usually incentivized through stock options and bonus. For example, Google CEO Sundar Pichai made $100.5 million in 2015, among which only $652,500 was salary, and $99.8 million was in the form of restricted stock, which will vest fully in 2017. In other words, how much a CEO is being paid largely depends on shareholders’ wealth gains. If we assume that the CEO’s compensation is proportional to total dividends to shareholders, then the incentives of CEO would be largely aligned with that of shareholders, in which case we can think of $u(c_t)$ as utility of the shareholders. When $u$ takes the log form, CEOs being paid a fixed fraction of total dividends will act in the same way as if they were maximizing shareholder utility.}
Firms differ in their productivity which follows Markov processes with country-specific conditional density \( f_i(z'|z) \). A firm in country \( i \) can engage in the production of a homogeneous product and operate its affiliates at different host countries (including the home country \( i \)). The affiliate with productivity \( z_{ih} \) at the host \( h \) uses \( l \) units of labor and \( k \) units of capital to produce

\[
y = (z_{ih}k)^\alpha l^{1-\alpha}
\]

units of output, with \( 1 - \alpha \in (0, 1) \) being the labor share. We assume that the productivity of an affiliate depends partially on the productivity of the parent and partially on the host country it operates in, described by the increasing function \( z_{ih} = \tilde{z}_{ih}(z) \), with \( \tilde{z}_{ii}(z) = z \) as a normalization.

In the above setting, output is a homogeneous good so there is no scope for trade. As shown in the appendix, however, this setup is isomorphic to a closed-economy version of an environment with the CES preference and monopolistic competition, if we model capital stock as the fixed cost for production of horizontally differentiated varieties. In that alternative model, it is possible to explore the interaction between trade and FDI policies. Since this interaction is not the focus of the present paper, we stay with the above simple neoclassical production setting throughout.

**3.2 Affiliate Finance and Production**

Affiliates hire labor in perfectly competitive host country labor markets and finance their capital use with funds from the parent firm as well as funds raised from local investors. Because of the financial constraint at the corporate level, when the shadow price of internal capital is higher than the local cost of financing, parents have the incentive to use as much fund from the host country as possible, in either equity or debt. However, regardless of the instruments used, various imperfections in the financial market and the credit conditions in the host country might limit the extent to which a parent can rely on funding from the host country.\(^{14}\)

Given our focus on the aggregate FDI and its macro implications, we wish to capture this force in a simple environment. We assume that to raise each dollar from the host country, an exogenously given minimum investment from the parent must be made. This is similar to the ‘collateral constraint’ setup used in many macro finance models. Without confusion, we will refer to the funding from host country as local debt, but we interpret this broadly as capture both debt and equity held by local partners and will pin down the collateral constraint using data that include both sources. Cross-sectionally, in host countries with better financial institutions, both the enforcement problem for debt and the agency problem for equity will be less severed, so the parent could be more dependent on local funds. Over time, financial institutions are unlikely to

\(^{14}\)For example, if financed through debt, a standard enforcement problem might arise, forcing the parent to put in some collateral for borrowing; if financed through equity, an agency problem might exist—the inputs of the parent, who is the developer and original owner of a production technology, are crucial for successful deployment of the production technology. Because the inputs, such as the transfer of know-how, are hard to specify in a contract, if the parent does not hold enough stake of the affiliate, the incentive for technological transfer will be undermined. Another way of thinking about this channel is that, because of contractual frictions on parents’ input, there is no perfect market for technology. A substantial share of affiliates assets must be at parents’ stake for host country investors to be willing to invest. This mechanism has been examined in a micro-founded model and tested using the data of MNCs headquartered in the U.S. in Antras et al. (2009).
change in the short-run, but the availability of credit in a country will also affect the difficulty of accessing local funding. In quantification, we capture both by allowing this parameter to differ both across countries and over time, but suppress time subscript for now.

Formally, we assume that given the investment by a parent in a country \( h \) affiliate, \( e_h \), the amount borrowed by an affiliate from home \( i \) at host \( h \), denoted \( b^F_{ih} \), cannot exceed \( \mu_{ih} \cdot e_h \), where \( \mu_{ih} \geq 0 \) captures the maximum ‘leverage’ allowed in host country \( h \) given the financial institution and credit market conditions. The maximum capital that can be used is therefore \( (1 + \mu_{ih})e_h \).

The return from investing is the output plus non-depreciated capital, minus labor cost and interest expense. Denoting the wage and net interest rate in host \( h \) by \( w_h \) and \( r^b_h \) respectively, for given \( e_h \) and affiliate productivity \( z_{ih} = \tilde{z}_{ih}(z) \), the parent firm makes the affiliate financing and production decision to maximize the affiliate-specific return:

\[
\tilde{R}_{ih}(z, e_h) = \max_{b^F, k, l, y} \left[ y + (1 - \delta)k - w_h l - (1 + r^b_h)b^F \right],
\]

s.t. \[ y = [z_{ih}(z)k]^a l^{1-a} \]
\[ 0 \leq b^F \leq \mu_{ih}e_h \]
\[ 0 \leq k \leq e_h + b^F. \]

Cross-border investment projects are characterized by significant frictions, such as the barriers to communications and transfer of knowledge (Keller and Yeaple, 2013), the risk of extortion by corrupted foreign officials (Wei, 2000) or the expropriation of foreign governments (Thomas and Worrall, 1994), as well as idiosyncratic risks involved in individual projects. To capture these, we assume that the parent can seize only a fraction of the return, denoted \( \eta_{ih} \tilde{R}_{ih}(z, e_h) \), with the remaining ‘melting’ in the repatriation process much like in the ‘iceberg’ trade cost specification. \( \eta_{ih} \equiv \bar{\eta}_{ih} \zeta_h \), in which \( \eta_{ih} \) is the deterministic component common for all firms from country \( i \) investing in host \( h \), while \( \zeta_h \) is the idiosyncratic component that is i.i.d. across parents and affiliates. The idiosyncratic components capture heterogeneous ‘match’ quality between the production technology and potential host countries. The literature has documented that MNCs are more productive than domestic firms and rationalized this with a fixed cost of setting up affiliate so that the average return from opening up a foreign affiliates increases in productivity (Helpman et al., 2004). In quantification, we capture this force by allowing \( \eta_{ih} \neq h \) to be increasing in \( z \) and parameterize this function using firm-level data.

### 3.3 Parent Firm Finance and Investment

In each period, after seeing the realizations of the investment return shocks from each potential host country, \( \eta_i = (\eta_{i1}, \eta_{i2}, ..., \eta_{iN}) \), and the current productivity, \( z \), firms first decide whether to produce or stay idle for the period. An idle firm loans out its net worth \( a \) at the market interest rate. An active parent firm, on the other hand, decides whether to borrow risk-free bond domestically using its net worth as collateral to scale up, and which host country to invest in.

The rate for lending and borrowing in country \( i \) is denoted \( r^b_i \). For active firms, the amount
they can borrow is subject to the collateral constraint

\[ b^H \leq \lambda_i a, \]

which says that the external funds cannot exceed \( \lambda_i \) fraction of the parent firm’s net worth. The total fund at the parent, \( a + b^H \), will then be allocated to affiliates to maximize the total return.

Formally, the Bellman equation for the value function (adding time subscript explicitly) of firm owners from country \( i \) reads

\[
v_{i,t}(z, \eta, a) = \max_{c, a', (c_h)_{h=1}^H} \log(c) + \beta \mathbb{E}v_{i,t+1}(z', \eta', a') \tag{3}
\]

\[
s.t. \quad \sum_h e_h = a + b^H
\]

\[
-c + a' = \sum_h \tilde{R}_{ih,t}(z, e_h) \eta_{ih} - (1 + r^{H}_i)b^H,
\]

where the first constraint says that funds allocated to affiliates should sum to net worth plus borrowing raised in the home country. The second constraint says that, (1) an idle parent firm can loan out all, but not more than, its net worth; and (2), funds raised by an active parent firm cannot exceed the collateral constraint. In the third constraint, \( \eta_{ih} \tilde{R}_{ih}(z, e_h) \) denotes the net return from investing in host country \( h \), which is net of wages, return to investors in the host country and the component melt on the way. This constraint says that the total repatriated profit from affiliates are split among: domestic lenders, retained earnings, and current consumption of firm owners.

### 3.4 Characterizing Affiliate- and Firm-Level Decisions

In the above problem, firms make a dynamic decision for capital accumulation and a static decision in allocating investment among host countries. The incomplete-market setting, while natural, also means that in solving the model, we need to keep track of the joint distribution of \((z, a)\) for each country. To bring the model to the data, we characterize all firm-level decisions analytically and provide some aggregation results, under two additional assumptions.

We start by solving for the return and policy functions for each affiliate, specified in Equation 2. Because both the objective function and the constraint in the problem are homogeneous of degree one in \( e_h \), the affiliate decision and return will be linear in \( e_h \), too, and can be characterized by the following lemma:
Lemma 1. The affiliate return defined in (2) satisfies \( \tilde{R}_{ih}(z, e_h) = R_{ih}(z)e_h \) where

\[
R_{ih}(z) = \max_{\hat{y}, \hat{k}, \hat{l}, \hat{y}} \hat{y} + (1 - \delta)\hat{k} - w_h \hat{l} - (1 + r_h^b)\hat{b}^F, \\
\text{s.t.} \quad \hat{y} = [\tilde{z}_{ih}(z)\hat{k}]^{\alpha} \hat{l}^{1 - \alpha} \\
0 \leq \hat{b}^F \leq \mu_{ih} \\
0 \leq \hat{k} \leq 1 + \hat{b}^F.
\]

The solutions to (4) satisfy \( X_{ih}(z, e_h) = \hat{X}_{ih}(z)e_h \) for \( X = (b^F, k, l, y) \), where \( \hat{X}_{ih}(z) \) are solutions to (4) and can be explicitly characterized by

\[
\hat{b}_{ih}(z) = \begin{cases} 
\mu_{ih}, & \forall \tilde{z}_{ih}(z) \geq z_{ih}^* \\
0, & \forall \tilde{z}_{ih}(z) < z_{ih}^* 
\end{cases} \\
\hat{k}_{ih}(z) = [1 + \hat{b}_{ih}(z)] \\
\hat{l}_{ih}(z) = \tilde{z}_{ih}(z)\hat{k}_{ih}(z)\left(\frac{1 - \alpha}{w_h}\right)^{1/\alpha} \\
\hat{y}_{ih}(z) = \tilde{z}_{ih}(z)\hat{k}_{ih}(z)\left(\frac{1 - \alpha}{w_h}\right)^{(1 - \alpha)/\alpha},
\]

with the cutoff \( z_{ih}^* \) determined implicitly by \( \pi_{ih}(z_{ih}^*) = 1 + r_h^b \) and \( \pi_{ih}(z_{ih}) \) defined as:

\[
\pi_{ih}(z_{ih}) = a z_{ih} \left(\frac{1 - \alpha}{w_h}\right)^{(1 - \alpha)/\alpha} + 1 - \delta.
\]

The first part of the lemma specifies that, the affiliate-level total return is simply the return to each unit of investment times total investment, with the unit-investment return given by the solution to Equation 4. The second part of the lemma shows that affiliates’ decision follow a cutoff rule: affiliates whose productivity are above the threshold, \( z_{ih}^* \), will leverage the funding from local investors to the maximum amount possible; on the other hand, affiliates whose productivity are below the threshold will choose not to use any funding from the host country. The cutoff is given by the condition that equals the cost of borrowing, \( 1 + r_h^b \), to the return from an additional unit of capital, given by \( \pi_{ih}(z_{ih}) \). The selection channel as in the Melitz model operates here: as the wage goes up, the cutoff will also increase, so fewer active affiliates will seek funding from local investors for expansion.

Lemma 1 gives an explicit solution to the affiliates’ problem, so we now know \( \tilde{R}_{ih,t}(z, e_h) \) in Equation 3. However, it still remains a challenge to characterize the solution to the dynamic problem at the firm-level. Following a large literature in macro economics, we assume the utility function of firms’ owner takes the log form, that is,

Assumption 1. \( u(c) = \log(c) \).

Under Assumption 1, the solution to Equation 3 can be characterized by Lemma 2, using the homogeneity property of the problem.
Lemma 2. The value function of firm owners (Equation 3) satisfies
\[ v_{i,t}(z, \eta, a) = \hat{v}_{i,t}(z, \eta) + \frac{1}{1 - \beta} \log(a) \]
for some \( \hat{v}_{i,t}(z, \eta) \). Policy functions for consumption and investment satisfy
\[ c_{i,t}(z, \eta, a) = \hat{c}_{i,t}(z, \eta) a, \quad \hat{c}_{i,t}(z, \eta) = (1 - \beta) R_{i,t}^a(z, \eta) \]
\[ d_{i,t}(z, \eta, a) = \hat{d}_{i,t}(z, \eta) a, \quad \hat{d}_{i,t}(z, \eta) = \beta R_{i,t}^\delta(z, \eta), \]
where
\[ R_{i,t}^\delta(z, \eta) = \begin{cases} \max_{h'} R_{ih't}(z) \eta_{h'} (1 + \lambda_{i,t}) - (1 + r_{i,t}^b) \lambda_{i,t}, & \text{if } \max_{h'} R_{ih't}(z) \eta_{h'} \geq 1 + r_{i,t}^b \\ (1 + r_{i,t}^b), & \text{if } \max_{h'} R_{ih't}(z) \eta_{h'} < 1 + r_{i,t}^b \end{cases} \] (5)

with \( R_{ih't} \) defined in Lemma 1.

According to Lemma 2, the value function of a firm owner is the sum of two components, one depending solely on parent productivity and the idiosyncratic return draws for its potential affiliates across the world, the other log of a firm’s net worth. Moreover, a fixed share (\( \beta \)) of the total end-of-period wealth will be reinvested, with the remaining used for consumption. This end-of-period wealth takes into account the net return firms will be making from either active production or staying idle loaning out the net worth. Given the linear return structure at the affiliate level (Lemma 1), it follows that active firms will invest all resource into the most profitable affiliate, which might be profitable for either fundamental reasons or a lucky \( \zeta_h \) draw. Given the collateral constraint at the firm level, if the highest net return is above the cost of borrowing, the firm will max out the credit allowed \((\lambda_{i,t} \cdot a)\); else, the firm owner will lend the net worth, in which case the return to wealth is the market interest rate.

3.5 Aggregation

Together, Lemmas 1 and 2 express firms’ decisions as functions of their states \((z, a, \eta_i)\) after the uncertainty about the idiosyncratic \((\zeta_h)_{h=1}^N\) draws has been resolved. Keeping track of the evolution of the net worth distribution in each country and the aggregate FDI between each country pair requires integrating across firms with all possible realizations of \((\zeta_h)_{h=1}^N\), which is in general a daunting task. To overcome this problem we make the following assumption:

Assumption 2. The CDF for \((\zeta_h)_{h=1}^N\) is given by: 15
\[ G(\zeta_1, ..., \zeta_N) = 1 - \sum_{h} \frac{1}{N} [\zeta_h]^{\theta}, \text{ for } \zeta_h \geq 1, \forall h. \]

15For ease of exposition, we assume the underlying correlated Pareto distribution is standardized, so for any given parent, draws from different hosts are symmetric. In this setting, \( \bar{\eta}_{ih} \) do not enter the distribution as a parameter but directly affect the return conditioning on the draws.
This distribution is the special case (when $\rho = 0$) of the multivariate distribution introduced in Arkolakis et al. (2017a)\textsuperscript{16}. One attractive property of this distribution is that $\max_{h'}(\zeta_{ih'})^N$ has a Pareto tail. As we show below, this will allow us to analytically aggregate across firms with different $(\zeta_{ih'})^N$ draws in the presence of a cutoff in firm policy function.\textsuperscript{17}

Specifically, define $\Xi(z) \equiv \max_{h'}\eta_{ih'}R_{ih'}(z) = \max_{h'}\eta_{ih'}R_{ih'}(z)\tilde{\zeta}_{ih'}$. From Equation 5, the realization of $\Xi(z)$ determines whether a firm with productivity $z$ will be active and if so, which destination to invest. The CDF for $\Xi(z)$, denoted $H(\zeta|z)$, is given by the following:

$$H(\zeta|z) \equiv Pr(\Xi(z) \leq \zeta) = \begin{cases} 1 - (\frac{\zeta}{R_i(z)})^{-\theta}, & \text{for } \zeta \geq R_i(z) \\ 0, & \text{for } \zeta < R_i(z), \end{cases}$$

where $R_i(z) \equiv \max_{h'}\eta_{ih'}R_{ih'}(z)$, and $\tilde{R}_i(z) \equiv \left(\frac{1}{N} \sum_{h'}[\eta_{ih'}R_{ih'}(z)]^\theta\right)^{\frac{1}{\theta}}$.

Because all $\zeta_{ih}$ draws are greater than or equal to 1, the support of $\Xi(z)$ is $[\tilde{R}_i(z), \infty)$. Above $R_i(z)$, the distribution of $\Xi(z)$ has a Pareto tail; at $R_i(z)$, there is a measure $1 - \frac{\tilde{R}_i(z)}{R_i(z)}$ mass point. This measure is zero if $\tilde{R}_i(z) = \tilde{R}_i(z)$, that is, if all countries offers the same mean return for firms.

\textsuperscript{16}In the general distribution, parameter $\rho$ governs correlation among $(\zeta_{h'})^N_{h=1}$, with $\rho \to 1$ corresponds to higher correlation and $\rho \to 0$ the lowest. Because our model captures the correlation in productivity among affiliates of the same parent through $z_{ih}(z)$, we interpret $\zeta_{ih}$ as capturing solely the residual idiosyncratic ‘match’ quality between the parent’s technology and a host country. We therefore assume $\rho = 0$.

\textsuperscript{17}That is, these shocks only matter for return if firms decide to be active; and firms only decide to be active, if the most profitable affiliate generates higher net return than risk-free bond (see Lemma 2). This cutoff renders a widely used strategy to smooth firm decisions—adding a Frechet shock—intractable, because it is not invariant to truncations. One way to restore the tractability of the Frechet distribution is to assume that each firm operates a continuum of projects and each with a draw from the Frechet distribution. Firms make investment decisions before seeing the realizing of the shocks and then choose the host country for each project. This setup, however, generates the counterfactual predictions that all firms operate in all host countries; the assumption that upon seeing the realization of the draws, firms still must carry out all projects is also at odds with reality.
with productivity \( z \).\(^{18}\)

Given \( H(\xi | z) \), we can characterize the aggregate investment decision and return on net worth for firms with productivity \( z \) under two different scenarios. First, firms whose productivity \( z \) is such that \( \tilde{R}_i(z) < 1 + r_i^b \). This is the case illustrated in Figure 1. Firms will stay active if and only if the realization of \( \Xi(z) \) falls to the right of the vertical blue line. Second, when \( \tilde{R}_i(z) \leq 1 + r_i^b \), in which case all firms will be active. Lemma 3 summarizes the results:

**Lemma 3.** Consider the pools of firms with productivity \( z \),

(i) if \( \tilde{R}_i(z) < 1 + r_i^b \), the share of active firms is

\[
[\tilde{R}_i(z)/(1 + r_i^b)]^\theta.
\]

The share of active firms investing in host \( h \), denoted by \( \tilde{e}_{ih}(z) \), is

\[
\tilde{e}_{ih}(z) = [\tilde{R}_i(z)/(1 + r_i^b)]^\theta \chi_{ih}(z), \text{ where } \chi_{ih}(z) \equiv \frac{1}{N} \left( \frac{\eta_{ih}R_{ih}(z)}{\tilde{R}_i(z)} \right)^\theta.
\]

The expected return to the net worth of these firms is

\[
\mathbb{E}[R_i^a(z, \eta)|z] = \left( 1 - [\tilde{R}_i(z)/(1 + r_i^b)]^\theta \right)(1 + r_i^b) + [\tilde{R}_i(z)/(1 + r_i^b)]^\theta \left( \frac{\theta}{\theta - 1} (1 + r_i^b)(1 + \lambda_i) - (1 + r_i^b) \lambda_i \right).
\]

(ii) If \( \tilde{R}_i(z) \geq 1 + r_i^b \), the share of active firms is 1. If the set \( \overline{\mathbf{H}} = \arg \max_{h'} \eta_{ih'}R_{ih'}(z) \) is a singleton, the share of active firms investing in \( h \) is

\[
\tilde{e}_{ih}(z) = \begin{cases} 
1 - [1 - \chi_{ih}(z)][\tilde{R}_i(z)/\tilde{R}_i(z)]^\theta, & \text{if } h \in \overline{\mathbf{H}}, \\
\chi_{ih}(z)[\tilde{R}_i(z)/\tilde{R}_i(z)]^\theta, & \text{if } h \notin \overline{\mathbf{H}},
\end{cases}
\text{ where } \chi_{ih}(z) \text{ defined in part (i)}.
\]

The expected return to the net worth of these firms is

\[
\mathbb{E}[R_i^a(z, \eta)|z] = \left( 1 - [\tilde{R}_i(z)/\tilde{R}_i(z)]^\theta \right) \tilde{R}_i(z)(1 + \lambda_i) + [\tilde{R}_i(z)/\tilde{R}_i(z)]^\theta \left( \frac{\theta}{\theta - 1} \tilde{R}_i(z)(1 + \lambda_i) - (1 + r_i^b) \lambda_i \right).
\]

The above lemma circumvents the need to integrate across idiosyncratic return shocks in quantification. The first part of the Lemma follows directly from the fact that \( H(\xi | z) \) has a Pareto tail.

\(^{18}\)To see this, note that \( \text{Prob}(\Xi(z) \leq \xi) = \text{Prob}(\xi_1 \leq \frac{\xi}{\eta_{11}R_{11}(z)}, \xi_2 \leq \frac{\xi}{\eta_{12}R_{12}(z)} \ldots, \xi_N \leq \frac{\xi}{\eta_{1N}R_{1N}(z)}) \). When \( \xi = \tilde{R}_i(z) \equiv \max_{h'} \eta_{ih'}R_{ih'}(z) \), we have \( \frac{\xi}{\eta_{ih'}R_{ih'}(z)} \geq 1, \text{ for all } h' \), which is in the support of \( G \), so

\[
\text{Prob}(\xi_1 \leq \frac{\xi}{\eta_{11}R_{11}(z)}, \xi_2 \leq \frac{\xi}{\eta_{12}R_{12}(z)}, \ldots, \xi_N \leq \frac{\xi}{\eta_{1N}R_{1N}(z)}) = G(\xi_1 \leq \frac{\xi}{\eta_{11}R_{11}(z)}, \xi_2 \leq \frac{\xi}{\eta_{12}R_{12}(z)}, \ldots, \xi_N \leq \frac{\xi}{\eta_{1N}R_{1N}(z)}) = 1 - \left( \frac{\xi}{\tilde{R}_i(z)} \right)^{-\theta}.\]
The second part needs to, in addition, explicitly consider the firms whose realization of \( \Xi(z) \) are right at the mass point \( R_i(z) \equiv \max_{h'} \bar{\eta}_{ih'} R_{ih'}(z) \) is achieved by only one host. When it is achieved in more than one country, then a positive measure of firms will be indifferent between hosts, in which case we need to impose a tie-breaking rule.\(^{19}\)

At each period \( t \), the aggregate state of the economy is the joint distribution over \((z,a)\) in each country \( i \), characterized by joint density functions \((\Phi_{i,t}(z,a))_{i \in I}\). Since policy functions conditional on \( z \) are linear in \( a \), for aggregation, it is sufficient to track the aggregate net worth by each productivity level. Formally, define the marginal net worth over productivity as

\[
\phi_{i,t}(z) = \int_0^\infty \Phi_{i,t}(z,a) da. \tag{6}
\]

The transition of \( \phi_{i,t}(z) \) is then given by the following equation:

\[
\phi_{i,t+1}(z') = \int_0^\infty \phi_{i,t}(z) \beta \mathbb{E}[R_{i,t}^a(z, \eta)] |z| f_i(z'|z) dz, \tag{7}
\]

where \( \mathbb{E}[R_{i,t}^a(z, \eta)|z] \) is explicitly characterized in Lemma 3.

With this, we can express the aggregate objects in the model as follows. In our model, FDI emerges as within-firm transfer of capital. With the policy functions derived in Lemma 1 and Lemma 2, and the marginal net worth defined in Equation 6, the aggregate FDI stock from \( i \) to \( h \) at time \( t \) is

\[
[FDI]_{ih,t} = \int_0^\infty \hat{e}_{ih,t}(z) \phi_{i,t}(z) dz.
\]

The aggregate multinational production conducted by parent firms from \( i \) in host \( h \) at time \( t \) is

\[
Y_{ih,t} = \int_0^\infty \hat{y}_{ih,t}(z) \hat{e}_{ih,t}(z) \phi_{i,t}(z) dz.
\]

We also write out the total capital in a host \( h \) by aggregating domestic and foreign investment

\[
K_{h,t} = \sum_i K_{ih,t} = \sum_i \int_0^\infty \hat{k}_{ih,t}(z) \hat{e}_{ih,t}(z) \phi_{i,t}(z) dz,
\]

and total productions

\[
Y_{h,t} = \sum_i Y_{ih,t}.
\]

\(^{19}\)In quantitative implementation, however, this does not matter. Unless two hosts \( h \) and \( h' \) have the exact same primitives, equilibrium wage, and bond interest rate, \( \bar{\eta}_{ih} R_{ih}(z) \) crosses \( \bar{\eta}_{ih'} R_{ih'}(z), h \neq h' \) for only finite values of \( z \). Because in the quantitative exercise, \( z \) will have a continuous density, outside the special case with two identical host countries—which does not arise in the data—\( \max_{h'} \bar{\eta}_{ih'} R_{ih'}(z) \) can be achieved by more than one country for only zero measure of firms.
3.6 Equilibrium

Given an initial distribution of parent firms \((\Phi_{i,0}(z, a))_{i \in I}\), a competitive equilibrium is a sequence of (a) wages and bond interest rates, (b) parent firm value and policy functions, affiliate return and policy functions, and (c) distribution of firms, such that at every period (i) value, return and policy functions solve firms’ problems; (ii) labor and credit (bond) markets clear in each country; (iii) the distributions of firms are consistent with the transitions implied by the firms’ policy functions and the exogenous productivity process.

3.7 FDI, MP, and Financial Market Conditions

We now discuss the model’s predictions on the relationship between financial market conditions and the two measures of MNC activities—FDI and MP. As country \(i\) becomes financially more developed (or as it experiences a boom in credit markets), accessing external finance will be easier for both affiliates and parent companies, i.e., \(\mu_i\) and \(\lambda_i\) will both increase. A higher \(\mu_i\) allows productive affiliate to use more external finance, which increases the return to parent investment in that affiliate. This leads to an increase in inward FDI. A higher \(\lambda_i\) will have three effects. First, an increase in \(\lambda_i\) helps channel credits into the most productive firms, who are more likely to become MNCs. Second, this reallocation increases the wage and interest rate in the domestic economy, which pushes domestic firms to invest abroad. Finally, with better access to credits, productive firms will be able to accumulate capital faster. Because productivity is persistent, this reallocation increases the aggregate efficiency and investment of the economy in the long run, resulting in a higher outward FDI level as well. All three channels imply that financial development is a push factor for FDI.

In addition to connecting FDI and financial market conditions, the model also links FDI with MP through the following proposition:

**Proposition 1.** For every \((i, h)\) at period \(t\),

\[
\frac{Y_{ih,t}}{Y_{h,t}} = \frac{[FDI]_{ih,t}}{K_{h,t}} \times \bar{lev}_{ih,t} \times \frac{z_{ih,t}}{\bar{z}_{h,t}}
\]

where \(\bar{lev}_{ih,t}\) is the average leverage (total assets / parent financing) of affiliates in host \(h\) from home \(i\), \(z_{ih,t}\) is the average (production weighted) productivity of affiliates in host \(h\) from home \(i\), and \(\bar{z}_{h,t}\) is the average productivity of all affiliates in host \(h\).

\(\frac{Y_{ih,t}}{Y_{h,t}}\) is the commonly used measure of MP in the literature: share of foreign affiliates in total domestic production. \(\frac{[FDI]_{ih,t}}{K_{h,t}}\) measures the share of FDI in total domestic capital stock. Because affiliates only have a finite capacity to raise external funding, MP share is tightly connected to FDI share, consistent with Fact 1. However, these two measures are intermediated by two other components: first, how much local source of financing the affiliates uses, summarized by the average leverage and ultimately depending on \(\mu_{ih}\). Given FDI share, MP share will be higher if the host country allows affiliates to leverage more on local financing, consistent with Fact 2, as well as the empirical finding that inward FDI brings more benefits in countries with a more
developed financial market (Alfaro et al., 2004). Second, the productivity of these affiliates relative to indigenous firms also matters: when affiliates are more productive, given the FDI share, MP share will be higher.

Moreover, note that conditional on the FDI share and the productivity distribution, according to Equation 8, MP share does not depend on the financial market condition of the host ($\lambda_i$), in line with the empirical finding from the last Column of Table 3.

### 3.8 Model Discussion

Before moving into quantitative exercises, we discuss the validity of model assumptions and the implications if they are violated. One of the key assumptions is that, firms face short-run financial frictions, so the shadow value of capital differs from the cost of external credits. This assumption motivates FDI as within-firm capital flow and allows the model to deliver empirically consistent relationships. Admittedly, the specific way in which we introduce the financial constraint is ad-hoc, yet as long as some forms of financial constraint exist, the qualitative channel will remain.

One might also be skeptical that whether MNCs, which are typically large conglomerates, still face financial constraint. In reality, even though large firms can borrow from banks or the bond market, as their leverage increases, the default risk and agency cost usually leads to a higher cost of borrowing (Baxter, 1967). Our model captures this idea in a parsimonious way. It is worth noting that our model only restricts the short-term debt equity ratio. Productive firms can still expand by accumulating more equity and leveraging it to borrow more. Consistent with our assumption on the existence of financial constraints, researchers have shown empirically that large multi-unit firms, both nationals and multinationals, use internal factor market to reallocate resources when one unit or plant receives a shock (Klein et al., 2002; Almeida et al., 2015; Giroud, 2013; Biermann and Huber, 2018).

Our current setup also abstracts from the fixed cost of setting up foreign affiliates, so the ‘proximity-concentration’ tradeoff emphasized in the literature (Brainard, 1997) does not play a role. Moreover, by assuming each affiliate is an independent producer of a homogeneous good, the model does not allow for the interaction between affiliates through demand cannibalization (Tintelnot, 2016). On the other hand, the model incorporates the transfer of technology to affiliate and the cannibalization between affiliates in their competition for the scarce internal factor (capital), thus capturing better the ‘brown-field’ investment in the form of mergers and acquisitions. Given that the latter is the dominant form of foreign investment, we view the current framework as suitable for analyzing aggregate FDI between countries.

### 4 Accounting for the Dynamics of FDI

Through the lens of the model, we now explore the importance of various fundamental factors in explaining the dynamics of FDI. We keep Assumptions 1 and 2. Our current framework is

---

20In particular, Giroud (2013) shows the exogenous improvement in productivity of one plants in a multi-plant firm lead to a decrease in investment in other plants.
flexible on the depth of international bond market integration. Country-specific interest rates correspond to a segregated world bond market; a common interest rate across countries, on the other hand, corresponds to a fully integrated bond market. The reality likely lies between the two polar cases. Given that a large literature in international finance has explored the welfare consequences of moving from country-specific bond market to a global bond market (Gourinchas and Jeanne, 2006b), to focus on FDI and its welfare implications, our quantitative exercises will assume the bond market is fully integrated. In the concluding section, we discuss how the model can be extended to study the interaction between bond market frictions and FDI.

As is well-known, the past two decades saw rapid growth of activities by MNCs. Figure 2 plots the aggregate cumulative outward FDI flow from a number of countries over the period 2001 to 2012 (we normalize the 2001 U.S. GDP to be 1). Two patterns stand out. First, throughout the sample period, firms from these countries are investing an increasing amount of resources abroad. The aggregate growth is mainly accounted for by large growth by countries which were already investing heavily abroad in 2001. For example, the cumulative investment in overseas affiliates increased by 170% and 90% for MNCs based on the U.S and U.K., the two largest home country of MNCs, respectively. In smaller and less developed countries, while the level of outward FDI is not as significant, the growth rate has been pronounced. Second, the aggregate FDI growth slowed down since the financial crisis. The U.K. and Germany, for example, see their outward FDI stock flattening out after 2008.21

Our model provides a natural starting point to quantitatively decompose the effects of changes in various fundamental factors, including financial institution for parent companies ($\lambda_{it}$) and affiliates ($\mu_{iht}$), and firms’ average productivity.

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21 These two patterns are also consistent with the finding by Alviarez et al. (2017) based on sales of affiliates.
The spirit of our quantification exercise follows that of Eaton et al. (2016). We will pick the model fundamental parameters and the residual wedges to match the data along a number of dimensions during the period of 2001-2012 for 36 major developed and developing countries. If we feed all of these changes into the model, the model will produce time series of GDP, bilateral investment, credit use, etc., exactly as in the data. We will then switch off changes in certain fundamental factors to see the contribution of each to the change in FDI.

In the appendix, we provide additional information on the data sources and computational algorithm. Below we describe how we pin down the model parameters. Many of our parameters are jointly identified, but some are more crucial for matching certain empirical target than others, so we discuss the most direct target for each parameter, before describing our numerical procedures.

4.1 Parameterization

4.1.1 Targets and values

We start with the parameters that are kept constant over time. The entrepreneurs’ discount rate, $\beta$, determines the saving rate. We set $\beta = 0.9$, so the world interest rate is 3% at the initial state. We set capital share $\alpha = 0.4$ and depreciation rate $\delta = 4.5\%$ based on the average number in the Penn World Table for our sample countries. The dispersion parameter of the multivariate Pareto distribution, $\theta$, determines the sensitivity of firms’ investment decision to host-country specific returns. Using variation on international tax, Wei (2000) estimates this elasticity to be 4.6, which is also the median value according to a recent meta analysis (De Mooij and Ederveen, 2003). We set $\theta = 5$ as a benchmark and experiment with alternative values.

We next parameterize parent firms’ idiosyncratic productivity $z$ to follow the AR(1) process below:

$$\log(z') = \bar{z}_i + \rho z \log(z) + \epsilon,$$  \hspace{1cm} (9)

in which $\epsilon$ is a mean-zero innovation term with variance $\sigma^2_{\epsilon,i}$ and $\bar{z}_i$ is the fundamental productivity in country $i$. Using firm-level data, Asker et al. (2014) estimate the productivity process for a large number of developed and developing countries. We take the median estimate from their sample of countries and set $\rho_z = 0.85$, $\sigma^2_{\epsilon,i} = 0.69$. We allow $\bar{z}_i$ to vary over time and will use it to match the country-level output, so later we will also use notation $\bar{z}_{it}$.

When an MNC opens an affiliate in a host country, the productivity of the affiliate depends on not only the productivity of the parent, but also on host-specific factors, such as infrastructures and knowledge embodied in local firms. We capture this by assuming that the productivity of an affiliate in country $h$ is the following:

$$z_h(z) = z^{\gamma} \bar{z}_h^{1-\gamma},$$ \hspace{1cm} (10)

in which $z$ is the productivity of the parent and $\bar{z}_h^{1-\gamma}$ is the aggregate TFP of the host country. We set $\gamma = 0.4$ based on a recent estimate by Cravino and Levchenko (2017), using a data set that covers parents and affiliates in a large number of countries.
The literature has documented that more productive firms are more likely to become MNCs. We capture this in a reduced-form way by assuming that the international investment wedge has a time-invariant component that depends on $z$:

$$\bar{\eta}_{ih,t}(z) = \bar{\eta}_{ih,t}z^\eta_{z}, h \neq i. \quad (11)$$

Given the idiosyncratic component of the return shock, the probability a firm finds it optimal to open an affiliate overseas can be shown to be a log linear function of $z$, with $\eta_z$ being the corresponding elasticity. This allows us to estimate $\eta_z$ using a Multinomial Logit specification. Based on a representative survey of manufacturing firms in a number of countries (Bloom et al., 2012), we estimate $\eta_z = 0.4$.

The remaining parameters are allowed to change over time, and disciplined correspondingly using time-varying targets. Parameter $\lambda_{i,t}$ determines to the extent to which a company can use net worth as collaterals for external borrowing. In the long-run, this parameter depends on the financial institution of a country, but its short-run fluctuation is likely driven by the availability of credits in a country. We therefore use $\lambda_{i,t}$ to match the time series of credit over GDP ratio in our sample countries, interpreting its over time change as capturing the evolving credit market conditions.

Parameter $\mu_{ih,t}$ determines the extent to which a parent company can rely on local partners for finance. Its short-term fluctuations likely depend on the availability of credits in a country. The response of $\mu_{ih,t}$ to local credit conditions could be potentially different from that of $\lambda_{i,t}$. On the one hand, affiliates of foreign firms are backed by the reputation of their well-known parents; on the other hand, they might lack the connection to local financial institutions compared to local-grown firms. To allow for differential responses, we discipline $\mu_{ih,t}$ independently with another time series. Recall that $\mu_{ih,t}$ pins down the share of an affiliates’ balance sheet financed by host country partners. We use the BEA data to construct the empirical counterpart of this object. Specifically, the BEA reports the total assets of U.S. affiliates in each country in each year, and the position of U.S. parents in these affiliates. This allows us to measure the share of affiliates’ balance sheet financed by host-country partners and pin down $\mu_{ih,t}$. We construct this ratio for the U.S., using data from BEA on foreign affiliates in the U.S.

The domestic investment wedge and the remaining component of international investment wedge determine the evolution of domestic investment and FDI, respectively. We set the capital stock in each country ($K_{ht}$) to the data at the beginning of our sample period. We then use $\bar{\eta}_{iit}$ and $\bar{\eta}_{ih,t}, h \neq i$ to match the evolution of domestic and foreign investment, respectively.\(^{22}\) This procedure ensures that the model matches perfectly the evolution of capital stock in each country and the distribution of their ownership across home countries.

The labor endowment in each country, $L_{it}$ is set to the effective employment from the Penn World Table, which takes into account changes in population, labor force participation, and effec-
Table 4: Model Parameterization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target/Source</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>Capital share</td>
<td>PWT</td>
<td>0.4</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Capital depreciation rate</td>
<td>PWT</td>
<td>4.5%</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Elasticity of FDI w.r.t. return</td>
<td>Wei (2000)</td>
<td>5</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Firm productivity autocorrelation</td>
<td>Asker et al. (2014)</td>
<td>0.85</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>Firm productivity innovation variance</td>
<td>Asker et al. (2014)</td>
<td>0.69</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Parent weight in affiliate productivity</td>
<td>Cravino and Levchenko (2017)</td>
<td>0.4</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Relationship between MNC status and productivity</td>
<td>Estimated using Bloom et al. (2012) data</td>
<td>0.4</td>
</tr>
<tr>
<td>{ L_t }</td>
<td>Effective employment</td>
<td>PWT</td>
<td>-</td>
</tr>
</tbody>
</table>

B: Parameters Calibrated in Equilibrium

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target/Source</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>{ \lambda_t }</td>
<td>Credit market conditions for parent companies</td>
<td>Credit/Capital ratio</td>
<td>Figure 3</td>
</tr>
<tr>
<td>{ \mu_{ht} }</td>
<td>Credit market conditions for affiliates</td>
<td>Share of affiliates balance sheet financed by parents</td>
<td>Figure 3</td>
</tr>
<tr>
<td>{ \eta_{ht} }</td>
<td>Return wedge for domestic and foreign direct investments</td>
<td>( { K_{ht} }, { K_{iht} } )</td>
<td>-</td>
</tr>
<tr>
<td>{ \zeta_t }</td>
<td>Fundamental TFP</td>
<td>GDP</td>
<td>-</td>
</tr>
</tbody>
</table>

tive human capital of labor force.

With the above parameterization, our model matches the aggregate capital and labor input in each period. To match the evolution of GDP in each country, we can directly adjust the intercept of firm productivity process, specified in Equation 9. The resulting term, which we label \( \hat{Z}_{it} \), could be thought of as the measured aggregate TFP, absorbing all the variation of output beyond those of aggregate inputs. But in our heterogeneous firm model, this is different from the fundamental TFP, \( \bar{Z}_{it} \)—in addition to the fundamental TFP, \( \bar{Z}_{it} \) also captures the changes in allocative efficiency in response to financial market conditions. For example, in response to increases in \( \lambda_{it} \) and \( \mu_{ht} \), efficient firms will gain market share, resulting in an upward bias in the measured aggregate TFP. We isolate the fundamental TFP as the aggregate productivity needed to match the aggregate output, assuming that the relative distribution of capital over firms with different productivities, \( \frac{\phi_{ht}(z)}{K_{ht}} \), stay at the same level of 2001. Then the measured aggregate TFP change due to the allocative efficiency is \( \hat{z}_{it} = \frac{\bar{Z}_{it}}{\bar{Z}_{it}} \). In some counterfactual experiments, to understand how fundamental technological progress affects the dynamics of FDI, we will change \( \bar{Z}_{it} \).

Table 4 summarizes the model parameters and how we pin them down. As indicated in the Table, parameters in Panel A could be pin down externally without solving the model. Parameters in Panel B are determined jointly. Importantly, in addition to these parameters, the dynamics of the model also depends on the joint distribution of asset and productivity at the beginning of the period. Ideally, we would like to measure the joint distribution directly. Without access to a comprehensive firm-level dataset that covers all countries in our sample, we assume that, for each country, its density distribution of net worth across productivity is the same as its steady-state distribution corresponding to the parameters in 2001.

---

23 By construction, the contribution of allocative efficiency to the aggregate productivity is zero in 2001.

24 Note that we only assume the density is in steady state and then scale the volume of domestic net worth to match the capital use in the data in 2001, so our calibration does not impose that countries are already in their steady states.
4.1.2 Numerical Procedures

The calibration algorithm works as follows. For a given set of Panel-B parameters, we first solve for the steady-state joint density distribution between asset and productivity, assuming all parameters stay constant at their 2001 values. For that given set of parameters and taking the steady-state joint distribution as the initial distribution, we then solve for the transitional path of the model. We compare the moments of the model, such as the credit over GDP ratios, the dynamics of domestic and foreign investment, etc., along this transition, and adjust parameters until all these moments match their empirical counterparts during the period of 2001-2012.

In the above process, we need to compute the joint distribution of firms and the transition of the distribution numerous times. One specific challenge to our context is that, when making the discrete decisions of whether to stay active and which countries to enter, firms with the same productivity will have the same probabilistic decision rule. If a set of firms are indifferent between two decisions, aggregate quantities might be sensitive to the tie-breaking rule imposed. To avoid this possibility, rather than discretizing the firm productivity distribution, we specify it as continuous, so that firms with indifference are contained in a zero-measure set. In the Online Appendix, we describe an efficient numerical algorithm to compute the transition of $\phi_{i,t}(z)$ when the productivity follows an AR(1) process.

4.1.3 The Path of Financial Market Conditions

Figure 3 plots the calibrated sequences of $\lambda_{it}$ (left panel) and $\mu_{ht}$ (right panel). The colored curves highlight selected countries and the dotted line denotes the evolution of the mean value across all countries. The left panel shows two patterns. First, there is great heterogeneity across countries. The U.S. has the highest average $\lambda_{it}$, with a mean value of 1.2. Turkey, on the other hand, has an average value of around 0.1. This long-run difference across countries reflects different financial
market development across countries. Second, the curves show a common pattern across most sample countries. They are on an upward trend in the first half of the sample period, corresponding to a period of each credit in many countries. Then the trend is met by a sharp downturn around 2008, mirroring credit crunch since the financial crisis we document. This drop is more pronounced for some countries—for the U.S., for example, $\lambda_{it}$ declined from its peak value of 1.8 to 1.2 within just two years. Given the reduced-form evidence that links available of funding to FDI, the credit crunch likely impacts FDI severally. We will explore this quantitatively in counterfactual experiments.\footnote{Existing works have shown that a tightened credit constraint, calibrated to match the decline in aggregate credit use, can explain the decrease in domestic investment (Buera et al., 2015), but have not studied its implication for cross-border investment.}

The right panel of Figure 3 plots the evolution of $\mu_{ht}$ for each host country, which captures the dependence of foreign affiliates on local partners in financing their investment. Again, host countries are very heterogeneous along this dimension. The over-time pattern of this variable is less clear-cut than before, but in many countries, we can still see a decrease in this parameter in 2008.

### 4.2 Model Validation

We assess the validation of the model by comparing our calibration to some external measures.

#### 4.2.1 Measurements of financial market development

The long-run average of $\lambda_{it}$ and $\mu_{ht}$ reflect the quality of financial institutions in a country. To the extent that in countries with better financial institutions, parent firms can more easily borrow and affiliates of foreign firms can also rely more heavily on local partners, we should expect these two measures to be correlated. Figure 4 shows this is indeed the case.

In Table 5 we test if the recovered parameters correlate systematically with external measures of financial development. Each observation is a country. In both columns, the independent variable is the log financial institution index, same as in the main empirical exercise. The dependent variables are average of $\lambda$ (first column) and $\mu$ (second column) over the sample period. The regressions show that both measures are strongly and statistically significantly correlated with the external measure of financial development.

#### 4.2.2 Relationship Between MP and FDI

Through Proposition 1, the model links the two related concepts of MNC activities—production of their overseas affiliates (MP) and the finance of affiliate production (FDI). Our calibration uses only information on FDI, but not on MP. We validate the model by testing if the model-implied relationship between MP and FDI, given by Equation 8, holds up in the data. Because both MP and FDI are well approximated by a gravity equation, directly comparing them will misleadingly
Figure 4: The Dynamics of FDI
Note: The figure plots, for each country, the average value of $\mu_{it}$ (vertical axis) against that of $\lambda_{it}$.

Table 5: Correlation Between Calibration and External Measure

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (Financial Institution Index)</td>
<td>0.416***</td>
<td>0.197*</td>
</tr>
<tr>
<td></td>
<td>(0.150)</td>
<td>(0.117)</td>
</tr>
<tr>
<td>Observations</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.185</td>
<td>0.078</td>
</tr>
</tbody>
</table>

Note: Notes: The dependent variables are the average $\mu_{it}$ and $\lambda_{it}$ over the sample period. The independent variable is the financial institution index of a country.

Robust standard errors in parenthesis in parenthesis.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. 
show close fit. We consider the following transformed equation instead:

$$\log\left( \frac{MP \ Share_{ih}}{FDI \ Share_{ih}} \right) = \log(\text{lev}_{ih}) - \log(\bar{z}_{ih}) - \log(\bar{z}_{ih}).$$  \hspace{1cm} (12)$$

The left-hand side of Equation 12 is the log ratio between MP and FDI. This ratio increases with the average dependence on host funding of foreign affiliates and decreases with productivity of indigenous firms in country $h$ relative to that of foreign affiliates. Since our measure of the host share of affiliate finance does not vary by home country, we aggregate MP and FDI shares and run the regression at host country level. In the absence of firm-level affiliate data, we use FDI-weighted source country fundamental TFP as a proxy for $\bar{z}_{ih}$.  

Table 6 reports the results of the test, using both the model-implied ratio between MP and FDI (first column) and its empirical counterpart (second column). The first column shows that in the model, the above relationship holds strongly. In addition to having consistent signs, the two measures also capture most of the variation in the model, as indicated by the high R-squared. The second column shows that in the data, the affiliate leverage ratio has a positive and statistically significant effect on the dependent variable, with the point estimate very close to that predicted by the model. Consistent with the theoretical prediction, the log relative TFP is estimated to have a negative effect on the outcome variable, as predicted by the model, although the coefficient is not statistically significant.

### 4.2.3 Cross-border investment return wedges and their relationship to observables

Our calibration takes the model structure as given and attributes anything not already in the model to the cross-border investment wedges, which in principle includes both fundamental frictions impeding FDI and mis-specifications of our model. We test if the calibrated wedges are correlated with measurable frictions and policies.

Table 7 report the results. The dependent variable is the bilateral wedge. Because for some policies, our measurement is at the beginning of the sample period, with no variation over time, we take the wedges from the first period only. In Column 1, we examine the extent to which well-known geographic frictions, such as bilateral distance, common language, colonial tie, affects the
Table 7: FDI Return Wedges and Measurable Outcomes

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log distance</td>
<td>-0.229***</td>
<td>-0.186***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>common border</td>
<td>0.010</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
<td>(0.050)</td>
</tr>
<tr>
<td>colonial tie</td>
<td>0.249***</td>
<td>0.246***</td>
</tr>
<tr>
<td></td>
<td>(0.047)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>common language</td>
<td>0.127***</td>
<td>0.180***</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.061)</td>
</tr>
<tr>
<td>Low Tax Country</td>
<td>0.329***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td></td>
</tr>
<tr>
<td>profit tax</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>ln_fdi_restriction_oecd_d</td>
<td>-0.391***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.136)</td>
<td></td>
</tr>
<tr>
<td>log DFD index</td>
<td>0.042</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>(0.108)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>log GDP</td>
<td></td>
<td>0.651***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.163)</td>
</tr>
<tr>
<td>(mean) ctfp</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>1048</td>
<td>1007</td>
</tr>
<tr>
<td>R²</td>
<td>0.770</td>
<td>0.672</td>
</tr>
<tr>
<td>Host country fe</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Home country fe</td>
<td>Yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Notes: robust standard errors in parenthesis
* p < 0.10, ** p < 0.05, *** p < 0.01

bilateral return wedges, controlling for home and destination fixed effects. The results suggest that increasing the distance between home and parent countries decrease the net return; sharing a colonial tie or a common language increases the net return, while sharing a common border does not have a statistically significant impact.

The second column replaces the host country fixed effects by various host characteristics, including measures of tax rates and policy restrictions on inward FDI. We find that, while being labeled as a low-tax country has a significant positive effect, conditional on that, the effect from a lower profit tax rate is not significant. This is in-line with that many low-tax countries not necessarily have low statutory tax rates, but instead rely on special subsidies to attract foreign business. Policy restrictions of the host country on inward FDI also have an economically and statistically significant effect on the return wedge, consistent with our interpretation of the wedges as residual variation in frictions and policies not in the model. Finally, financial institution index is no longer important to explain the residual wedges, suggesting that the effect of host financial development on inward FDI is entirely captured by the model mechanisms.

Overall, the above exercises show that our model generates consistent predictions with the data in dimensions not directly targeted. Below we use the model to perform a structural accounting exercise.
4.3 Financial Markets and the Patterns of FDI

We examine the role of various ‘fundamental’ shocks in accounting for the dynamics of FDI. Empirical studies have found that access to credit has a significant impact on outward FDI, usually focusing on individual countries using a diff-in-diff design (Klein et al., 2002). Our first set of counterfactual experiment is to assess whether the calibrated financial shocks can generate significant impacts on country-specific patterns of FDI. Given the trend break in 2007 (see Figure 2), we split the counterfactual experiments into two periods: before and after 2007.

4.3.1 FDI Growth During 2001-2007

We examine first the extent to which the easing access to credit in the lead-up to the financial crisis can account for the increase in FDI during this period. The focus of our investigation is the outward FDI from each country, shown in Figure 5. Each bar in the figure is for a country; the height of the bar corresponds to the cumulative net outward FDI flows from 2002 to 2007—or equivalently, the level increase in outward FDI stock from 2001 to 2007. Consistent with the increase in aggregate FDI shown in Figure 2, most countries see increasing outward FDI stock during the sample period. The level increase tends to be higher for larger and more developed countries—for example, the U.S., the U.K., and France are the top three source countries of FDI.

We structurally decompose the cumulative outflow from each country into four components. To isolate the impact of different shocks, for experiments on country $i$, we change only the targeted parameter of country $i$, keeping all other parameters for country $i$ and the rest of countries at the calibrated values.

In the first set of experiments, we set $\lambda_{it}$ to the 2001 value and solve for the counterfactual transitional path, one country at a time. With the calibrated $\lambda_{it}$ on the upward trend for most sample countries during the period, this experiment should reduce outward FDI through the static and dynamic channels discussed in Section 3.7. The pink bars in Figure 5 demonstrate the strength of this force for individual countries. More precisely, the height of the pink bar indicates by how much the outward FDI would have been lower, had $\lambda_{it}$ stayed at the value of 2001 for country
i. A positive value indicates the change in $\lambda_{ht}$ between 2001 and 2007 contributes positively to outward FDI growth; a negative value indicates the opposite. For most countries, the contribution from home country financial market change is positive, yet the importance of this channel differs. In the U.S., U.K., Israel, and Switzerland, for example, this force alone accounts for half or more of the FDI increase; in Belgium and Singapore, on the other hand, it hardly matters. Because the importance of this force in the top FDI sending countries, the sum of the pink bars across all countries account for around half of world FDI increase during the period.

To gauge the scope of the host-side financial market factors to influence FDI, in the second set of experiments, we set the $\mu_{ht}$ for affiliates from home country $i$ to the 2001 value for all $h \neq i$, while keeping them at the benchmark sequence for affiliate in country $h$ from $i' \neq i$. This exercise captures the impact on FDI through the ‘pull’ force of increasing credit availability in a host country. The green bars in Figure 5 show that, by making it easier for parent companies to rely on funding from host country partners, financial market conditions elsewhere could have a quantitatively significant impact on the investment decisions of MNCs. In fact, in some countries, such as Belgium, foreign financial shocks play a more important role than domestic financial shocks. Together, the sum across all countries of the influence of home and host country financial market conditions on outward FDI account for the majority of increase in world FDI, highlighting the importance of modeling the capital channel in understanding decisions of multinational firms.

In the last set of experiment, we explore the influence of domestic productivity growth on outward FDI. The blue bars in the figure indicates the importance of this channel, which differ significantly across countries. For the U.S., domestic productivity growth plays an important role; this is not the case in many European countries, such as Spain and France. This heterogeneity primarily reflects the difference in TFP growth rate across countries.

Finally, the white bars in Figure 5 are the remaining net FDI outflow during this episode after deducting the above three channels. This term encompassing changes in the investment and FDI return wedges—which are not formally modeled and could be driven by policy, technology, or mis-specifications in the model—as well as the interaction among countries.

### 4.3.2 Slowdown in FDI: 2008-2012

Having shown that the credit boom in the leading-up of the crisis can explain an important share of outward FDI growth of individual countries, we now investigate the role of the credit crunch during and immediately after the financial crisis on the slowdown in FDI.

As before, we perform three sets of experiments. Instead of keeping $\lambda_{ht}$, $\mu_{ht}$, and $z_{ht}$ constant at their 2001 values, however, we feed in the calibrated values until 2007 and freeze them afterwards. Again, the experiments are performed for one country, one parameter at a time. Our exercises here are slightly different from before—instead of asking how much the FDI growth can be explained by each factors in the previous section, the question we ask here is, for example, how much more

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26 Statically, more productive firms earn higher net returns on investing overseas; higher domestic wage as a result of the productive increase also pushes firms to move production abroad; dynamically, firms grow faster and accumulate more capital.
outward FDI we would see, had $\lambda_{it}$ stayed at the 2007 peak value.\footnote{An alternative to this exercise, which is perhaps more natural given the exercise for 2001-2007, is to construct a counterfactual path on which FDI will keep the same speed of growth as before, and then decompose the difference between this counterfactual and the benchmark. The difficulty in implementing this decomposition is that the fundamental shocks that lead to a constant FDI growth rate are not unique.}

Figure 6 present the results. The white bars show the factual cumulative FDI outflow during 2008-2012. The red bars show the additional outward FDI from country $i$, if $\lambda_{it}$ stays at the value of 2007 for subsequent periods. Eliminating the credit crunch can double outward FDI from the U.S. and Canada, and increase it by more than 30% in a number of other developed countries. In countries whose financial market was less interrupted by the crisis, such as China, this counterfactual barely makes any difference.

The green bars shows that, deteriorations in the financial market in host countries reduce the incentive for MNCs to invest abroad. As the biggest sending country of FDI, U.S. is the most affected, but this channel is also important for Netherlands, Switzerland, the U.K.—these countries send a large amount of investment to other EU countries, which were significantly affected by the European debt crisis. Finally, the blue bars show the role of domestic productivity, which is minimum in most countries.

Taking stock, the counterfactual exercises show that the changes in financial market conditions can have significant impacts on the dynamics of outward FDI from individual countries. Country heterogeneity notwithstanding, financial factors can explain more than half of the cumulative FDI outflow during 2002-2007; had the access to credit remained at the peak level of 2007, the cumulative FDI outflow during 2008-2012 could almost double.

This result, however, is only partial-equilibrium because it is performed one country at a time, holding fix the evolution of other parameters of all other countries at the calibrated sequences. In the next subsection, we investigate the importance of the changes in world-wide financial market conditions on aggregate FDI.
4.4 Aggregate FDI Dynamics and the Importance of the General Equilibrium

We analyze the two periods separately. Figure 7a plots the results for the cumulative FDI between 2002-2007. The solid blue line is the data. The solid red line is the counterfactual world FDI, if for all countries, $\mu_{ht}$ and $\lambda_{ht}$ are kept at their respective 2001 values. In line with findings from country-specific experiments, if the world economy had not experienced the credit boom, the aggregate FDI would have grown by less. The difference between the blue and red lines accounts for about a third of the cumulative FDI flows during this period.

The yellow dash line and gray dash-dotted line focus on the two sub-components of the credit boom that matters for the parents and affiliates, respectively. These two components contribute about equally to the growth in overall FDI—without either component, the cumulative FDI flows during 2002-2007 would have been one-sixth lower. In contrast, the growth in fundamental productivity had a negligible effect, as shown by the dotted green line.
Figure 7b plots similar exercises focusing on 2008-2012, an episode of financial market disruptions and global FDI slow down. In counterfactual experiments, we feed in the calibrated sequences of parameters until 2007 and keep the relevant set of parameters constant for subsequent years. The blue solid line is the data. The red solid line shows that, had $\lambda_{ht}$ and $\mu_{ht}$ all remained at their peak values in 2007, the cumulative world FDI flow would have been around 40% higher. The yellow dash line and gray dash-dotted line decompose this effect into for parents and affiliates, respectively. We find that both forces are quantitatively relevant, but the deteriorating access to credit for parents mattered more towards the end of the period. As before, shutting down the productivity dynamics has a very minor impact on the aggregate FDI.

Together, the above exercises show that the changing financial market conditions play an important role in explaining aggregate FDI. However, compared to the sum of country-specific results, the impacts on aggregate dynamics are substantially smaller. To understand the source of
the differences, Figure 8a plots a few outcome variables corresponding to an experiment in Germany that fixes its \( \lambda_{it} \) at 2001 throughout the sample period. The blue solid line indicates that, without the improvement in the financial market, the cumulative FDI outflows from Germany would decrease. Because the credit boom in Germany is only moderate, by the end of 2007, the decrease accounts for around 8% of the cumulative FDI outflow during the period.

This change, however, still overstates the influence on the aggregate FDI for two reasons. The first is a general equilibrium effect within the German economy: efficient firms are more constrained in the absence of the credit boom, which reduces aggregate productivity and wage in Germany. This attracts more inward FDI, alleviating the decrease in the World FDI. The second force is a third-country effect. As German firms decrease investment overseas, the labor market in the destination countries become less competitive, drawing more foreign investment from third countries. To gauge the strength of these two forces, the red dash line in Figure 8a plots the net change in the sum of inward and outward FDI in Germany, whereas the yellow dotted line indicated the net change in FDI across all countries. The difference between the blue and red lines is the strength of the first force; the difference between the red and yellow lines is due to the second channel. In the case of Germany \( \lambda_{it} \) shock, both forces play a similar role; together, they reduce the decrease in world FDI by half.

The above analysis focuses on \( \lambda_{it} \) changes in Germany only. Figure 8b shows that the domestic general equilibrium and third country interactions are important in other countries as well. The blue solid line in the figure is the sum across the country-specific \( \lambda_{it} \) experiments, each of which focuses on one country by setting its \( \lambda_{it} \) at the 2001 value, while keeping all other parameters of the country and all parameters for all other countries as before. The absolute value of the sum at 2007 is thus equivalent to the sum of all the pink bars in Figure 5. The red dash line is the sum across all these experiments of the net change in inward and outward FDI for the country of focus. In 2007, this value is only 60% of the blue curve, indicating a significant domestic general equilibrium effect. The dotted yellow curve is the sum of the net change in world FDI across all experiments. The difference between this curve and the dash red curve suggests that the third country effect is also quantitatively significant. Finally, the gray line corresponds to the change in world FDI, when \( \lambda_{it} \) in all countries are set to their 2001 values at the same time. The gray line shows a bigger decrease than the dotted yellow line—because the changes in financial markets were largely a global trend, when we fix \( \lambda_{it} \) for all countries, third-country effects from different countries offset each other. This interaction reduces the third country effect by around half.

In summary, consistent with our analysis on country-specific FDI, we find that the booming credit market can explain up to a third of the world FDI increase before 2007; in the absence of the credit crunch in the crisis, the cumulative FDI flow between 2008 and 2012 would be 40% higher. While still significant, these numbers are much smaller than the sum of impact on individual countries. These results demonstrate the value of a quantitative general equilibrium framework in evaluating the effects of various shocks on FDI.
5 Welfare Analysis

We now discuss the implications of the model for the welfare gains from MNC activities. Following a growing literature on the gains from trade and MP (see, e.g., Arkolakis et al., 2012), we focus on the ex-post effect. To demonstrate the channels at play, we proceed in two steps.

5.1 The Static Effect

In the first step, we consider the static effects of international integration and compare them to existing studies of multinational firms. Characterizing analytically the broad impact of a general policy change is difficult; instead, the following proposition focuses on the wage impact of a specific type of policy change, which liberalizes inward FDI while keeping outward FDI restricted.

**Proposition 2.** Assume that the net worth distribution in period t for host country h, denoted $\phi_h(z)$, is Pareto with tail index $\gamma > 1$ and that outward FDI from country h is restricted, then the contemporaneous change in workers’ wage in country h in response to a change in inward FDI policy is:

$$\Delta \log(w_{h,t}) = -\alpha \Delta \log\left(\frac{Y_{hh,t}}{Y_{h,t}}\right) + \alpha \gamma - \frac{1}{\gamma} \Delta \log\left(\frac{K_{hh,t}}{W_{h,t}}\right),$$

where $Y_{hh,t} / Y_{h,t}$ is the share of production conducted by domestic firms and $K_{hh,t} / W_{h,t}$ is the share of domestic wealth used by domestic firms, which can be expressed as:

$$\Delta \log\left(\frac{K_{hh,t}}{W_{h,t}}\right) = -\gamma \Delta \log(r_{h}^\delta + \delta) - \frac{\gamma(1 - \alpha)}{\alpha} \Delta \log(w_{h,t}).$$

The thought experiment considered in Proposition 2 is between two equilibria with different degrees of inward FDI, which could be the result of, for example, an increase of $\eta_{ih}$ for country h. The restriction on outward FDI is strong and violated in most countries, but it allows us to focus on inward FDI as a benchmark. Besides this restriction, the thought experiment needs not take a stand on whether the credit market in country h is integrated with the rest of the world, or whether this has changed between the equilibria.

Equation 13 relates the wage impact to two sufficient statistics. The first, $Y_{hh,t} / Y_{h,t}$, depends only on the MP shares and captures the importance of foreign firms. A policy change that makes country h more attractive to foreign firms will reduce $Y_{hh,t} / Y_{h,t}$, which would lead to an increase in wage. This effect can happen for two reasons, both subsumed in the MP share. First, foreign affiliates might be much more efficient than the average domestic producer, so their entry might increase aggregate productivity. Second, given the credit constraint faced by domestic producers, the entry of foreign firms increases the capital used in domestic production.

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28The former holds exactly if productivity process is i.i.d. and follows Pareto distribution, as in, e.g. Itskhoki and Moll (2014); the latter holds if a policy restriction makes the return from operating overseas too low, e.g., if $\zeta_{hh'} = 0, \forall h', \neq h$.

29The Pareto distributional assumption is a technical one. With the exit of foreign firms, workers previously working with them switch to domestic firms. The output of these workers after the switch depends crucially on the productivity and access to capital of domestic firms, which in turn is entirely captured by $\phi_h(z)$. Under the Pareto assumption, the above Proposition shows that the wage impacts can be characterized by two sufficient statistics.
Comparison to Ricardian models of multinational production. In Ricardian models of multinational production, knowing the MP shares is sufficient to conduct ex-post welfare evaluations. However, in our model, conditioning on the MP shares, the composition of the ‘technology’ and ‘capital’ content of foreign firms also affect the wage gains. This composition effect is captured by the second term in Equation 13, in which $K_{hh,t}$ is the capital used in production by domestic firms and $W_{h,t}$ is their net worth. Their ratio is closely connected to inward FDI. Intuitively, if affiliates rely more heavily on internal funds from their parents (in the form of FDI), then they will use fund from partners in country $h$ less intensively, resulting in a higher $K_{hh,t}/W_{h,t}$.

The term $\Delta K_{hh,t}/W_{h,t}$ enters Equation 13 because it captures the crowding out effect of foreign affiliates in the domestic credit market. An increase in the ratio means that a higher fraction of entrepreneur net worth is used by indigenous producers and a lower fraction by foreign affiliates or flow out in the form of bond, so the crowding out effect is weaker. Given the MP shares, the wage gains are higher, if the MP share is driven by either high productivity of foreign affiliates or parent investment in these affiliates; the wage gains are lower if the affiliates use a lot of credit from the host country.

This effect can be best understood by inspecting Equation 14, which shows that $\Delta K_{hh,t}/W_{h,t}$ depends on the equilibrium change in wage and interest rate. If the policy change decreases the interest rate, then the wage gains for workers will be higher because domestic firms will have a higher demand for workers; this effect, however, has a negative equilibrium mediating effect—as wage increases, fewer domestic firms will be active, resulting in lower labor demand hence lower wage. This second effect is captured by the wage term in Equation 14. Combining Equations 13 and 14, we can write equilibrium wage change as a function of the MP shares and the change in interest rate only:

$$
\Delta \log(w_{h,t}) = -\frac{\alpha}{1 - (1 - \gamma)(1 - \alpha)} \Delta \log\left(\frac{Y_{hh,t}}{Y_{h,t}}\right) - \frac{\alpha(\gamma - 1)}{1 - (1 - \gamma)(1 - \alpha)} \Delta \log(r^b_h + \delta) \quad (15)
$$

In the special case that country $h$ is a small open economy, the equilibrium interest rate does not change, so the second term above vanishes. In this case, the MP shares are sufficient for inferring the wage impacts, as in existing studies. The elasticity for the MP shares depends on both the capital share, and the Pareto parameter governing the joint distribution of productivity and net worth, which is different from in, for example, Ramondo and Rodríguez-Clare (2013).  

Comparison to neoclassical models of FDI. In the above special case, the MP share still composes of the ‘technology’ and ‘capital’ channel, the latter directly corresponding to the inferred wage gains from FDI, if one were to follow a different tradition of FDI studies that treats it as just another form of international capital flow (Mundell, 1957; Feldstein, 1995). To see this, note that

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30 Because the strength of this force depends crucially on domestic firms’ willingness (their productivity) and capacity (their net worth) to expand, the assumption on the joint distribution of productivity and net worth matters. Under the Pareto distribution assumption, this channel could be entirely captured by $\Delta K_{hh,t}/W_{h,t}$, with the corresponding elasticity dependent on the tail parameter of the Pareto distribution $\gamma$.

31 This is related to recent findings in international trade that different models might yield the same ex-post gains, but imply different theory-consistent ways of estimating trade elasticity. See, e.g., Arkolakis et al. (2012) and Melitz and Redding (2015).
the equilibrium wage in a host country $h$ is:

$$w_{h,t} = (1 - \alpha) \left( \frac{A_{h,t}K_{h,t}}{L_{h,t}} \right)^{\alpha},$$

where

$$A_{h,t} = \sum_i \int_0^\infty \frac{\hat{k}_{ih,t}(z)\hat{e}_{ih,t}(z)\phi_i(z)}{K_{h,t}} z_{ih}(z) dz$$

is the average productivity of foreign affiliates and home-grown firms in country $h$. Thus, fixing $L_{h,t}$, the change in wage between any two equilibria can be decomposed into that from $K_{h,t}$ or $A_{h,t}$, i.e.,

$$\Delta \log(w_{h,t}) = \alpha \Delta \log(A_{h,t}) + \alpha \Delta \log(K_{h,t}).$$

(16)

The first component above captures the improvement in productivity distribution because of technology embedded in affiliates; the second component corresponds to the wage gains in response to the increase capital stock. This channel exits despite the world credit market being fully integrated because domestic firms might lack the collateral to borrow. If measured through the lens of a neoclassical Model that treats FDI inflow as an increase in capital stock, the static wage increase as a result of FDI is exactly $\alpha \Delta \log(K_{h,t})$, so it is clear that this will underestimate the true wage effect.

We examine the significance of the underestimation and its heterogeneity across countries. As a baseline, we perform a perhaps extreme experiment in which countries ban foreign firms from entry. Specifically, for each country $h$, we shut down the inward FDI by setting the bilateral investment return wedge $\eta_{ih,t} = 0, \forall i \neq h$. We then measure the static gains from inward FDI by the change in equilibrium wage from the counter-factual equilibrium to the benchmark and apply the decomposition described in Equation (16). We run this experiment for each country independently.

Table 8 presents the results for several representative countries for year 2001. The first column of the table reports the wage gains from inward FDI. Given that we assume the world credit market is fully integrated, the inferred wage gains in our model is tightly connected to the MP share. Workers in countries with significant inward FDI, such as Canada and Indonesia, benefits more. The second column reports the contribution of the capital channel to the wage gains. On average, the technology channel account for more than half of the welfare gains, so treating FDI as simply another form of capital flow would significantly underestimate the inferred static gains.

The extent of the bias differs significantly from countries, ranging from 25% for Sweden to 80% for India. This heterogeneity arises because host countries receive investment from different origins, some more productive than others. In general, countries already with productive firms benefit less from the technological channel, while the opposite is true for developing countries.

To systematically account for what countries have a higher static gain, and from which channels, Table 9 regresses the total wage gains and the channels on two country characteristics: their
capital scarcity, as measured by the ratio between capital stock (excluding inward FDI stock) and GDP, and their aggregate TFP. The first column shows that countries relative scarce in capital tend to enjoy a greater benefit. The second and third column relate the gains from the capital and technology channels to the same set of country characteristics. The coefficient suggests that countries scarce in capital (holding constant total productivity) tend to benefit more from the capital channel, whereas countries with a lower productivity tend to benefit more from the technological channel. This systematic patterns suggests that while focusing only on the capital channel will lead to underestimation of the wage gains for all, such bias would be larger for less developed countries.

Table 8: Static Welfare Gains from Inward FDI for Selected Countries

<table>
<thead>
<tr>
<th>ISO</th>
<th>Δ log(wage)</th>
<th>Fraction from change in A (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARG</td>
<td>0.08</td>
<td>51.2</td>
</tr>
<tr>
<td>CHN</td>
<td>0.08</td>
<td>86.0</td>
</tr>
<tr>
<td>CAN</td>
<td>0.16</td>
<td>57.0</td>
</tr>
<tr>
<td>GBR</td>
<td>0.14</td>
<td>43.0</td>
</tr>
<tr>
<td>IDN</td>
<td>0.17</td>
<td>81.1</td>
</tr>
<tr>
<td>JPN</td>
<td>0.01</td>
<td>54.5</td>
</tr>
<tr>
<td>MEX</td>
<td>0.11</td>
<td>63.9</td>
</tr>
<tr>
<td>SWE</td>
<td>0.06</td>
<td>23.0</td>
</tr>
<tr>
<td>RUS</td>
<td>0.01</td>
<td>76.2</td>
</tr>
<tr>
<td>USA</td>
<td>0.04</td>
<td>25.0</td>
</tr>
</tbody>
</table>

Note: Δ log(wage) is calculated for year 2001, from the counter-factual equilibrium shutting down a single country’s inward FDI to the benchmark equilibrium. The last column is the fraction of welfare gains from embedded technology following the decomposition described by Equation (16). By definition, one minus the last column corresponds to the fraction of welfare gains from increased capital.

Table 9: Static Welfare Gains and Country Characteristics

<table>
<thead>
<tr>
<th>log(K/Y), no FDI</th>
<th>Δ log(wage)</th>
<th>Change in K</th>
<th>Change in A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.241***</td>
<td>-0.321**</td>
<td>0.0799</td>
</tr>
<tr>
<td></td>
<td>(0.0808)</td>
<td>(0.128)</td>
<td>(0.0523)</td>
</tr>
<tr>
<td>log(Host TFP)</td>
<td>0.00287</td>
<td>0.0420*</td>
<td>-0.0391***</td>
</tr>
<tr>
<td></td>
<td>(0.0247)</td>
<td>(0.0244)</td>
<td>(0.0143)</td>
</tr>
<tr>
<td>Observations</td>
<td>36</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.445</td>
<td>0.548</td>
<td>0.411</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

Note:

5.2 Dynamic Effects

The previous discussion centers on the contemporaneous impact on wage of inward FDI, holding domestic wealth and its distribution across firms constant. Because the decisions of domestic
firms will react to the exit of foreign firms, the evolution of wealth and productivity distribution will also be changed, leading to a dynamic impact. In the second step of the welfare analysis, we show that once this is factored in, the full dynamic wage impact will differ substantially from the static effect.

To illustrate this point, we consider two sets of counterfactual experiments for each country. In the first experiment, we calculate the full dynamic gains from inward FDI, by solving a counterfactual equilibrium with $\eta_{ih,t} = 0, \forall i \neq h, t \geq 2001$ and comparing the path of wage in this counterfactual equilibrium to that in the benchmark economy. In the second set of experiments, we calculate the static wage gains from inward FDI, by shutting down inward FDI into the country at each point in time, assuming up to that point, the world economy has evolved as in the benchmark economy. For example, the static wage gains for 2006 could be calculated by letting the economy evolve as before in 2001-2005, and then applying a sudden change by setting $\eta_{ih,2006} = 0, \forall i \neq h$.

Results from these two sets of experiments for China are shown in Figure 9. Panel (a) plots the wage gains. The black line shows the period-by-period static gains for the ‘in sample’ period (2001-2012). After an increase from 8% to 10% in the first year, the static wage gain has stayed mostly stable, suggesting that the intensity of inward FDI to China has not increased much. The red line shows the dynamic gains calculated by solving the transitional path without inward FDI. Given the nature of this experiment, we can also solve for the gains ‘out-of-sample’ by comparing the evolution of the counterfactual and factual economies after 2012 (dotted line shows the difference between these two scenarios). For this out of sample analysis, our assumption is that the TFP growth of each country converges linearly from their average growth rate between 2001 and 2012 to a long-run growth rate of 2% and all other fundamentals remain the same as in 2012.

By construction, the dynamic wage gains are the same as the static gains in the first period; subsequently, however, the two curves diverge, with the dynamic gains declining over time and even turning negative after 2014—the entry of foreign firms might lead to wage loss in the long run.
Two channels explain this result. The first is a domestic capital accumulation channel—by pushing up the domestic wage, foreign firms crowd out the market share of domestic firms. Over time, this leads to slower capital accumulation of domestic firms, reducing the demand for labor. Panel (b) of Figure 9 plots period-by-period the change in aggregate capital used by domestic active firms after the FDI entry. Domestic capital is crowded out on impact because the entry of foreign affiliates raises wage in the host country, lowers the return from production, and drives out active but unproductive firms. The lower return from production also depresses capital accumulation of active domestic firms, leading to lower net worth in the long run. The significance of such long-run effect can be shown by comparing the paths of domestic capital between the ‘Dynamic’ and ‘Static’ experiments. For the case of China, domestic capital could be 20% lower by year 2012 than if FDI were turned off from year 2002 onward. Domestic capital would be only 4% lower than if FDI were turned off in year 2012 only.

Second, conditioning on the level of domestic capital, the entry of foreign firms also affects the distribution of the capital uses across firms, and thus the aggregate productivity. Specifically, on impact, the entry of foreign firms will drive out some unproductive domestic firms, increasing the productivity of the economy. This ‘selection’ effect, however, dissipates over time—with the wage higher, the return differentials between productive and unproductive firms narrows. Among the active firms, the fraction of net worth in the hands of the most productive ones shrinks, so the allocative efficiency deteriorates and the aggregate productivity decreases.

To demonstrate the importance of this channel, Panel (b) of Figure 9 shows the static impact of FDI on the aggregate TFP (black), which includes only the ‘selection’ channel, and the full dynamic effect (red). The static channel gradually increases over the sample period, so the ‘selection’ effect is positive and increasing; despite this, because of the dynamic channel, the full effect actually declines. By 2012, the difference between the two channels account for around 2% of the aggregate TFP.

A perhaps surprising finding from our experiment is that, the long-run wage impact can be negative. In particular, standard economic intuition might suggest that, the lower long-run wage in the FDI equilibrium should have been arbitraged out with inflow of profit-seeking foreign capital. Two elements of our model, both of which are needed to generate FDI as within-firm capital flow, are responsible for the reversal. First, the existence of financial frictions. Because of this, even though the international credit market is fully integrated, firms’ net worth in a period limits their scale of operation. Second, the imperfect substitution between different host countries for a given firm. As can be seen from Lemma 3, the elasticity with respect to the return from country $h$ of the probability that a firm opens up an affiliate in country $h$ is $\theta$. That $\theta$ is finite captures the difference between FDI and general capital flow—because FDI is tied to technology, the substitution between different hosts are imperfect.

A natural question, then, is, under what situations will a long-run wage loss occur. In our model, firms from different countries operate under different credit market conditions and have

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32Recall for each ‘Static’ experiment, inward FDI is shutdown for the current period only. Firms’ productivity distribution at the current period thus agrees between the benchmark and no-inward-FDI equilibrium, so the static experiment captures only the selection effect.
different productivity growth rates. From the perspective of workers in a country, the growth rate of wage is tied to the growth rate of labor demand, which in turn depends on the size and productivity growth rate of the employers. Given that for most countries, domestic firms account for the bulk of employment, the wage losses would arise, only when the crowd-out effect from FDI leads to a bigger decline in domestic labor demand than could be made up for by foreign affiliates. From the above discussion, this is more likely to occur, when the productivity growth of domestic firms exceeds that of foreign firms. To put in comparative advantage terms, because the size of domestic firms and foreign affiliates in the future depends on their current market share, to maximize long-run wage, it makes sense to give market share to firms that are growing faster.\footnote{One implication of our model is that, in this situation, subsidizing domestic firms might be welfare-improving. In a closed-economy setting with financial frictions and incomplete market, Itskhoki and Moll (2014) shows in terms of policy implications, this channel is isomorphic to the learning-by-doing externality emphasized in the literature (Young, 1991).}

In our model, the domestic producer as a whole can exhibit faster productivity growth for two reasons. First, the fundamental productivity might be growing faster; second, the reallocation process might be more efficient—that is, the domestic financial market is more efficient. Figure 10 shows this is indeed the case.

The exercises in this section demonstrate that the dynamic impacts of FDI differ markedly different the static impacts. It also shows that to understand the full effects of FDI, it is necessary to micro-found FDI as within-firm capital flow.

6 Concluding Remarks

This paper integrates two distinct approaches in studies of FDI through a unified model, which enables us to perform the first quantitative analysis of FDI in a setting where FDI embodies both technology and capital. We show that financial market conditions play an important role in ex-
plaining the dynamics of FDI. We further show that in our setting the welfare implications of FDI
differ substantially from those predicted by existing models.

Our model abstracts from a number of interesting channels. From the micro side, we overlook
potential technological spillovers from foreign to domestic firms, which many empirical studies
find important; aside from financial constraints, we also abstract away from firm-level distor-
tions, which have been documented to be large in many developing countries. With increasingly
available micro-data, these elements could be easily incorporated for a more comprehensive un-
derstanding of FDI. From the macro side, the household sector is deliberately kept simple in our
model. Extending the model to incorporate household saving would allow it to simultaneously
match the dynamics of current account, which consists of mainly portfolio investment, and FDI
flows.\footnote{International portfolio investment flows, similar to trade and FDI flows, have been shown to follow a gravity
equation (Portes and Rey, 2005). It is relatively straightforward to introduce the multivariate shock into the households’
portfolio choice problem to allow for partially integrated international bond market.} Such an extension can then be used to study the interaction between these two types of
capital flows and the effects of various capital control policies.
Appendix A  Data and Facts

This appendix describes the data used in Section 2 and results from additional robustness tests.

Our primary source of data is from Ramondo et al. (2015), which compiles data on bilateral FDI stock and MNC affiliates sales for a large number of countries. The same dataset also provides information on various measures of bilateral distance, size of a country. All variables are averaged over the period of 1996 and 2001. We supplement the above dataset with additional information. To be consistent in timing, to the extent possible, we average the data over 1996 and 2001. Our additional sources of data include the following:

*The World Development Indicator*: we obtain business tax rate, financial development index and total credit use from the World Bank Database.

*The Penn World Table*: we obtain the aggregate TFP (ctfp) and capital stock of our sample countries from the Penn World Table.

*Additional sources*: we define tax-heaven indicators based on information from Wikipedia page for tax heavens. According to our reading of the materials, we define the following countries in our sample to be tax-heaven countries—BRB, PAN, LUX, CHE, SGP, IRL, BHS, and NLD.

Table 10 reports robustness analysis for Fact 2. In Table 2, we use the index developed by the World Bank to measure financial institutions of a country. A strand of empirical and quantitative research (Manova, 2013; Buera et al., 2011) uses credit over GDP ratio as the measure for financial institution. The advantage of this measure is that it is available for a large number of countries and covers a long time period. The drawback is that, since this measure is an outcome, it captures not only the financial institution, but also other aspects of the economy. As a robustness test, in the first column of Table 10 we use this alternative measure. The coefficient associated with log credit/GDP is 0.24, which is economically and statistically significant.

Second, to rule out the possibility that our finding is driven by a few small outlying country pairs, in the second column of Table 10, we reports the result from a regression weighted by log GDP of host countries. The coefficient barely changes. Finally, Column 3 of the table shows that our results are robust to using least square estimation, as opposed to PPML.

Table 11 reports the robustness tests for Fact 3. As above, the first column use credit over GDP ratio as the measure for financial development. Column 2 uses the same benchmark specification as in Table 3, but weights each observation by the size of the home country. Column 3 uses least square estimations. Our results are robust to these alternative choices.

---

### Table 10: Additional Robustness Exercise for Fact 2

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (Financial development index)</td>
<td>0.422*</td>
<td>0.322**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.220)</td>
<td>(0.138)</td>
<td></td>
</tr>
<tr>
<td>Log (credit/GDP)</td>
<td>0.244**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.112)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (FDI)</td>
<td>0.611***</td>
<td>0.578***</td>
<td>0.903***</td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
<td>(0.064)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>Observations</td>
<td>982</td>
<td>982</td>
<td>1201</td>
</tr>
<tr>
<td>R²</td>
<td>0.962</td>
<td>0.963</td>
<td>0.961</td>
</tr>
<tr>
<td>Home country FE</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Additional host characteristics</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Bilateral distance measures</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Note: See Table 2 for definition of variables. The first column is the same as Column 3 in Table 2, but use Log (credit/GDP) as the measure for financial development. The second column is PPML weighted by host GDP. The third column is estimated using OLS. Standard errors (clustered at host-country level) in parenthesis. * p < 0.10, ** p < 0.05, *** p < 0.01.

### Table 11: Additional Robustness Exercise for Fact 3

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (credit/GDP)</td>
<td>0.895***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.225)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (Financial development index)</td>
<td>1.005***</td>
<td>1.026**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.273)</td>
<td>(0.436)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>1717</td>
<td>1717</td>
<td>1792</td>
</tr>
<tr>
<td>R²</td>
<td>0.882</td>
<td>0.885</td>
<td>0.707</td>
</tr>
<tr>
<td>Host country FE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Additional home characteristics</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Bilateral distance measures</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Note: See Table 2 for definition of variables. The first column is the same as Column 3 in Table 2, but use Log (credit/GDP) as the measure for financial development. The second column is PPML weighted by home GDP. The third column is estimated using OLS. Standard errors (clustered at host-country level) in parenthesis. * p < 0.10, ** p < 0.05, *** p < 0.01.
Appendix B  Quantification

B.1 Data

The following are additional sources of data in our calibration.

**Bilateral FDI** We combine two sources, Ramondo et al. (2015) and the bilateral FDI flow from UNCTAD.

**Credit/GDP ratio** The data is based on the World Bank Database

**Parent position/ asset** Based on BEA data. Describe the imputation procedure

**GDP per-capita** Penn World Table 8.0

Appendix C  Theory

C.1 Proofs

**Proof of Lemma 2.**

Proof. Guess the value function takes the form

\[ v_{i,t}(z, \eta, a) = \hat{v}_{i,t}(z, \eta) + B \log(a). \]

where \( \hat{v}_{i,t}(z, \eta) \) and \( B \) are functions and coefficients to be determined. Plug the guess into the right hand side of the parent firm’s Bellman equation (3) we have

\[ v_{i,t}(z, \eta, a) = \max_{c, a', (e_h)_{h=1}^N} \log(c) + \beta \mathbb{E}[ (\hat{v}_{i,t+1}(z', \eta') + B \log(a')) | z] \]

\[ s.t. \quad \sum_h e_h = a + b^H \]

\[ -a \leq b^H \leq \lambda_{i,t} \cdot a \]

\[ c + a' = \sum_h R_{ih,t}(z) \eta_h e_h - (1 + r_{i,t}^b) b^H. \]

Since the objective is linear in \( b^H \) and \( e_h \), if \( 1 + r_{i,t}^b > \max_{h'} R_{ih',t}(z) \eta_{h'} \), the firm chooses to be idle and loan out all the net worth by setting \( b^H = -a \). If \( 1 + r_{i,t}^b < \max_{h'} R_{ih',t}(z) \eta_{h'} \), the firm chooses to be active, borrows to the maximum and allocate \( e_h \) to hosts that attains \( \max_{h'} R_{ih',t}(z) \eta_{h'} \).
If \( 1 + r^b_{i,t} = \max_{h'} R_{ih',t}(z) \eta_{h'} \), the firm is indifferent between being idle and active, since both generate the same returns. Therefore, we have \( \tilde{R}_{i,t}^a(z, \eta, a) = R_{i,t}^a(z, \eta) a \) with

\[
R_{i,t}^a(z, \eta) = \begin{cases} 
\max_h R_{ih,t}(z) \eta_h (1 + \lambda_{i,t}) - (1 + r^b_{i,t}) \lambda_{i,t}, & \text{if } \max_h R_{ih,t}(z) \eta_h \geq 1 + r^b_{i,t}, \\
(1 + r^b_{i,t}), & \text{if } \max_h R_{ih,t}(z) \eta_h < 1 + r^b_{i,t}.
\end{cases}
\]

Therefore the right hand side of equation (17) reduces to

\[
\max_{a'} \log(R_{i,t}^a(z, \eta) a - a') + \beta \mathbb{E}((\hat{\vartheta}_{i,t+1}(z', \eta') + B \log(a'))|z).
\]

First order condition w.r.t. \( a' \)

\[
1
R_{i,t}^a(z, \eta) a - a' = \frac{\beta B}{a'}
\]

\[
\Rightarrow a' = \beta B \frac{R_{i,t}^a(z, \eta) a}{1 + \beta B}.
\]  

Plug (18) into (17) we have

\[
v_{i,t}(z, \eta, a) = \log(\frac{R_{i,t}^a(z, \eta)}{1 + \beta B}) + \log(a) + \beta \mathbb{E}((\hat{\vartheta}_{i,t+1}(z', \eta')|z) + \beta B \log(\frac{\beta B}{1 + \beta B} R_{i,t}^a(z, \eta) a)).
\]

Comparing with guessed functions and coefficients to be determined we have verified \( B \) and \( \hat{\vartheta}_{i,t}(z) \) that satisfy the following

\[
1 + \beta B = B
\]

\[
\hat{\vartheta}_{i,t}(z, \eta) = \log(\frac{R_{i,t}^a(z, \eta)}{1 + \beta B}) + \beta \mathbb{E}((\hat{\vartheta}_{i,t+1}(z', \eta')|z) + \beta B \log(\frac{\beta B}{1 + \beta B} R_{i,t}^a(z, \eta))
\]

solve the Bellman equation. Thus we have

\[
B = \frac{1}{1 - \beta}
\]

and

\[
a' = \beta R_{i,t}^a(z, \eta) a
\]

\[
c = R_{i,t}^a(z, \eta) a - a' = (1 - \beta) R_{i,t}^a(z, \eta) a.
\]

Before proving Lemma 3, we first characterize several properties of the correlated Pareto distribution. Some of these properties are also covered in Arkolakis et al. (2017b).

**Lemma 4.** Suppose \( \zeta = (\zeta)^N_{n=1} \) follows the standarized correlated Pareto distribution with cumulative
distribution function (cdf)

\[
Pr(\xi_1 \leq \xi_1, \ldots, \xi_h \leq \xi_h) = \begin{cases} 
1 - \left( \sum_h \frac{1}{N} \left( \frac{\xi_h - \theta}{1 - \rho} \right)^{1-\rho} \right)^{1-\rho}, & \text{if } \xi_h \geq 1, \forall h, \\
0, & \text{if } \exists h \text{ s.t. } \xi_h < 1.
\end{cases}
\]

Given any constants \( \{A_h\}_{h \in H} \) s.t. \( A_h > 0 \ \forall h \), define \( \Xi = \max_{h'} A_{h'} \xi_{h'} \).

1. The cdf for \( \Xi \) is

\[
Pr(\Xi \leq B) = \begin{cases} 
1 - \bar{A} B^{-\theta}, & \text{if } B \geq \max_{h'} A_{h'} \\
0, & \text{if } B < \max_{h'} A_{h'}.
\end{cases}
\]

where \( \bar{A} = \left( \frac{1}{N} \sum_{h'} A_{h'} \right)^{\frac{1}{\theta - 1}} \). This immediately implies

\[
Pr(\Xi = \max_{h'} A_{h'}) = 1 - \bar{A}^{\theta} [\max_{h'} A_{h'}]^{-\theta}.
\]

2. The bottom truncated mean of \( \Xi \) is

\[
\mathbb{E}[\Xi | \Xi \geq B] = \begin{cases} 
\frac{\theta}{\theta - 1} B, & \text{if } B > \max_{h'} A_{h'} \\
(1 - \bar{A}^\theta [\max_{h'} A_{h'}]^{-\theta}) \max_{h'} A_{h'} + \frac{\theta}{\theta - 1} \max_{h'} A_{h'}, & \text{if } B \leq \max_{h'} A_{h'}
\end{cases}
\]

3. The conditional probability of each choice attaining maximum

\[
Pr(\arg \max_{h'} A_{h'} \xi_{h'} = h | \Xi \geq B) = \frac{A_h^{\theta / (1-\rho)}}{\sum_{h'} A_{h'}^{\theta / (1-\rho)}}, \text{ if } B > \max_{h'} A_{h'}.
\]

and if \( \overline{H} = \arg \max_{h'} A_{h'} \) is a singleton, then for \( \overline{h} \in \overline{H} \),

\[
Pr(\arg \max_{h'} A_{h'} \xi_{h'} = \overline{h} | \Xi = \max_{h'} A_{h'}) = 1.
\]

Proof. 1. Consider

\[
Pr(\Xi \leq B) = Pr(A_1 \xi_1 \leq B, \ldots, A_h \xi_h \leq B) = Pr(\xi_1 \leq \frac{B}{A_1}, \ldots, \xi_h \leq \frac{B}{A_h}) = 1 - \left( \sum_{h'} \frac{1}{N} \frac{B}{A_{h'}} \right)^{1-\rho}, \text{ for } \frac{B}{A_{h'}} \geq 1, \forall h'
\]

\[
= 1 - \left( \frac{1}{N} \sum_{h'} A_{h'}^{\theta / (1-\rho)} \right)^{1-\rho} B^{-\theta}, \text{ for } B \geq \max_{h'} A_{h'}
\]
If $B < \max_{h'} A_{h'}$, then $\exists h$ s.t. $\frac{B}{A_h} < 1$. Therefore,

$$Pr(\Xi \leq B) = Pr(\zeta_1 \leq \frac{B}{A_1}, \ldots, \zeta_h \leq \frac{B}{A_h}) = 0.$$  

Therefore,

$$Pr(\Xi = \max_{h'} A_{h'}) = Pr(\Xi \leq \max_{h'} A_{h'}) - \lim_{B \to \max_{h'} A_{h'}} Pr(\Xi \leq B) = 1 - \left(\frac{1}{N} \sum_{h'} A_{h'}^{\rho}\right)^{1-\rho} \left[\max_{h'} A_{h'}\right]^{-\theta}.$$  

2. For $B > \max_{h'} A_{h'}$, from part 1, $\forall C \geq B$

$$Pr(\Xi > C | \Xi > B) = \left(\frac{C}{B}\right)^{-\theta}.$$  

Therefore, $\Xi | \Xi > B$ follows a Pareto distribution with tail parameter $\theta$ and scale parameter $B$, and thus we have

$$\mathbb{E}[\Xi | \Xi > B] = \frac{\theta}{\theta - 1} B.$$  

Since for $B > \max_{h'} A_{h'}$, $Pr(\Xi \leq B)$ is continuous in $B$, we have $Pr(\Xi = B) = 0$ and therefore

$$\mathbb{E}[\Xi | \Xi \geq B] = \mathbb{E}[\Xi] = \frac{\theta}{\theta - 1} B.$$  

For $B \leq \max_{h'} A_{h'}$, since $Pr(\Xi \geq B) = 1$, we have

$$\mathbb{E}[\Xi | \Xi \geq B] = \mathbb{E}(\Xi) = Pr(\Xi = \max_{h'} A_{h'}) \mathbb{E}(\Xi | \Xi = \max_{h'} A_{h'}) + Pr(\Xi > \max_{h'} A_{h'}) \mathbb{E}(\Xi | \Xi > \max_{h'} A_{h'}) = (1 - A_{\theta}^{\theta} \left[\max_{h'} A_{h'}\right]^{-\theta}) \max_{h'} A_{h'} + \frac{\theta}{\theta - 1} \max_{h'} A_{h'}.$$  

3. For $B > \max_{h'} A_{h'}$,

$$Pr(\arg \max_{h'} A_{h'} \zeta_{h'} = h \land \Xi \geq B) = \int_{B}^{\infty} Pr(A_{h'} \zeta_{h'} \leq u, \forall h' \neq h | A_{h} \zeta_{h} = u) g_{h}(u) du$$

where $g_{h}(u)$ is the marginal distribution of $A_{h} \zeta_{h}$. For $u > \max_{h'} A_{h'}$,

$$Pr(A_{h'} \zeta_{h'} \leq u, \forall h' \neq h | A_{h} \zeta_{h} = u) g_{h}(u) = \frac{\partial Pr(A_{1} \zeta_{1} \leq u, A_{h} \zeta_{h} \leq C, A_{h'} \zeta_{h'} \leq u)}{\partial C} \bigg|_{C = u}$$

$$= A_{h}^{\rho} \left(\frac{1}{N} \sum_{h'} A_{h'}^{\rho}\right)^{-\rho} \theta u^{-\theta - 1}.$$  

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Therefore,
\[
\Pr(\arg \max_{h'} A_{h' \bar{\eta}} \bar{z}_{h'} = h \land \Xi \geq B) = \frac{A_h^{\phi \rho}}{N} \left( \frac{1}{N} \sum_{h'} A_{h'}^{\phi \rho} \right)^{-\rho} B^{-\theta}.
\]

And
\[
\Pr(\arg \max_{h'} A_{h' \bar{\eta}} \bar{z}_{h'} = h \bigg| \Xi \geq B) = \frac{\Pr(\arg \max_{h'} A_{h' \bar{\eta}} \bar{z}_{h'} = h \land \Xi \geq B)}{\Pr(\Xi \geq B)} = \frac{A_h^{\phi \rho}}{N} \left( \frac{1}{N} \sum_{h'} A_{h'}^{\phi \rho} \right)^{-\rho} B^{-\theta} \left( \frac{1}{N} \sum_{h'} A_{h'}^{\phi \rho} \right)^{-1} B^{-\theta} = \frac{A_h^{\phi \rho}}{\sum_{h'} A_{h'}^{\phi \rho}},
\]

where the second equality applies \(\Pr(\Xi \geq B) = \left( \frac{1}{N} \sum_{h'} A_{h'}^{\phi \rho} \right)^{-1} B^{-\theta} \) from part 1 of the lemma.

If \(\overline{H} = \arg \max_{h'} A_{h'}\) is a singleton, then consider for \(h \notin \overline{H}\), we have \(A_h < \max_{h'} A_{h'}\), and
\[
\Pr\left( A_{h \bar{\eta}} \bar{z}_h = \max_{h'} A_{h'} \land \Xi \leq \max_{h'} A_{h'} \right) \leq \Pr\left( \bar{z}_h = \frac{\max_{h'} A_{h'}}{A_h} \right) = 0,
\]
since \(\max_{h'} A_{h'} > 1\). Therefore,
\[
\Pr\left( \Xi \leq \max_{h'} A_{h'} \right) = \sum_{h} \Pr\left( A_{h \bar{\eta}} \bar{z}_h = \max_{h'} A_{h'} \land \Xi \leq \max_{h'} A_{h'} \right) = \sum_{\tilde{h} \in \overline{H}} \Pr\left( A_{\tilde{h} \bar{\eta}} \bar{z}_{\tilde{h}} = \max_{h'} A_{h'} \land \Xi \leq \max_{h'} A_{h'} \right).
\]

Since \(\overline{H}\) is a singleton, for the only element \(\tilde{h} \in \overline{H}\), we thus have
\[
\Pr\left( A_{\tilde{h} \bar{\eta}} \bar{z}_{\tilde{h}} = \max_{h'} A_{h'} \land \Xi \leq \max_{h'} A_{h'} \right) = \Pr\left( \Xi \leq \max_{h'} A_{h'} \right).
\]
And since \(\Pr(\Xi < \max_{h'} A_{h'}) = 0\) and \(\Pr(\Xi = \max_{h'} A_{h'}) > 0\) (from part 1), we have
\[
\Pr(\arg \max_{h'} A_{h' \bar{\eta}} \bar{z}_{h'} = \tilde{h} \bigg| \Xi = \max_{h'} A_{h'}) = 1.
\]

\[\square\]

**Proof of Lemma 3.**

**Proof.** Omit time subscript for short notations. Define \(\Xi = \max_{h'} \eta_{h'} R_{ih'}(z) \bar{z}_{h'}\). Notice we have
defined \( \bar{R}_i(z) \equiv \max_{h' \neq h} H_{ih'} R_{ih'}(z) \) and \( \tilde{R}_i(z) \equiv \left( \frac{1}{N} \sum_{h'=[H]} [H_{ih'} R_{ih'}(z)]^\theta \right)^{\frac{1}{\theta}} \). If \( 1 + r^b_i > \bar{R}_i(z) \), apply part 1 of Lemma 4 we have

\[
Pr(\Xi \geq 1 + r^b_i | z) = [\bar{R}_i(z)]^\theta (1 + r^b_i)^{-\theta}.
\]

Apply part 3 of Lemma 4 we have the probability of investing in \( h \) conditional on being active

\[
\chi_{ih}(z) = \frac{1}{N} \left( \frac{\eta_{ih} R_{ih}(z)}{\bar{R}_i(z)} \right)^\theta.
\]

Apply part 2 of Lemma 4 we have the return conditional on \( z \)

\[
E[R^\theta_i(z, \eta) | z] = \left( 1 - [\bar{R}_i(z) / (1 + r^b_i)]^\theta \right) (1 + r^b_i) + [\bar{R}_i(z) / (1 + r^b_i)]^\theta \left( \frac{\theta}{\theta - 1} (1 + r^b_i) (1 + \lambda_i) - (1 + r^b_i) \lambda_i \right).
\]

This proves part (i).

For part (ii), if \( 1 + r^b_i < \bar{R}_i(z) \), from part 1 of Lemma 4

\[
Pr(\Xi \geq 1 + r^b_i | z) = 1,
\]

i.e., all firms with productivity \( z \) are active.

If set \( \bar{H} = \arg \max H_{ih'} R_{ih'}(z) \) is a singleton, apply part 3 of Lemma 4 we have

\[
Pr(\arg \max_{h' \neq h} H_{ih'} R_{ih'}(z) \xi_{ih'} = \bar{h} | \Xi = \bar{R}_i(z), z) = 1,
\]

which says that conditional on \( \max_{h'} H_{ih'} R_{ih'}(z) \xi_{ih'} = \bar{R}_i(z) \), with probability one the investment goes to the host that attains the maximum. And we thus have for \( h \in \bar{H} \), the share of active firms investing in \( h \)

\[
\hat{e}_{ih}(z) = \left( 1 - \frac{\bar{R}_i(z) / \bar{R}_i(z)}{Pr(\Xi = \bar{R}_i(z) | z)} \right) + \frac{\bar{R}_i(z) / \bar{R}_i(z)}{Pr(\Xi > \bar{R}_i(z) | z)} \chi_{ih}(z)
\]

\[
= 1 - \left[ 1 - \chi_{ih}(z) \right] [\bar{R}_i(z) / \bar{R}_i(z)]^\theta,
\]

and for \( h \notin \bar{H} \),

\[
\hat{e}_{ih}(z) = \chi_{ih}(z) [\bar{R}_i(z) / \bar{R}_i(z)]^\theta.
\]

Apply part 2 of Lemma 4, the return conditional on \( z \)

\[
E[R^\theta_i(z, \eta) | z] = \left( 1 - \frac{\bar{R}_i(z) / \bar{R}_i(z)}{Pr(\Xi = \bar{R}_i(z) | z)} \right) \bar{R}_i(z) (1 + \lambda_i) + \frac{\bar{R}_i(z) / \bar{R}_i(z)}{Pr(\Xi > \bar{R}_i(z) | z)} \left( \frac{\theta}{\theta - 1} \bar{R}_i(z) (1 + \lambda_i) - (1 + r^b_i) \lambda_i \right).
\]

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Proof of Proposition 1

Proof. The proof utilizes constant return to scale production functions and applies a series of definitions. To shorten notations, we suppress the time subscript throughout the proof.

Define

\[ \psi_{ih}(z) = \hat{\theta}_{ih}(z) \phi_i(z) \]

the marginal mass of investment in host country \( h \) by parent firms with productivity \( z \) from home country \( i \). Define

\[ y(w_h) = \left( \frac{1 - \alpha}{w_h} \right)^{(1-\alpha)/\alpha} \]

output per unit of efficient capital given wage \( w_h \). Then we have

\[ Y_{ih} = \int_0^\infty \hat{y}_{ih}(z) \hat{\theta}_{ih}(z) \phi_i(z) dz \]

\[ = \int_0^\infty \psi_{ih}(z) \hat{k}_{ih}(z) z_{ih}(z) y(w_h) dz. \]

Therefore,

\[ \frac{Y_{ih}}{\sum_j Y_{jh}} = \frac{\int_0^\infty \psi_{ih}(z) \hat{k}_{ih}(z) z_{ih}(z) dz}{\sum_j \int_0^\infty \psi_{jh}(z) \hat{k}_{jh}(z) z_{jh}(z) dz}. \]

Rewrite

\[ Y_{ih} = \int_0^\infty \psi_{ih}(z) \hat{k}_{ih}(z) z_{ih}(z) dz \]

\[ = \int_0^\infty \int_0^{z''} \frac{\psi_{ih}(z')}{\int_0^\infty \psi_{ih}(z'') dz''} \hat{k}_{ih}(z') dz' \times \int_0^\infty \int_0^{z''} \frac{\psi_{ih}(z''')}{\int_0^\infty \psi_{ih}(z''') dz'''} \hat{k}_{ih}(z''') dz''' \times \int_0^\infty \psi_{ih}(z') dz'. \]

And

\[ \sum_j Y_{jh} = \int_0^\infty \sum_j \psi_{jh}(z) \hat{k}_{jh}(z) z_{jh}(z) dz \]

\[ = \int_0^\infty \sum_j \psi_{jh}(z') \hat{k}_{jh}(z') dz' \times \int_0^\infty \frac{\sum_j \psi_{jh}(z')}{\int_0^\infty \sum_j \psi_{jh}(z') dz'} \hat{k}_{jh}(z') z_{jh}(z) dz. \]
Therefore

\[
\frac{Y_{ih}}{\sum_j Y_{jh}} = \frac{\int_0^\infty \psi_{ih}(z') dz'}{\int_0^\infty \sum_j \psi_{jh}(z') \hat{k}_{jh}(z') dz'} \frac{\int_0^\infty \psi_{ih}(z'') \hat{k}_{ih}(z'') dz''}{\int_0^\infty \sum_j \psi_{jh}(z') \hat{k}_{jh}(z') dz'} \times \left[ \int_0^\infty \frac{\psi_{ih}(z)}{\int_0^\infty \psi_{ih}(z') dz'} \hat{k}_{ih}(z) dz \right] \left[ \int_0^\infty \frac{\psi_{ih}(z') dz'}{\int_0^\infty \psi_{ih}(z') dz'} \right] \left[ \int_0^\infty \frac{\hat{k}_{ih}(z'') dz''}{\int_0^\infty \psi_{ih}(z') dz'} \right] \left[ \int_0^\infty \frac{\hat{k}_{ih}(z') dz'}{\int_0^\infty \psi_{ih}(z') dz'} \right] \left[ \int_0^\infty \frac{\psi_{ih}(z) dz}{\int_0^\infty \psi_{ih}(z') dz'} \right] \left[ \int_0^\infty \frac{\psi_{ih}(z') dz'}{\int_0^\infty \psi_{ih}(z') dz'} \right]
\]

(19)

Notice that from definitions in Section 3.7,

\[
\int_0^\infty \psi_{ih}(z') dz' = [FDI]_{ih}
\]

\[
\int_0^\infty \sum_j \psi_{jh}(z') \hat{k}_{jh}(z') dz' = K_h.
\]

And \(\int_0^\infty \frac{\psi_{ih}(z'')}{\int_0^\infty \psi_{ih}(z') dz'} \hat{k}_{ih}(z'') dz''\) is the average leverage of affiliates in host \(h\) from home \(i\), denoted by \(\bar{\psi}_ih\), since \(\hat{k}_{ih}(z'')\) is the leverage (affiliate total asset over parent financing) of affiliates from parent with productivity \(z''\), and \(\frac{\psi_{ih}(z'')}{\int_0^\infty \psi_{ih}(z') dz'}\) is the density of affiliates from parent firms with productivity \(z''\) conditional on entering host \(h\). Similarly, \(\int_0^\infty \frac{\psi_{ih}(z') dz'}{\int_0^\infty \psi_{ih}(z') dz'} \hat{k}_{ih}(z') dz\) is the average productivity of affiliates in host \(h\) from home \(i\) weighted by capital uses, denoted by \(\bar{z}_{ih}\). And \(\int_0^\infty \frac{\hat{k}_{ih}(z') dz'}{\int_0^\infty \psi_{ih}(z') dz'} \bar{z}_{ih}\) is the average productivity of all affiliates in host \(h\) weighted by capital uses, denoted by \(\bar{z}_h\). With the above (re)definitions of variables, Equation (19) can be rewritten as

\[
\frac{Y_{ih}}{Y_h} = \frac{[FDI]_{ih}}{K_h} \times \bar{\psi}_ih \times \bar{z}_{ih} \times \bar{z}_h.
\]

Proof of Proposition 2

Proof. Define

\[
\psi_{ih}(z) = \hat{\psi}_{ih}(z) \phi(i)
\]

the marginal mass of investment in host country \(h\) by parent firms with productivity \(z\) from home country \(i\). Define

\[
y(w_h) = \left( \frac{1 - \alpha}{w_h} \right)^{1 - \alpha}/\alpha
\]

output per unit of efficient capital given wage \(w_h\) and

\[
l(w_h) = \left( \frac{1 - \alpha}{w_h} \right)^{1/\alpha}
\]

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labor demand per unit of efficient capital given wage $w_h$.  

Since country $h$ is kept to be outward-FDI autarky in that the country is restricted to be operating domestically only. From Lemma 3, the investing decision of a parent firm in country $h$ reduces to a threshold rule: the firm borrows to the maximum, operates and invests domestically if $z > z^*_h$, and stays idle if $z < z^*_h$, where $z^*_h$ is determined by

$$a z^*_h y(w_h) = r^*_h h + \delta.$$ 

Using the policy function and aggregate wealth and wealth share distribution defined in Section 3.6, the total capital used by domestic firms is

$$W_h \int_{z^*_h}^{\infty} \phi_h(z)(1 + \lambda_h) dz = K_{hh},$$

Plug in the the Pareto pdf $\hat{\phi}_h(z) = \gamma z^\gamma_h z^{1-\gamma}$ we have

$$z^\gamma_h [z^*_h]^{-\gamma} (1 + \lambda_h) = \frac{K_{hh}}{W_h}. \tag{20}$$

From labor market clear we have

$$\left[ \sum_{i \neq h} \psi_{ih}(z)[1 + b_{ih}(z)] \bar{z}_{ih}(z) dz + W_h \int_{z^*_h}^{\infty} \phi_h(z)(1 + \lambda_h) \bar{z}_{hh}(z) dz \right] I(w_h) = L_h.$$

Apply the normalization $\bar{z}_{hh} = z$, and notice the production conducted by affiliates home $i$ in host $h$ is

$$Y_{ih} = y(w_h) \int_{0}^{\infty} \psi_{ih}(z)[1 + b_{ih}(z)] \bar{z}_{ih}(z) dz,$$

and hence we have

$$\left[ \sum_{i \neq h} Y_{ih} + y(w_h) W_h \int_{z^*_h}^{\infty} \phi_h(z)(1 + \lambda_h) z dz \right] \frac{I(w_h)}{y(w_h)} = L_h. \tag{21}$$

Apply the Pareto pdf $\hat{\phi}_h(z) = \gamma z^\gamma_h z^{1-\gamma}$ we have

$$\int_{z^*_h}^{\infty} \phi_h(z) z dz = \frac{\gamma}{\gamma - 1} z^\gamma_h [z^*_h]^{1-\gamma}. \tag{22}$$

Combine (20), (21) and (22) we have

$$W_h (1 + \lambda_h) \frac{\gamma}{\gamma - 1} \left( \frac{K_{hh}}{W_h} \right) \frac{z^\gamma_h}{\gamma - 1} z^\gamma_h \frac{Y_{hh}}{Y_h} I(w_h) = L_h.$$
Apply the definition of $l(w_h)$ and take log both sides, we have

$$\text{Cons} - \frac{1}{\alpha} \log(w_h) + \frac{\gamma - 1}{\gamma} \log \left( \frac{K_{hh}}{W_h} \right) - \log \left( \frac{Y_{hh}}{Y_h} \right) = 0,$$

where $\text{Cons}$ is a constant (notice that $W_h$ does not change at the impact of the reform). Moving from autarky ($\frac{K_{hh}}{W_h} = 1, \frac{Y_{hh}}{Y_h} = 1$) to FDI openness we have

$$\Delta \log(w_h) = -\alpha \log \left( \frac{Y_{hh}}{Y_h} \right) + \alpha \frac{\gamma - 1}{\gamma} \log \left( \frac{K_{hh}}{W_h} \right).$$

For the case of small open economy, in which interest rate is fixed at $r^*$. The productivity of the marginal productive firm should satisfy

$$\alpha z_h^k y(w_h) = r^* + \delta. \quad (23)$$

Combine (21), (22) and (23) we have

$$W_h (1 + \lambda_h) \frac{\gamma}{\gamma - 1} \left( \frac{r^* + \delta}{\alpha y(w_h)} \right)^{\frac{1 - \gamma}{\gamma}} \frac{X_h}{Y_h} z_h^\gamma l(w_h) = L_h.$$

Apply the definition of $l(w_h)$ and $y(w_h)$, and take log both sides, we have

$$\text{Cons}_2 - \frac{1}{\alpha} \log(w_h) + (1 - \gamma) \frac{1 - \alpha}{\alpha} \log(w_h) = \log \left( \frac{Y_{hh}}{Y_h} \right).$$

Moving from autarky ($\frac{K_{hh}}{W_h} = 1, \frac{Y_{hh}}{Y_h} = 1$) to FDI openness we have

$$\Delta \log(w_h) = -\frac{\alpha}{1 - (1 - \gamma)(1 - \alpha)} \log \left( \frac{Y_{hh}}{Y_h} \right).$$

\[\square\]

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