A Monopolistic Competition Model of Heterogeneous Firm Matches in Cross-Border Mergers & Acquisitions

Steven Brakman, Harry Garretsen, Michiel Gerritse & Charles van Marrewijk

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Abstract
Models of cross-border mergers and acquisitions (M&As) assume that highly productive acquirers transfer their productivity level to that of the target. This is at odds with stylized facts according to which less productive firms only partially catch-up via an M&A, and that post-merger firm productivity is an average of pre-merger productivity levels. Using the Melitz (2003) model of heterogeneous firms, we develop a model of matching in cross-border M&As that permits imperfect knowledge transfers. With imperfect transfers, (weak) assortative matching in productivity arises for firms in cross-border M&As, without strict productivity ordering. This is in line with the empirical evidence since M&As occur at all productivity ranges and crucially also between firms of similar productivity levels. Allowing for M&As when productivity transfers are imperfect raises the overall average productivity and welfare, but the welfare benefits are smaller if knowledge transfers are less than perfect.

Keywords: Cross-border M&As, heterogeneity, knowledge transfers, productivity differences

JEL classification: F23, L23, F12, G34

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1 Introduction

Firms that pair up in international mergers and acquisitions often match closely in their productivity levels, but models of international trade and investment generally cannot explain that matching pattern. We develop a monopolistic competition model of matching between heterogeneous firms in cross-border mergers & acquisitions (M&As). The model allows for both perfect and imperfect knowledge (and hence productivity) transfers between the M&A partners. To motivate our analysis, we first provide stylized facts to illustrate that these transfers are not perfect and that there is (weak) positive assortative matching in M&As – such that M&As are more likely between firms where the initial, pre-merger productivity gap is not too large. Recent theories of cross-border M&As assume perfect knowledge transfers and are thus at odds with the stylized facts. In our model, perfect knowledge transfers are a special case. Moreover, we show that M&As raise the firm viability cut-off level and welfare more as the quality of knowledge transfers improves.

Conventional models of foreign direct investments (FDI) focus on greenfield investments, but most foreign investments are through cross-border M&As. The majority of these cross-border investments is accounted for by international acquisitions, followed by mergers (Barba Navaretti and Venables, 2004; Antras and Yeaple, 2014; Davies et al., 2018). A small but growing literature that builds on trade models of monopolistic competition analyses cross-border M&As explicitly. This literature is often inspired by the Melitz (2003) model of firm heterogeneity (for surveys see Bernard et al., 2012; Yeaple, 2013; Antras and Yeaple, 2014). In the Melitz model, a firm’s international activity is restricted to exports, but it can be easily extended to incorporate horizontal (greenfield) foreign direct investment (see Helpman et al., 2004; Chen and Moore, 2010; Yeaple, 2013). The model introduced in this paper is linked to this literature and the underlying model of monopolistic competition is used to analyse horizontal or market-seeking cross-border M&As.

To be clear from the outset, this recent literature and its reliance on the market structure of monopolistic competition differs from an older, well-established literature where (cross-border) M&As are analysed in a model of oligopolistic competition and where the main motive of the M&As is to change/reduce the degree of competition (see Salant, Switzer, and Reynolds, 1983; Bertrand and Zitouna, 2006). One feature of these models is that M&As do no take place when firms are identical, that is if they have the same productivity level. In contrast, in our model two identical firms can engage in a cross-border M&A if the productivity level passes a threshold to make it profitable to bypass the trade costs that go along with the M&A alternative of exporting.

2 The role of differences between countries, which is central in the international trade literature, is mostly neglected in the cross-border M&A literature. Neary (2004, 2007) takes these country-wide influences on cross-border M&As into account by linking cross-border M&As to comparative advantage, but does so in a model of oligopolistic firms as opposed to monopolistic competition like in the present paper (see for empirical support of Neary (2007), Brakman et al., 2013, and Blonigen et al., 2014).
This finding shows that our model and motivation to study cross-border M&As originate in the international trade literature and not in the industrial organisation literature (as in Salant et al., 1983) where anti-competitive motives drive the M&A decision. This also differentiates our approach from Falvey (1998) and Neary (2007), who study cross-border M&As from an international economics perspective but do so in a setting of oligopolistic markets.

The bulk of the literature on cross-border M&As assumes a perfect knowledge (and thus productivity) transfer in the process of the M&A (see Neary, 2004, 2007; Guadalupe et al., 2012; Ramondo and Rodríguez-Clare, 2013; Blonigen et al., 2014; Antras and Yeaple, 2014). As the post-M&A firm assumes the productivity of the most productive partner, the productivity of the less productive partner becomes irrelevant for the profits of the merged firm (although it does determine the value of the acquired firm). Most empirical evidence, however, rejects the assumption that productivity is perfectly transferred. The international business literature in particular documents that the post-M&A productivity is generally lower than that of the acquirer, and the combined value after the M&A is often lower than one would expect based on the expansion of the (more productive) acquirer (for surveys see King et al. 2004; Moeller et al. 2005; McCarthy, 2011). There are many reasons for imperfect knowledge transfers, including costly transfers which prevent full transmission of the knowledge base and less productive firms which lack the ‘absorptive capacity’ to efficiently transfer knowledge (see Teece, 1977; Leahy and Neary, 2007). Once the target firm does not perfectly catch up with the more productive parent, the characteristics of the target firm can determine post-merger performance.

The relevance of target characteristics is highlighted in a smaller set of papers. In Nocke and Yeaple (2007, 2008) acquirers look for assets in (potential) targets, including special ‘capabilities’ of the target firm that are specific to the host country – such as knowledge of the local market, or distribution networks. These capabilities indicate that firms are heterogeneous across various dimensions. Access to such immobile assets, coupled with low target productivity, makes an acquisition more likely. In their empirical application, Nocke and Yeaple (2008) stress that the parent’s productivity determines the likelihood of choosing an M&A over a greenfield investment (see also Anand and Delios, 2002, for empirical support). Head and Ries (2008) argue that the headquarter’s management characteristics of a multinational need to match the characteristics of the target, for M&As to be profitable.

Direct matching in productivity levels in cross-border M&As is documented in a more limited set of studies. Braguinsky et al. (2015) document, in a historical setting, that despite differences in profitability, acquirers and targets often have similar physical productivities, refuting the idea that highly productive firms acquire unproductive firms. Similarly, Rhodes-Kropf and Robinson (2008) show that acquisitions typically occur between firms with similar market-to-book ratios.

When presenting stylized facts below, we also document that the productivity of a firm after the cross-border M&A is generally in between the pre-M&A productivities of the two partners – indicating imperfect productivity transfers. Relaxing the assumption of perfect productivity
transfers introduces – in a stylized manner – that post-M&A firms are not always characterized by the highest productivity of the constituting firms, and that they can be less productive than the most productive firm prior to the M&A. From an empirical point of view this finding is not new, because it has been found to apply more generally to conglomerate firms as compared to stand-alone firms (Schoar, 2002). We also know that mergers may lead to lower post- compared to pre-merger profits (Fridolfsson and Stennek, 2005).

The contribution of this paper is that we incorporate a process of matching between firms in the Melitz type monopolistic competition model of international trade. Random firm pairs are given the opportunity to engage in an M&A, after which the common firm productivity is a mix of the pre-M&A productivities of the partners. After the M&A, the new firm continues to produce the two unique varieties that both individual firms were selling in the domestic market – and possibly also exporting – before the M&A. In this sense we include – in a simple way – firms that expand by producing more varieties, which is a substantial margin of adjustment to trade opportunities (Breinlich and Cuñat, 2014, Breinlich, Nocke and Schutz, 2019). Firms engage in an M&A if the combined profit after the M&A exceeds the sum of individual pre-M&A profits. Profits and expenses in the new firm are perfectly shared between the two firms, that is, we abstract from distributional effects that could have been negotiated between the original firms. We also allow for synergies in production, though that is not fundamental to our results. The new firm might improve average productivity by combining the technologies, depending on the degree of knowledge transfers. Furthermore, we show that M&As are welfare improving, even if knowledge transfers are not perfect.

Our model can explain several stylized facts on cross-border M&As. First, we observe weak positive assortative matching: a match between two firms is only viable, if the productivity of the least productive firm is not too different from the productivity of the most productive firm (see Chade, et al., 2017). Second, our model is consistent with the fact that cross-border M&As occur across the spectrum of productivities. Our model thereby opens up the possibility for less productive firms to be active in M&As, albeit with other partners than highly productive firms.

The degree to which productivity can be transferred between partners has several noteworthy implications. The range of potential matches increases with the strength of productivity transfers, suggesting that the flow of international M&As depends on the transferability of factors that can affect productivity such as, knowledge spillover, quality of management, production routines and other determinants of productivity. As in related models of M&As, introducing the possibility of cross-border M&As strictly increases welfare. That holds for all degrees of productivity transfer. However, the magnitude of welfare gains varies with the transferability of knowledge: the welfare benefits of observed flows of international investment are lower as knowledge transfers become less perfect.

The structure of the paper is as follows. In section 2, we use firm-level data to document some stylized facts about cross-border M&As. The descriptive statistics show that the productivity of a
firm after an M&A is generally somewhere in between the productivities of the firms involved in the M&A. This suggests that knowledge transfers are not perfect. It also shows that the probability of an M&A occurring between two firms falls with the productivity difference of those two firms. Section 3 sets out our M&A matching model in detail. Section 4 analyses M&A viability for a given parameter setting, while section 5 discusses the probability of M&As, the distribution of firm productivity, and the endogenous impact of M&As on the firm cut-off viability with associated welfare consequences. Finally, section 6 concludes.

2 Stylized Facts about Firm Matches

We first provide stylized facts on cross-border M&A data for 22 advanced and middle income countries (see below). The data reveal some well-understood regularities, such as the common finding that firms that engage in FDI are, on average, more productive. However, our data also shows some less established patterns in the literature on cross-border M&As. First, there is a large overlap in the distributions of productivities of firms engaged in M&As as an acquirer, as a target, or not engaging in M&As at all. Despite higher prevalence among more productive firms, M&As occur between firms in all productivity levels in our data. Second, we document that the post-merger firm generally does not assume the productivity of the most productive partners. Third, we show that mergers or acquisitions often take place between firms in similar positions in the productivity ordering: high-high matches or low-low matches are widespread, and the pattern of a high-productive acquirer matching with a low-productive target is relatively rare.

We draw on two datasets of *Bureau van Dijk: Orbis* for firm-level data and *Zephyr* for data on M&As. In the firm-level data, we restrict ourselves to firms that have a physical address, more than one employee, and more than $1,000 in assets and turnover (automatically excluding firms for which we do not observe employment, sales, or assets). We consider firms that are part of a country’s NACE (rev. 2) 4-digit sector that has at least 50 firms in our data. We draw our observations on mergers and acquisitions from the *Zephyr* database. We use completed M&As only and do not include other international transactions like IPOs, buy-ins, and joint ventures. The database of potential mergers and acquisitions consists of a cross of all firms with all potential targets. We consider an observation potential for M&A to exist if the potential acquirer and the potential target operate in the same 4-digit sector. As *Bureau van Dijk* follows the same firms in the *Zephyr* and *Orbis* datasets, we assume the M&A information is comprehensive for the set of firms observed in *Orbis*. We merge the dataset of mergers with accounting information from *Orbis*. The dataset spans the years 2007-2016, covering firms in 780 NACE (4 digit) sectors from 22 countries, namely 15 European countries plus Australia, Brazil, China, Japan,
Taiwan, and the USA.\textsuperscript{3} \textit{Orbis} is one of the few sources of harmonized firm-level data that allow an international comparison. It does not cover the universe of firms, the coverage seems adequate to approximate the productivity ranks of firms in their populations (Alfaro and Chen, 2018). After our selection criteria for firm information and sector size, we find our required financial information for around 52\% of all observed legal entities. As an approximation of firm productivity denoted by $\varphi$, we take the log of sales (in thousand dollars) per employee. This measure of labour productivity is consistent with the firm heterogeneity framework that we will use in our model. We deflate the nominal sales by the U.S. GDP deflator from the World Bank.\textsuperscript{4}

2.1 Patterns in the productivity of firms matched in a cross-border M&A

How do firms pair up in mergers and acquisitions? Table 1 summarizes the pattern by tabulating, for all cross-border M&As, the productivity quintiles of the two firms within their sector-country group. The first quintile (I; 0-20 per cent) thus indicates the 20 per cent least productive firms relative to their peers in the same sector, country and year; and the fifth quintile (V; 80-100 per cent) indicates the 20 per cent most productive firms in their sector-country-year group. The quintiles are ranked for the high-productive partner in the rows of Table 1 and for the low-productive partner in the columns. For example, the bottom-left cell in the table indicates that 4.8 per cent of all M&As occur when the high-productive partner is in the fifth quintile V and the low-productive partner is in the first quintile I.

What would the distribution in Table 1 look like if M&As were a purely random process from a firm productivity perspective of the involved firms? In that case, the diagonal entries would all be four per cent and the bottom-left off-diagonal entries would all be eight per cent.\textsuperscript{5} Table 1 indicates that M&As do not occur randomly. More specifically, if firm productivity differences are relatively small M&As occur more frequently (as indicated by the cells II-II, III-II, III-III, IV-III, and IV-IV in the centre of the table), whereas if firm productivity differences are relatively large M&As occur less frequently (as indicated by the extreme bottom-left cells IV-I, V-I, and V-II of the table).

Note that the proximity-concentration trade-off argument implies that M&As occur more frequently in the extreme bottom-left cells (between highly productive firms that seek to establish a presence and less productive firms that can be bought at low cost) and thus less frequently closer to the diagonal. In contrast, the empirical observations show that only 4.8 per

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\textsuperscript{3} The European countries are: Belgium, Bulgaria, Czech Republic, Germany, Spain, Estonia, Finland, France, UK, Italy, Netherlands, Norway, Russia, Sweden, Switzerland, and Ukraine.

\textsuperscript{4} From the World Bank National Accounts data, see \url{https://data.worldbank.org/indicator/NY.GDP.DEFL.ZS}.

\textsuperscript{5} Based on this random distribution; for probability $p$ and $n$ independent observations, shading is light-yellow below $p - 2\sqrt{p(1-p)}/n$ and dark-green above $p + 2\sqrt{p(1-p)}/n$. 

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6
cent of the M&As belong to that V-I category, 5.7 per cent to the IV-I category, and 6.7 per cent to the V-II category. The most frequent combinations occur when productivity differences are relatively small, namely 10.6 per cent in the IV-III category, 9.0 per cent in the III-II category, and (compared to the random benchmark of four per cent) if the two firms are in the same quintiles II-II, III-III, and IV-IV.6

Table 1 Frequency distribution of cross-border M&As by productivity quintiles; % of total M&As

<table>
<thead>
<tr>
<th>By productivity quintile of the low-productive and high-productive partner in an M&amp;A</th>
<th>Low-productive firm quintile</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>I (0-20)</td>
<td>3.4</td>
<td>3.4</td>
</tr>
<tr>
<td>II (20-40)</td>
<td>7.5</td>
<td>12.6</td>
</tr>
<tr>
<td>III (40-60)</td>
<td>7.4</td>
<td>31.6</td>
</tr>
<tr>
<td>IV (60-80)</td>
<td>5.7</td>
<td>30.2</td>
</tr>
<tr>
<td>V (80-100)</td>
<td>4.8</td>
<td>32.2</td>
</tr>
<tr>
<td>Total</td>
<td>28.7</td>
<td>100</td>
</tr>
</tbody>
</table>

Source: authors’ elaborations on data from Zephyr and Orbis; 3065 observations; grey shading is more than two st.errors below and dark-green shading is more than two st.errors above random distribution, see main text for details.

To further investigate how productivity differences affect M&A choices, we explain the M&A status for a given firm pair from their respective productivities. We organize our firm data in country pairs for every 4-digit sector. Firms in each sector in a given year and country may consider cross-border M&A partners in the population of firms in the same sector and year, but in another country. We cross all firms in a sector for every country pair to consider the M&A opportunities (ensuring that there are no double pairs by symmetry), so that every observation is a potential match. In the regression, subscripts \( l \) and \( h \) signal the two firms, which are – by construction – in the same sector and year but in a different country. Next, we explain whether a match occurred (1 for yes, 0 for no) from the productivity difference of the two firms.

Table 2 reports the results of regressing the M&A status on the absolute difference in log productivity of the two partners. In the first regression (column 1) we use firm-year fixed effects

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6 The pattern is similar for horizontal M&As in the same 4-digit sector (see Appendix A) and when excluding observations from sectors in a country that has less than 5 or 50 firms operating (available on request).
for both participating partners to rule out that any individual firm characteristics explain the coefficient – including the possibility that some firms are more likely to engage in M&As because they are more productive, for instance. Informally, we compare different potential partners for a given firm while keeping the characteristics of that firm constant, so we cannot attribute the impact of productivity differences on M&As to such firm-specific characteristics. For readability, we have divided the independent variable – the absolute log productivity difference percentile – by 1000. The results in column 1 indicate that for a given firm pair, doubling the productivity differences (one log point; 100 per cent) leads to a significant decline in the likelihood of an M&A of about 0.03 per cent. For comparison, the average log productivity difference between firm pairs in our sample is 1.66.

Table 2 The impact of productivity differences on the likelihood of M&As

<table>
<thead>
<tr>
<th>Dependent variable: M&amp;A occurrence (0,1)</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute difference in productivity</td>
<td>-0.32***</td>
<td>-0.0018**</td>
</tr>
<tr>
<td>percentile difference of partners / 1000</td>
<td>(0.12)</td>
<td>(0.00075)</td>
</tr>
<tr>
<td>Productivity of the high-productive partner percentile / 1000</td>
<td>0.0015***</td>
<td>(0.00086)</td>
</tr>
<tr>
<td>Observations</td>
<td>190,517</td>
<td>27,183,613</td>
</tr>
<tr>
<td>FE partner 1 × year</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>FE partner 2 × year</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FE country pair × sector × year</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Odds ratio (of one log point partner difference)</td>
<td>0.85</td>
<td>0.90</td>
</tr>
<tr>
<td>(Odds ratio se)</td>
<td>(0.05)</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>

Source: authors’ elaborations on data from Zephyr and Orbis; note that column 1 includes partner-year fixed effects, such that all variation for firms who are not engaged in an M&A in a given year is excluded, hence the large difference in the number of observations.

The impact of higher productivity differences on a given pair’s M&A probability is small in absolute terms, but the impact applies to many potential matches and an M&A is a rare event. Hence, we also report the odds ratio of M&As implied by a 100 percentage point additional productivity difference between the partners. The odds ratio is significantly different from one, and it shows a decline of around 15 per cent in the probability of an M&A occurring in each firm pair, if the productivities of the partners are one log point further apart.

Column 2 of Table 2 reports a similar regression, this time using fixed effects at the country pair-sector-year level instead of the firm-year level. It controls less precisely for firm-specific shocks, but it allows identification of the coefficient on the level of productivity of the most productive firm in the M&A. Moreover, as the identification can now include firms that never engage in M&As, the sample is broader. The results suggest that a higher productivity of the most productive firm in a pair is linked to an increased probability that the pair of firms engages in an
M&A. Note that, conditional on this effect, productivity differences are still associated with a lower probability of M&As.

**Stylized fact 1**

**M&As occur for all ranges of firm productivity, but are less likely if the difference in productivity between the potential partners is larger.**

These statistics suggest that M&As may not be reserved for the most productive firms only, but that they occur across a wide range of productivities. Figure 1 plots the distributions of the log of sales per employee for three sets of firms: the complete set (all firms); firms that acquired another firm (acquirers); and firms that were the target in an acquisition (targets). The average productivity is highest in the group of acquirers (5.41), somewhat lower for firms in general (5.06), and lowest for the targets (4.64). Further summary statistics (based on firms operating in a country in a sector with at least 50 observations that year) are provided in Table A1 in Appendix A.⁷

**Figure 1. Productivity distribution of acquirers, targets, and all firms**

![Graph showing productivity distribution](image)

Source: authors’ calculations based on microdata from *Orbis* and *Zephyr* (see main text for details).

The distributions of the targets and the acquirers are distinct and statistically different: a Kolmogorov-Smirnov test for similarity of the distributions rejects at the one per cent level. This suggests that the firms are not drawn from the same overall distribution. Note, however, that the three distributions show substantial overlap. For example, 42 per cent of the targets has a higher productivity than the median productivity of acquirers (which is 5.16).

### 2.2 Productivity transfers within merged firms

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⁷ One might also correct the log of sales per employee for global trends in addition to deflating. This leads to virtually the same graph as depicted in Figure 1 (available upon request).
How does productivity change after two firms have completed an M&A? A standard assumption is that the (high-productivity) acquirer transfers its knowledge to the (low-productivity) target. Another intuitive case might be that the post-M&A firm is simply a legal unit without any changes. In that case, the post-M&A firm productivity is a combination of the productivities of the two partners. From a productivity perspective, perfect knowledge transfers occur if the low-productivity firm reaches the same productivity level as the high-productivity firm. From now on, we therefore no longer use the acquirer-target terminology, but focus instead on the productivity-match of the two firms engaged in the M&A, where the sub-index $h$ denotes the high-productivity firm and sub-index $l$ denotes the low-productivity firm.

We estimate the parameter for the productivity development in the data described above. We treat every M&A as an event, for which we observe both the productivities of the firms that participated and the combined productivity after the M&A has taken place. Let $\varphi_m$ denote the productivity of the merged firm, $\varphi_h$ denote the productivity of the high-productive firm, and $\varphi_l$ denote the productivity of the low-productive firm, then we estimate the following regression:

\[
\ln(\varphi_m) = \beta_1 \ln(\varphi_h) + \beta_2 \ln(\varphi_l) + \varepsilon_1
\]

Coefficient $\beta_1$ is an indicator of the extent of knowledge transfers from the high-productive firm to the low-productive firm. With perfect knowledge transfers we have: $\beta_1 = 1$ and $\beta_2 = 0$.

Alternatively, a plausible benchmark for the evolution of productivity is that there is no knowledge transfer. To capture this, we construct the hypothetical combined-firm pre-merger productivity $\varphi_{pm}$ by simply adding pre-merger sales of the two firms and dividing by the pre-merger number of employees for the two firms. We then estimate the regression:

\[
\ln(\varphi_m) = \beta_3 \ln(\varphi_{pm}) + \beta_4 \ln(\varphi_h) + \varepsilon_2
\]

Table 3 Post-M&A productivity explained by pre-merger productivity

<table>
<thead>
<tr>
<th>M&amp;As all or within sector</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>all</td>
<td>all</td>
<td>within</td>
<td>within</td>
</tr>
<tr>
<td><strong>Equation (1) estimate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High-productive partner parameter $\beta_1$</td>
<td>0.66***</td>
<td>0.64***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.050)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low-productive partner parameter $\beta_2$</td>
<td>0.29***</td>
<td>0.32***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.058)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Equation (2) estimate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Combined pre-M&amp;A parameter $\beta_3$</td>
<td>0.84***</td>
<td>0.75***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
<td>(0.13)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
We do this to test if there are any knowledge transfers at all, that is if the productivity of the merged firm is affected by the productivity of the high-productive partner above and beyond its contribution in the simple combination of the accounts. Without any such knowledge transfers we have: $\beta_3 = 1$ and $\beta_4 = 0$.

Table 3 reports the estimation results. Columns 1 and 3 explain the post-M&A productivity based on the pre-merger productivity of the two partners, where column 1 includes all M&As and column 3 only includes M&As within the same sector. The coefficient for the low-productive partner is about 0.3 and for the high-productive partner is close to 0.7. The high-productive partner is therefore more important for determining post-merger productivity; a 10 per cent higher productivity of the less productive partner raises post-merger productivity by about 3 per cent, compared to 7 per cent for the high-productive partner. It is important to note that the coefficient for the most productive partner is significantly less than one and of the least productive partner is significantly larger than zero. Both firms are therefore important in determining post-merger productivity and knowledge transfers are not perfect.

Columns 2 and 4 of Table 3 analyse the impact of the high-productive firm in addition to the pre-merger hypothetical combined-firm productivity, where column 2 includes all M&As and column 4 only includes M&As within the same sector. The coefficient of 0.84 for all firms and 0.75 for within sector (horizontal) M&As indicate that pre-merger combined productivity is important for determining post-merger productivity. However, since both $\beta_3$ coefficients are significantly lower than 1 and the $\beta_4$ coefficients for the high-productive firms are significantly positive, post-merger productivity also benefits from some knowledge transfers from the high-productive partner.\(^8\)

Stylized fact 2
The productivity of the merged firm is a combination of the productivity of the two partners and lower than that of the high-productive partner. Knowledge transfers are not perfect.

\(^8\) One might argue that productivity shocks at the global or sector level or output price fluctuations could explain fluctuations in the pre-and post-merger productivity. In Table A3 in Appendix A we report the same regressions while deflating firm productivity globally, or by sector-specific annual shocks. This does not change the results.
We can summarize our findings from this empirical section in two stylized facts. First, M&As do not occur predominantly between partners at the extremes of the distribution, as part of the literature suggests. Instead, M&As are more likely to occur between firms with similar productivity levels. Despite somewhat higher likelihoods for more productive firms, M&As occur for all productivities, and the productivity distribution of participating firms shows substantial overlap with firms that never engage in M&As. Second, post-M&A firm productivity clearly benefits from knowledge transfers from the high-productive partner, but knowledge transfers are not perfect and post M&A productivity is also determined by the low-productive partner. As explained in the introduction, the models of international firms and trade using perfect knowledge transfers cannot explain these stylized facts for cross-border M&As. Motivated by our stylized facts, the next section therefore develops a model of heterogeneous firm M&As in line with the M&A characteristics discussed above.

3 A Matching Model of Cross-Border Mergers & Acquisitions

Our general setup takes the Melitz (2003) model as its starting point, and augments it with the possibility of a cross-border M&A. Motivated by the observations in section 2, the merged firm operates at a productivity level that is an (geometric) average of the constituent firms’ productivity levels. This section analyses M&As for given demand parameters, while section 4 analyses the viability of M&As, whereas section 5 discusses the impact of M&As in terms of probability, distribution, cut-off value, and welfare implications.

3.1 Domestic and Exporting Firms

We start by introducing the monopolistic competition model developed by Melitz (2003) in which heterogeneity in (exogenous) firm productivity explains which firms survive in the market and which firms can export. Demand on the domestic market $q_\varphi$ for an individual firm which has productivity $\varphi$ and charges a price $p_\varphi$ is: $q_\varphi = A p_\varphi^{-\varepsilon}$. The iso-elasticity parameter $\varepsilon > 1$ leads to a constant mark-up $\varepsilon/(\varepsilon - 1)$ of price over marginal costs. Setting the wage rate as numéraire ($w = 1$), this implies that a firm with productivity level $\varphi$ which has marginal costs $1/\varphi$ charges an optimal price $p_\varphi = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \left(\frac{1}{\varphi}\right)^{\varepsilon}$. If we substitute the optimal price information in the demand function we can derive the firm’s domestic revenue $r_{\varphi,d}$ and domestic profits $\pi_{\varphi,d}$ (d for domestic) as:

\begin{align*}
(3) \quad r_{\varphi,d} &= p_\varphi q_\varphi = \left[\left(\frac{\varepsilon}{\varepsilon - 1}\right) \left(\frac{1}{\varphi}\right)^{\varepsilon}\right] A \left[\left(\frac{\varepsilon}{\varepsilon - 1}\right) \left(\frac{1}{\varphi}\right)^{\varepsilon}\right]^{-1} = \varepsilon(\varepsilon - 1)^{\varepsilon - 1} A \varphi^{\varepsilon - 1} = \varepsilon B \varphi^{\varepsilon - 1}, \\
(4) \quad \pi_{\varphi,d} &= p_\varphi q_\varphi - \left(f + \frac{q_\varphi}{\varphi}\right) = \frac{p_\varphi q_\varphi}{\varepsilon} - f = \frac{r_{\varphi,d}}{\varepsilon} - f = B \varphi^{\varepsilon - 1} - f,
\end{align*}

---

\textsuperscript{9} Again, as we already stated in the introduction of our paper, the fact that model set-up is based on market structure of monopolistic competition sets our analysis apart from the (bulk of) the literature on (cross-border) M&As where the market structure is oligopolistic (see Salant et al, 1983, for seminal example and see Neary, 2007 and Breinlich et al, 2019 for recent applications of an oligopoly model to (cross-border) M&As)
where the constant $B$ is defined as $B \equiv (\varepsilon - 1)\varepsilon^{-1}e^{-\varepsilon}A$ and $f$ is the per-period fixed cost.

We analyse the consequences for a model with two identical countries in which trade by exports is possible at an iceberg transport cost $\tau > 1$ and a fixed export costs per period $f_x$. The symmetry assumption ensures that the two countries have the same wage rates (normalized to unity) and the same aggregate variables. The price charged by an exporting firm is $\tau p_{\varphi}$ ($\tau$ times the price charged on the domestic market to cover the higher marginal costs), which implies the revenue in the foreign market is equal to $\tau^1 - \varepsilon B \varphi e^{-1}$ (since demand is iso-elastic). Since operating profits are $1/\varepsilon$ times revenue and a firm will engage in exporting activity if the associated operating profits are larger than the fixed exporting costs $f_x$, this means a firm will export if: $\tau^1 - \varepsilon B \varphi e^{-1} - f_x > 0$. Under the assumption $f_x > \tau^1 - \varepsilon f$, we will have both domestic firms and exporting firms active on the market. Identifying revenue and profits with a sub-index $x$ for exports, we have:

\begin{align*}
\tau_{\varphi,x} &= \tau^1 - \varepsilon B \varphi e^{-1} \\
\pi_{\varphi,x} &= \tau^1 - \varepsilon B \varphi e^{-1} - f_x
\end{align*}

Let a ‘*’ sub-index denote firm viability and let a ‘* $x$’ sub-index denote export viability. Note that $B \varphi^* e^{-1} = f$ and $\tau^1 - \varepsilon B \varphi^* e^{-1} = f_x$. We thus have the following profit function for all productivity ranges:

\begin{equation}
\pi_{\varphi} = \begin{cases}
0 & , \varphi < \varphi^*
, \\
\pi_{\varphi,a} = B \varphi e^{-1} - f & , \varphi^* \leq \varphi < \varphi_{x}\ \\
\pi_{\varphi,a} + \pi_{\varphi,x} = (1 + \tau^1 - \varepsilon)B \varphi e^{-1} - f - f_x, \varphi \geq \varphi_{x}
\end{cases}
\end{equation}

3.2 Mergers & Acquisitions

Next, we allow for the possibility of M&As. In a similar vein as the exogenous modelling of firm exit in the heterogeneous firm literature (using the parameter $\delta$, see Melitz, 2003, and section 5), the simplification we make is based on an exogenous probability of an M&A match, which ensures that the shape of the productivity distribution and ex ante survival probabilities are exogenously given (see also the discussion in section 6). Paraphrasing Melitz (2003, p. 1701, footnote 11): the increased tractability afforded by this simplification permits the detailed analysis of the impact of M&As on the endogenous range of productivity levels and on the distribution of market shares and profits across this range. Hence, two starting firms, each in a different country, are given a one-time opportunity with probability $\beta$ to consider a cross-border M&A. These firms take the macroeconomic environment, as summarized by the constant $B$, as given and determine if an M&A is viable and in their best interest. We use $h$ to denote the most productive firm and $l$ to denote the least productive firm, such that $\varphi_h \geq \varphi_l$ and $\pi_h \geq \pi_l$, where $\pi_h$ is short notation for $\pi_{\varphi_h}$ and similarly for $\pi_l$.

There are three logical combinations of firms engaging in an M&A, when ignoring the empty set of non-viable firms (and realizing that exporters are more productive than non-exporters):
1. Both firms are domestic firms: \( \varphi_* \leq \varphi_l \leq \varphi_h \leq \varphi_\lambda \).
2. The high-productive firm exports, but the other firm does not: \( \varphi_* \leq \varphi_l \leq \varphi_\lambda \leq \varphi_h \).
3. Both firms are exporting firms: \( \varphi_\lambda \leq \varphi_l \leq \varphi_h \).

The analysis below on the \textit{viability} of M&As needs to distinguish between the three cases even though the \textit{profit function} of the merged firm if the M&A is viable does not (see Proposition 1).

Note that in the Melitz model both firms produce a unique variety. They can develop and supply this unique variety because they already paid the fixed entry costs \( f_{en} \). Both firms are viable and generate a positive income stream from supplying their unique variety. In this setting, it is not profitable to stop producing either variety after the M&A, so we assume that the merged firm continues to produce both varieties (we thus analyse multi-product firms in a simple way). We assume that the merger involves a fixed per period merger cost \( f_m \geq 0 \) (defined in the same way as the export fixed cost \( f_\lambda \)).

3.3 \textit{Productivity and Profits}

We focus on the benefits of M&As through the (partial) transfer of knowledge from the high-productive firm to the low-productive firm by positing a new firm productivity (both at home and abroad) equal to the geometric mean of the pre-merger productivities (\( m \) for merged):

\begin{equation}
\varphi_m = \varphi_h^\omega \varphi_l^{1-\omega}.
\end{equation}

The merged firm productivity is maximal if \( \omega = 1 \) (with complete transfer of knowledge and minimal if \( \omega = 0 \) (with reverse transfer of knowledge). Our regression equation in section 2 follows this specification, and the results suggest that \( \omega \) is about 2/3. An \( \omega \) smaller than 1 suggests that post-merger firm performance is impaired if the low-productive partner is weaker. This observation is also borne out by evidence in King et al. (2004), Moeller et al. (2005), and McCarthy (2011).

The merged firm has two production locations, one at home and one abroad. The fixed cost for producing one variety at a location is \( f \), while the fixed cost of producing two varieties is \( (1+\alpha)f \), where \( \alpha \in [0,1] \). When \( \alpha = 1 \) there is no additional synergy from producing two different varieties within a single firm, while if \( \alpha = 0 \) two different varieties can be produced at no additional fixed costs. The merged firm thus can provide a variety locally at the additional fixed cost \( \alpha f \) and avoid the transport costs \( \tau \). We denote the merged firm profits by \( \pi_m \).

\textit{Proposition 1 (merged firm profits)}

A merged firm will always supply both varieties locally (not through exports) and earn per period profits \( \pi_m = 4B\varphi_m^{\epsilon-1} - 2(1+\alpha)f - f_m \).

We proceed in three steps. First, we note that it is profitable to supply both varieties domestically in each market since \( B\varphi_m^{\epsilon-1} - f \geq B\varphi_l^{\epsilon-1} - f \geq 0 \), where the first inequality arises because \( \varphi_m^{\epsilon-1} \geq \varphi_l^{\epsilon-1} \) and the second inequality because \( \varphi_l^{\epsilon-1} \geq \varphi_* \). Second, we note that it must be profitable to supply both varieties in the (initially) foreign market through local production since \( B\varphi_m^{\epsilon-1} - \alpha f \geq B\varphi_m^{\epsilon-1} - f \geq 0 \), as shown in step one. Third, we note that (post-merger) the
local production of the (pre-merger) foreign variety is even more profitable than of the domestic variety, which in turn is more profitable than exporting (because of the assumption \( f_x > \tau^{1-\varepsilon} f \), which ensures that there are both domestic and exporting firms active on the market). As a consequence, the merged firm will not export to the other market, and charges the same price for both varieties in both markets. As both varieties are sold in two markets, the total profits are equal to four times operating profits per variety sold in either market, minus total fixed cost.\(^{10}\)

### 4 M&A Viability

In this section, we analyse the viability of M&As (for given parameter values) in four steps.\(^{11}\) In sub-section 4.1 we introduce the M&A value function. In sub-section 4.2 we analyse the viability of a special case, namely symmetric (or identical) firm M&As. In sub-section 4.3 we illustrate and discuss the M&A value function for some numerical examples to get a better feel for the implications of the model. Finally, sub-section 4.4 analyses the viability of asymmetric M&As.

#### 4.1 M&A Value Function

Let us define \( \vartheta \) as the M&A value function: \( \vartheta(\varphi_h^{\varepsilon-1}, \varphi_l^{\varepsilon-1}|B, f, f_x, \tau^{1-\varepsilon}, \omega, \alpha, f_m) \equiv \pi_m - (\pi_h + \pi_l) \), which is equal to the total profits of the merged firm minus the stand-alone profits of the \( \varphi_l \) firm and \( \varphi_h \) firm. If \( \vartheta \geq 0 \) the two firms will merge because the match is viable, otherwise the merger will not materialize. Note that this set-up abstracts from any questions about how the firms distribute excess profits; if an M&A is viable the owners of the participating firms can be compensated such that they are at least as well off as before.

We determine the value of M&As for each of the three logical firm combinations.

- **Possibility 1:** both firms are domestic firms (\( \varphi_s \leq \varphi_l \leq \varphi_h \leq \varphi_x \))

  If both firms are domestic firms, the total pre-merger profits are: \( B(\varphi_h^{\varepsilon-1} + \varphi_l^{\varepsilon-1}) - 2f \). If the two firms merge, total profits are \( 4B\varphi_m^{\varepsilon-1} - 2(1 + \alpha)f - f_m \). The merger is thus attractive if:

  \[
  \vartheta(.) \equiv (4B\varphi_m^{\varepsilon-1} - 2(1 + \alpha)f - f_m) - (B(\varphi_h^{\varepsilon-1} + \varphi_l^{\varepsilon-1}) - 2f) \geq 0
  \]

- **Possibility 2:** the \( \varphi_h \) firm exports, but the \( \varphi_l \) firm does not (\( \varphi_s \leq \varphi_l \leq \varphi_x \leq \varphi_h \))

  In this case total pre-merger profits are: \( B((1 + \tau^{1-\varepsilon})\varphi_h^{\varepsilon-1} + \varphi_l^{\varepsilon-1}) - 2f - f_x \). If the two firms merge, total profits are \( 4B\varphi_m^{\varepsilon-1} - 2(1 + \alpha)f - f_m \). The merger is thus attractive if:

  \[
  \vartheta(.) \equiv (4B\varphi_m^{\varepsilon-1} - 2(1 + \alpha)f - f_m) - (B((1 + \tau^{1-\varepsilon})\varphi_h^{\varepsilon-1} + \varphi_l^{\varepsilon-1}) - 2f - f_x) \geq 0
  \]

- **Possibility 3:** both firms are exporting firms (\( \varphi_s \leq \varphi_l \leq \varphi_h \))

  In this case total pre-merger profits are: \( B(1 + \tau^{1-\varepsilon})(\varphi_h^{\varepsilon-1} + \varphi_l^{\varepsilon-1}) - 2f - 2f_x \). If the two firms merge, total profits are \( 4B\varphi_m^{\varepsilon-1} - 2(1 + \alpha)f - f_m \). The merger is thus attractive if:

\(^{10}\) The model abstracts from domestic M&As. Note, however, that proposition 1 implies that if an M&A is profitable, cross-border M&As would yield higher profits than domestic M&As. In the latter case the decision would depend on \( \pi_a = 2B\varphi_m^{\varepsilon-1} - 2(1 + \alpha)f - f_m \).

\(^{11}\) Note, that we do not analyse a second, or third, round of M&As. The analysis would be the same as presented in this section and would answer the question: can a M&A be viable for a given set of firms?
The potential benefits of a merger stem from three sources. As an example, consider the merger value function for two exporting firms in equation (11).

- First, there is a potential synergy in the fixed costs; the term \(-2(1 + \alpha)f - f_m\) represents potential savings in fixed costs for the combined firm, relative to the sum of fixed costs of the individual firms.

- Second, there is a transport cost saving motive; the term \(4B\phi_{m}^{\epsilon-1} - B(1 + \tau^{1-\epsilon})(\phi_{h}^{\epsilon-1} + \phi_{i}^{\epsilon-1})\) equals the change in operating profits. For example, if we abstract from any technological benefits by setting \(\phi_{h}^{\epsilon-1} = \phi_{i}^{\epsilon-1} = \phi_{m}^{\epsilon-1}\), the implied change in operating profits is: \(2B\phi_{m}^{\epsilon-1}(2 - (1 + \tau^{1-\epsilon}))\), which increases with rising transport costs. The reason is simple: the two merged firms avoid transport costs by producing locally. Note that transport costs savings are larger for domestic-domestic mergers (in which exporting was prohibitive for both firms) or domestic-exporter mergers (in which exporting was prohibitive for one of the two firms).

- Third, there is a potential benefit of knowledge transfers. Suppose, for the sake of argument, that transport costs are zero. The change in combined operating profits in that case is: \(4B(\phi_{m}^{\epsilon-1} - (\phi_{h}^{\epsilon-1} + \phi_{i}^{\epsilon-1})/2)\). If the post-merger (exponentiated) productivity of the firm is higher than the arithmetic average of the (exponentiated) productivities of its two partners, operating profits improve after the merger. Clearly, if knowledge transfers are perfect, \(\phi_{m}^{\epsilon-1}\) is equal to the top productivity of the two partners, and the mixing of technologies in the merger is always profit-increasing. When knowledge transfers are imperfect, this is less clear. This third source of benefits in (imperfect) knowledge transfers is key to our story: unlike the transport costs savings or fixed costs synergies in FDI, knowledge transfers make firms develop preferences about the characteristics of partner firms for M&As.

4.2 A Special Case: M&As between Identical Firms: \(\phi_{h}^{\epsilon-1} = \phi_{i}^{\epsilon-1}\)

In order to characterize the viability of M&As across all firms, it turns out to be helpful to first analyse the special case of identical firm M&As. If both firms have the same productivity level, then the merged firm’s productivity is \(\phi_{m}^{\epsilon-1} = \phi_{h}^{\epsilon-1} = \phi_{i}^{\epsilon-1}\), irrespective of the knowledge transfer \(\omega\). Using the merger value function for when both firms are domestic firms or both firms are exporting firms (equations 9 and 11), we get:

\[
\vartheta(\phi_{h}^{\epsilon-1}, \phi_{i}^{\epsilon-1}|B, f, f_x, \tau^{1-\epsilon}, \omega, \alpha, f_m) = \begin{cases} 
2B\phi_{h}^{\epsilon-1} - 2\alpha f - f_m & , \phi_{x}^{\epsilon-1} \leq \phi_{h}^{\epsilon-1} < \phi_{x_i}^{\epsilon-1} \\
2(1 - \tau^{1-\epsilon})B\phi_{h}^{\epsilon-1} + 2f_x - 2\alpha f - f_m & , \phi_{x_i}^{\epsilon-1} \leq \phi_{h}^{\epsilon-1}
\end{cases}
\]

The M&A value function above is clearly rising in \(\phi_{h}^{\epsilon-1}\) over the entire domain (since \(\tau^{1-\epsilon} < 1\) and continuous at \(\phi_{x_i}^{\epsilon-1}\) (since \(B\phi_{x_i}^{\epsilon-1} = f_x\)). This implies that two identical firms will merge once firm productivity exceeds a threshold level \(\phi_{m}^{\epsilon-1}\). A sufficient condition for all identical firms to merge is thus provided if the M&A value function is non-negative for \(\phi_{x}^{\epsilon-1}\). Since
$B \varphi_{m}^{\varepsilon-1} = f$, a sufficient condition is given if the merger costs are sufficiently small: $f_m \leq 2(1 - \alpha)f$. This is illustrated in Figure 2 for different merger costs $f_m$ and an example parameter configuration. Note that the value of $\varphi_{m}^{\varepsilon-1}$ is given by:

$$\varphi_{m}^{\varepsilon-1} = \begin{cases} \max \left\{ \varphi_{m}^{\varepsilon-1}, \frac{2\alpha f + f_m}{2B} \right\}, & \text{if } \frac{2\alpha f + f_m}{2B} \leq \varphi_{x}^{\varepsilon-1} \\ \frac{2\alpha f + f_m - 2f_x}{2(1-\tau^{1-\varepsilon})B}, & \text{otherwise} \end{cases}$$

(13)

Figure 2 M&A value as a function of productivity if $\varphi_{m}^{\varepsilon-1} = \varphi_{l}^{\varepsilon-1}$ for different merger costs

Note: $B = 3; f = 1; f_x = 1.4; \tau^{1-\varepsilon} = 0.8; \alpha = 0.6$

**Proposition 2 (symmetric / identical firm mergers)**

The M&A value for two identical firms is non-negative if their productivity exceeds a threshold value $\varphi_{l}^{\varepsilon-1} = \varphi_{h}^{\varepsilon-1} \geq \varphi_{m}^{\varepsilon-1}$. The threshold value is independent of the knowledge transfer parameter $\omega$. If merger costs are ‘sufficiently small’ (namely if $f_m \leq 2(1 - \alpha)f$), then all mergers by identical firms are viable: $\varphi_{m}^{\varepsilon-1} = \varphi_{x}^{\varepsilon-1}$.

4.3 Some Examples

To get a better feel for the main implications of the model regarding M&A viability, it is useful to depict and briefly discuss the M&A value function for some examples. We do so in two steps. First, we show in Figure 3 how the M&A value function changes for different knowledge transfers as a function of $\varphi_l$, the low-productivity partner’s productivity level (and thus for given $\varphi_h$). Second, we show in Figure 4 how the M&A value function changes for different knowledge transfers as a function of $\varphi_h$, the high-productivity partner’s productivity level (and thus for given $\varphi_l$). We round off this discussion with some observations regarding the behaviour of the M&A viability function as a function of high-productivity $\varphi_h$ for given low-productivity $\varphi_l$.

Figure 3 illustrates the M&A value function as the low-productivity level changes, given the level of high-productivity and under the assumption that the high-productivity firm is able to
export. The figure illustrates the M&A value function for different knowledge transfers, where \( \omega \in \{0; 1/4; 1/2; 3/4; 1\} \). The low-productivity level must be at least equal to the viability cut-off and can be at most equal to the high-productivity level \( \varphi_h \). As the low-productivity level reaches the export viability cut-off, the M&A value function is continuous, but not differentiable as the low-productivity firm switches regime as a stand-alone firm (the kinks in the curves in Figure 3). The figure illustrates that all M&A value curves converge to the same point as low-productivity approaches the high-productivity, irrespective of the degree of knowledge transfers. The figure also illustrates, as is easily verified, that the M&A value rises as the knowledge transfer parameter \( \omega \) rises (since higher \( \omega \) means higher merged firm productivity \( \varphi_m^{\varepsilon-1} \), which implies higher merged firm profits and a rise in \( \vartheta \)). We can formulate the following implications.

*Figure 3 M&A value function as low-productivity changes for different knowledge transfers*

\[ \varphi_h^{\varepsilon-1} = 2; \ B = 3; \ f = 1; \ f_x = 1.4; \ \tau^{1-\varepsilon} = 0.8; \ \alpha = 1; \ f_m = 2 \]

**Implication 1:** For a firm with a given productivity level the probability of an M&A match tends to decrease if the productivity difference (either up or down) rises (see Proposition 3 for details).

**Implication 2:** The viability of a match increases with the quality of knowledge transfers; the range of viable productivity differences rises with \( \omega \).

Based on Proposition 2, we now take a closer look at the M&A value function as the high-productivity level \( \varphi_h \) changes, for a given level of low-productivity \( \varphi_l \) and different knowledge transfers. This is illustrated in Figure 4 for two situations, labelled \( A \) and \( B \), for knowledge transfers \( \omega \in \{0; 1/4; 1/2; 3/4; 1\} \) with ‘large’ merger costs (such that \( \varphi_l^{\varepsilon-1} < \varphi_m^{\varepsilon-1} \)).

In situation \( A \) (panel a), the low-productivity level is equal to the viability level: \( \varphi_l^{\varepsilon-1} = \varphi_h^{\varepsilon-1} \). In this case, the M&A value for identical firms is negative (irrespective of knowledge transfers \( \omega \)), as indicated by point \( A \) in Figure 4a. As the high-productivity level rises, the M&A value
may rise and become positive if knowledge transfers $\omega$ are sufficiently high, as indicated by the graphs for $\omega = 1$ and $\omega = 3/4$ in the figure. Otherwise the M&A value function will remain negative irrespective of the high-productivity level. M&As may thus be viable if high-productivity $\phi_h$ exceeds a threshold level (and possibly up to some point), depending on the knowledge transfers (see below for details on this remark).

**Figure 4 M&A value function as high-productivity changes for different knowledge transfers**

Note: $B = 3$; $f = 1$; $f_x = 1.4$; $\tau^{1-\varepsilon} = 0.8$; $\alpha = 0.8$; $f_m = 1$; panel a: $\phi^{\varepsilon-1}_l = 1/3$; panel b: $\phi^{\varepsilon-1}_l = 1/2$.

In situation $B$ (panel $b$), the low-productivity level is higher than the threshold value for identical firms (but lower than the export cut-off): $\phi^{\varepsilon-1}_l < \phi^{\varepsilon-1}_m < \phi^{\varepsilon-1}_x$. In this case, the M&A value for identical firms is positive (irrespective of knowledge transfers $\omega$), as indicated by point $B$ in Figure 4b. As the high-productivity level rises, the M&A value will continue to be positive, unless knowledge transfers $\omega$ are sufficiently low (see again below for details), as indicated by the graphs for $\omega = 0$, $\omega = 1/4$, and $\omega = 1/2$ in the figure. In these cases, M&As are only viable if the high-productivity does not exceed a threshold level.

Since $\phi_m = \phi_h^\omega \phi_l^{1-\omega}$ we have $\frac{\partial \phi_l^{\varepsilon-1}}{\partial \phi_h^{\varepsilon-1}} = \frac{\omega \phi_l^{1-\varepsilon}}{\phi_h^{1-\varepsilon}}$, which implies $\frac{\partial \pi_m}{\partial \phi_h^{\varepsilon-1}} = \frac{\omega A B \phi_l^{\varepsilon-1}}{\phi_h^{\varepsilon-1}}$. Similarly,

$$\frac{\partial \pi_h}{\partial \phi_h^{\varepsilon-1}} = B$$

if $\phi_* \leq \phi_h \leq \phi_x$ and

$$\frac{\partial \pi_h}{\partial \phi_h^{\varepsilon-1}} = B(1 + \tau^{1-\varepsilon})$$

if $\phi_h \geq \phi_x$. We thus have:

$$\frac{\partial \phi_h^{\varepsilon-1}}{\partial \phi_h^{\varepsilon-1}} = \begin{cases} \omega A B \phi_m^{\varepsilon-1} / \phi_h^{\varepsilon-1} - B, & \phi_* \leq \phi_h \leq \phi_x \\ \omega A B \phi_m^{\varepsilon-1} / \phi_h^{\varepsilon-1} - B(1 + \tau^{1-\varepsilon}), & \phi_h \geq \phi_x \end{cases}$$

We analyse three possible values for the knowledge transfer parameter $\omega$.

a. If we have reverse knowledge transfers ($\omega = 0$), then the first term on the right-hand-side of equation (14) is zero and the M&A value function is monotonically declining in the level of high-productivity.
b. If we have complete knowledge transfers ($\omega = 1$) the first term on the right-hand-side of equation (14) is $4B$ and the M&A value function is monotonically rising in the level of high-productivity.

c. For intermediate values of the knowledge transfer parameter ($0 < \omega < 1$) the first term on the right-hand-side of equation (14) can be written as: $\omega 4B \varphi_l^{(e-1)(1-\omega)} / \varphi_h^{(e-1)(1-\omega)}$, which is monotonically declining in $\varphi_h^{e-1}$. Note that this term is equal to $\omega 4B$ for identical firms ($\varphi_h^{(e-1)} = \varphi_l^{(e-1)}$), which is lower than $B$ (if $\varphi_l < \varphi_{ex}$), respectively lower than $B(1 + \tau_1^{-\epsilon})$ (if $\varphi_l \geq \varphi_{ex}$), if $\omega$ is sufficiently small, in which case the M&A value function is monotonically declining. In all other cases the M&A value function is first rising and then declining in high-productivity. Finally, we note that under these circumstances:

$$
\lim_{\varphi_h^{e-1} \to \infty} \varphi_h^{e-1} \equiv \lim_{\varphi_h^{e-1} \to \infty} (\pi_m - (\pi_h + \pi_l)) = \lim_{\varphi_h^{e-1} \to \infty} B(4 \varphi_m^{e-1} - (1 + \tau_1^{-\epsilon}) \varphi_h^{e-1}) =
$$

$$
= \lim_{\varphi_h^{e-1} \to \infty} B(4 \varphi_l^{(e-1)(1-\omega)} \varphi_h^{(e-1)\omega} - (1 + \tau_1^{-\epsilon}) \varphi_h^{e-1}) = -\infty, \text{ implying that the M&A value function becomes negative for sufficiently large productivity of the high-productive partner.}
$$

4.4 Asymmetric M&As

The above three cases $a,b,c$ in combination with Proposition 2 allow us to completely characterize the productivity range in which mergers will take place. In case $b$, with complete knowledge transfer $\omega = 1$, the merger is viable if the high-productivity level is ‘sufficiently large’. In all other cases, the merger is not viable if the high-productivity level is ‘sufficiently large’. Note that there is always a range of viable high productivities if the low-productivity exceeds the threshold level for identical firms: $\varphi_l^{e-1} \geq \varphi_{e-1}$. Given the low-productivity level $\varphi_l^{e-1}$, we can define:

$$
\varphi_{m,L}^{e-1} \equiv \inf\{\varphi_{h}^{e-1} | \vartheta(\varphi_{h}^{e-1}, \varphi_{l}^{e-1}|B, f, f_{xx}, \tau_1^{-\epsilon}, \omega, \alpha, f_m) \geq 0\} \text{ and }
$$

$$
\varphi_{m,H}^{e-1} \equiv \sup\{\varphi_{h}^{e-1} | \vartheta(\varphi_{h}^{e-1}, \varphi_{l}^{e-1}|B, f, f_{xx}, \tau_1^{-\epsilon}, \omega, \alpha, f_m) \geq 0\}
$$

The above discussion shows that, for given $\varphi_l^{e-1}$, the set of high-productivities where M&As are viable is given by the interval $I_{\varphi_l} \equiv [\varphi_{m,L}^{e-1}, \varphi_{m,H}^{e-1}]$, with the properties summarized below.\(^\otimes\)

Proposition 3 (range of productivity for viable mergers)

Suppose two firms with productivities $\varphi_{l}^{e-1} \leq \varphi_{h}^{e-1} \leq \varphi_{e-1}^{e-1}$ are given the opportunity to merge. Given $\varphi_{l}^{e-1}$, the merger is viable if $\varphi_{h}^{e-1} \in I_{\varphi_l} \equiv [\varphi_{m,L}^{e-1}, \varphi_{m,H}^{e-1}]$, with the following characteristics:

- If there are complete knowledge transfers ($\omega = 1$), we have $\varphi_{m,H}^{e-1} = \infty$ and $I_{\varphi_l} = \emptyset$.
- If $\varphi_l$ is sufficiently large ($\varphi_l^{e-1} \geq \varphi_{e-1}^{e-1}$), we have $\varphi_{m,L}^{e-1} = \varphi_{l}^{e-1}$ and $I_{\varphi_l} = \emptyset$.
- If there are imperfect knowledge transfers ($\omega < 1$), we have $\varphi_{m,H}^{e-1} < \infty$.
- If there are reverse knowledge transfers ($\omega = 0$), we have $I_{\varphi_l} = \emptyset$ if $\varphi_{l}^{e-1} < \varphi_{e-1}^{e-1}$.

\(^\otimes\) Note, that these ranges can (but do not have to) include the export cut-off value.
The intuition behind the proposition is that with imperfect knowledge transfers, the productivity range for which a firm is willing to consider a merger is restricted. For a very productive firm, a merger with an unproductive firm implies a dilution of its productivity in the merger.

Implication 3: Merger viability decreases for sufficiently high-productivity firms when knowledge transfers are not perfect.

The proposition is illustrated in Figure 5 for a range of productivities for two firms that are given the opportunity to consider a merger. The firm from country 1 has productivity \( \varphi_1^{\varepsilon-1} \) and the firm from country 2 has productivity \( \varphi_2^{\varepsilon-1} \). Above the diagonal firm 1 is the low-productivity firm \( (\varphi_1^{\varepsilon-1} = \varphi_l^{\varepsilon-1}) \) and below the diagonal firm 2 is the low-productivity firm \( (\varphi_2^{\varepsilon-1} = \varphi_l^{\varepsilon-1}) \). The diagonal and the various cut-off levels divide this ‘spider’ figure in different parts, identified by dotted lines. Both productivities must be at least equal to the viability cut-off \( \varphi_\varepsilon^{\varepsilon-1} \) and may be higher or lower than the export cut-off \( \varphi_x^{\varepsilon-1} \). The threshold level for identical firm mergers \( \varphi_\varepsilon^{\varepsilon-1} \) (denoted identical in the figure, identified by dashed lines) gives rise to point \( M \) in the figure and plays a crucial role in our analysis and discussion, see also Proposition 2. In Figure 5 we assume this threshold to be in between the viability and export cut-offs: \( \varphi_x^{\varepsilon-1} < \varphi_\varepsilon^{\varepsilon-1} < \varphi_x^{\varepsilon-1} \), but this need not be the case (see below). The solid lines in the figure identify lower and upper bounds for the merger viability intervals summarized in Proposition 3. More specifically, for \( \omega = 1, \omega = 0.7, \) and \( \omega = 0.6 \) the lines indicate the lower bound of the merger interval \( \varphi_{m,L}^{\varepsilon-1} \), while for \( \omega = 0 \) and \( \omega = 0.2 \) the lines indicate the upper bound of the merger interval \( \varphi_{m,H}^{\varepsilon-1} \). For \( \omega = 0.45 \) the lower bound is indicated by low and the upper bound by up.
Figure 5 M&A viability for different productivities and different knowledge transfers

Note: $B = 3$; $f = 1$; $f_k = 1.4$; $\tau^{1-\varepsilon} = 0.8$; $f_m = 1.4$; $\alpha = 0.9$; if $m =$identical firm merger cut-off level; for $\omega = 1$, $\omega = 0.7$, and $\omega = 0.6$ the lines indicate the lower bound of the merger interval $\varphi_{m,L}^{\xi-1}$, while for $\omega = 0$ and $\omega = 0.2$ the lines indicate the upper bound of the merger interval $\varphi_{m,U}^{\xi-1}$; for $\omega = 0.45$ this is indicated by low and up.

As Figure 5 illustrates, whether a combination of firm productivities $(\varphi_1^{\xi-1}, \varphi_2^{\xi-1})$ gives rise to a viable merger depends mainly on the extent of knowledge transfers as measured by the parameter $\omega$, with two exceptions. The two exceptions are labelled never and always. In the never area M&As are not viable, independently of $\omega$. Similarly, in the always area, M&As are viable, independently of $\omega$. For all other combinations of firm productivities $(\varphi_1^{\xi-1}, \varphi_2^{\xi-1})$ M&As are only viable if the knowledge transfer parameter $\omega$ is sufficiently large. How large the knowledge transfer needs to be, depends on the specific combination of productivities under consideration. Note that the spider’s ‘body’ (point $M$) moves up or down the diagonal as $\varphi_{m}^{\xi-1}$ rises or falls. If, for example, mergers become less attractive because the associated cost as measured by $\alpha$ or $f_m$ rises, then the point $M$ moves up the diagonal and the area never becomes larger while the area always becomes smaller. The opposite occurs if $\alpha$ or $f_m$ falls.

To understand the role of productivity transfers, it may be instructive to consider the merger value function informally under the assumption that transfer is perfect – the first point of Proposition 3. The value function in that case ($\omega = 1$), taking a pair of exporters as an example, is:

$$\vartheta(.) \equiv (4B \varphi_h^{\xi-1} - 2(1 + \alpha)f - f_m) - (B(1 + \tau^{1-\varepsilon})(\varphi_{1}^{\xi-1} + \varphi_{2}^{\xi-1}) - 2f - 2f_x)$$
\[ B((3 - \tau^1)\varphi_h^{\varepsilon - 1} - (1 + \tau^1)\varphi_l^{\varepsilon - 1}) - 2(1 + \alpha)f - f_m + 2f + 2f_x \]

The term \(-2(1 + \alpha)f - f_m + 2f + 2f_x\) is simply the fixed costs difference between the merged and unmerged firms; and \(B((3 - \tau^1)\varphi_h^{\varepsilon - 1} - (1 + \tau^1)\varphi_l^{\varepsilon - 1})\) is the difference in operating profits between the merged and unmerged firms. For a merger to be viable, the operating profit difference needs to compensate the fixed costs difference. If productivity transfers are perfect, the operating profit difference between the merged and unmerged firms increases in the productivity of the most productive unmerged firm, and decreases in the productivity of the unmerged firm. Hence, the condition for merger viability is higher between combinations of very productive firms with very unproductive firms. The intuition is that in such mergers, it is not costly to drop the production technology of the least productive variety. This intuition contrasts starkly with imperfect transfers (\(\omega < 1\)), in which productivity differences reduce the value of the merger.

The model we developed so far has determined the productivity ranges for which M&As are viable. This suggests that giving an economy the opportunity to engage in M&As increases welfare. \textit{A priori}, this is not obvious as with imperfect knowledge transfers the average productivity of the new firm is less than that of the most productive pre-merger firm, and it is not always the case that a newly merged firm – which serves both markets – is a substitute for exporting as is the case with FDI in the Helpman et al. (2004) model; non-exporting firms can engage in viable M&A matches. This raises the question; why should an economy allow M&As? We now turn to the economy-wide consequences of M&As.

5 The Impact of M&As

To determine the economic impact of M&As we proceed in three steps. First, we determine the probability of a viable M&A in sub-section 5.1. Second, we determine the distribution of viable matches in the economy in sub-section 5.2. Third, we determine how this set of viable M&As affects the economy-wide cut-off viability (and hence economic welfare) in sub-section 5.3.

5.1 Probability of M&As

After determining which firm pairs wish to merge, we can now aggregate the successful mergers over the population of firms to study aggregated M&A flows. As in Melitz (2003), firms draw their productivity parameter \(\varphi\) from a common distribution \(g(\varphi)\), which has positive support over \((0, \infty)\) and a continuous cumulative distribution \(G(\varphi)\). With \(\varphi_*\) as the viability cut-off productivity, the productivity distribution conditional upon entry is \(\mu(\varphi) \equiv g(\varphi)/(1 - G(\varphi_*))\) for \(\varphi \geq \varphi_*\) and zero otherwise. Entering firms get the opportunity with probability \(\beta \in [0,1]\) to engage in a possible merger with a randomly chosen entering firm from the other country. The resulting joint distribution is thus \(\mu(\varphi_1)\mu(\varphi_2)\), where \(\varphi_i \geq \varphi_*\) for \(i = 1,2\).
Figure 6 Determining the probability of viable M&As

Note: $B = 3; f = 1; f_3 = 1.4; \tau^{1-\varepsilon} = 0.8; f_m = 1.4; \alpha = 0.9; \omega = 0.45$; the solid line indicates the upper bound of the merger interval $\varphi_{m,H}^{\varepsilon-1}$ and the dashed line the lower bound of the merger interval $\varphi_{m,L}^{\varepsilon-1}$; the shaded area indicates the range of viable M&As; at point A we have $\varphi_{m,L}^{\varepsilon-1} = \varphi_{m,H}^{\varepsilon-1}; \overline{\varphi}_a = 0.61$.

We want to determine the probability $p(\cdot)$ for a firm with productivity $\varphi$ of engaging in a viable merger, if given the opportunity to do so. Figure 6 helps us to understand this probability by repeating Figure 5 for a specific value of knowledge transfers ($\omega = 0.45$). The shaded area shows the range of viable M&As, as determined by $\varphi_{m,L}^{\varepsilon-1}$ and $\varphi_{m,H}^{\varepsilon-1}$ and explained in section 4.4.

We start by determining a minimum productivity level $\varphi_{min}$ for M&A viability. It is equal to the low-productivity level for which $\varphi_{m,L}^{\varepsilon-1} = \varphi_{m,H}^{\varepsilon-1}$, as illustrated by point A and its inverse $A_{inv}$ in Figure 6. Note that $\varphi_{min}$ may be equal to $\varphi_*$, namely if $\varphi_{m,L}^{\varepsilon-1} < \varphi_{m,H}^{\varepsilon-1}$ at $\varphi_*$, in which case all firms have a positive probability of engaging in a viable M&A. If not, then for all firms with a productivity between $\varphi_*$ and $\varphi_{min}$ the chance of a viable M&A merger is zero. Also note that $\varphi_{min}$ cannot be above the threshold level $\varphi_m$ for identical firm mergers, such that: $\varphi_* \leq \varphi_{min} \leq \varphi_m$. This is caused by the fact that point A coincides with point $M$ in Figure 6 if $\omega$ is

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$13$ Note that point A coincides with the export cut-off value in Figure 6. Given the other parameters of the figure, this happens for a range of knowledge transfers $\omega$ because of the kink at the export cut-off level. For different values of $\omega$ (such as $\omega = 0.55$) point A is not equal to the export cut-off value.
sufficiently small (for examples: see $\omega = 0$ and $\omega = 0.2$ in Figure 5; see also Propositions 2 and 3). Figure 6 gives an example with strict inequalities: $\varphi_+ < \varphi_{\text{min}} < \varphi_{\text{m}}$.

To determine the probability $p(.)$ we look at a vertical line (not drawn) for a given value of $\varphi_1$ (and thus of $\varphi_1^{\text{inv}}$) relative to the shaded M&A viability area.\(^{14}\)

1. If $\varphi_+ \leq \varphi_1 < \varphi_{\text{min}}$, then $p(\varphi_1) = 0$, as already explained above.
2. If $\varphi_{\text{min}} \leq \varphi_1 < \varphi_+$, then viability is in the range from $\varphi_{\text{m},L}$ to $\varphi_{\text{m},H}$ in Figure 6, which implies that the probability of a viable merger is equal to: $p(\varphi_1) = \int_{\varphi_{\text{m},L}(\varphi_1)}^{\varphi_{\text{m},H}(\varphi_1)} \mu(\varphi) d\varphi$.
3. If $\varphi_{\text{m}} \leq \varphi_1 < \varphi_{\text{min}}$, then viability is in the range from the inverse of $\varphi_{\text{m},L}$ (the diagonal reflection, which we will denote $\varphi_{\text{m},L}^{\text{inv}}$) to $\varphi_{\text{m},H}$ in Figure 6: $p(\varphi_1) = \int_{\varphi_{\text{m},L}(\varphi_1)}^{\varphi_{\text{m},H}(\varphi_1)} \mu(\varphi) d\varphi$.
4. If $\varphi_{\text{m}} \leq \varphi_1$, then viability is in the range from the inverse of $\varphi_{\text{m},H}$ (which we will denote $\varphi_{\text{m},H}^{\text{inv}}$) to $\varphi_{\text{m},H}$ in Figure 6: $p(\varphi_1) = \int_{\varphi_{\text{m},H}(\varphi_1)}^{\varphi_{\text{m},H}(\varphi_1)} \mu(\varphi) d\varphi$.

Note that (i) range 1 disappears if $\varphi_+ = \varphi_{\text{min}}$, (ii) ranges 2 and 3 disappear if $\varphi_{\text{min}} = \varphi_+$, and (iii) ranges 1, 2, and 3 disappear if $\varphi_+ = \varphi_{\text{min}} = \varphi_+$. Range 4 never disappears. It is clear from points 1-4 above that $p(\varphi)$ may either rise or fall as $\varphi$ rises.

The four ranges describe the probability of a viable M&A for any productivity level.

Consequently, the economy-wide probability of a viable M&A is equal to the firm density at a productivity level, multiplied by the probability that a firm of that productivity merges. Hence, the economy-wide probability $\bar{p}$ of a viable M&A for the fraction of firms given the opportunity for a merger is $\bar{p} = \int_{\varphi_+}^{\infty} p(\varphi) \mu(\varphi) d\varphi$. The fraction of firms entering the market that merges with another firm is thus $\beta \bar{p}$.

**Implication 4:** If knowledge transfers are not perfect, the absolute productivity of each of the matching firms cannot be too small for M&As to be viable; the lower bound is binding.

5.2 The Distribution of Viable M&As

The productivity distribution of merged firms may look different from the general productivity distribution. That is instrumental in the prediction of how mergers and acquisitions affect average productivity in the economy and as a consequence affects welfare. To describe the productivity distribution of post-M&A firms, we reason backwards: for every post-merger productivity level, we construct the set of consistent pre-merger productivity combinations. Given the probabilities of viable M&As for every pre-merger productivity level derived above, we characterize the probability distribution for every post-merger productivity outcome.

First, we collect all pre-merger productivity combinations that lead to $\varphi_\text{m}$. Since $\varphi_\text{m} = \varphi_\text{h} \varphi_1^{1-\omega}$ (see equation 8) and both $\varphi_\text{h} \geq \varphi_+$ and $\varphi_1 \geq \varphi_+$, we know that $\varphi_\text{m} \geq \varphi_+$. Obviously, different

\(^{14}\) We do not draw these lines to avoid cluttering the diagram.
combinations of $\varphi_h$ and $\varphi_1$ give rise to the same level of merged firm productivity, such that we can define iso-productivity curves, $\bar{\varphi}_m$ say, in $(\varphi_h, \varphi_1)$-space. These translate directly to $(\varphi_h^{e-1}, \varphi_1^{e-1})$-space since $\bar{\varphi}_m^{e-1} = (\varphi_h^{e-1})^\omega (\varphi_1^{e-1})^{1-\omega}$, as illustrated by the $\bar{\varphi}_m \bar{\varphi}_m$ curve in Figure 6 by also taking into consideration which firm has the highest productivity, hence the kink in the $\bar{\varphi}_m \bar{\varphi}_m$ curve at the diagonal (the curve can flex either way depending on $\omega$, and it has a kink for all values of $\omega$, except for $\omega = 0.5$). To continue the explanation based on Figure 6, define the set $X_{m^{\&}a}$ as given in equation (15), and the set $X_{\varphi_m}$ as given in equation (16). The set $X_{m^{\&}a}$ collects all combinations of $\varphi_1$ and $\varphi_2$ above the diagonal (with $\varphi_2 \geq \varphi_1$) for which mergers are viable. Taking the $e - 1$ powers of these productivities then gives the shaded M&A viability set above the diagonal in Figure 6. Similarly, the set $X_{\varphi_m}$ determines the sub-set of $X_{m^{\&}a}$ for which the merged firm productivity is equal to $\varphi_m$. This is illustrated in Figure 6 by the intersection of the shaded M&A viability area with the $\bar{\varphi}_m \bar{\varphi}_m$ curve above the diagonal (the thick and solid part of the $\bar{\varphi}_m \bar{\varphi}_m$ curve).

$$X_{m^{\&}a} = \{(\varphi_1, \varphi_2): \theta(\varphi_2^{e-1}, \varphi_1^{e-1}|B, f, f_x, \tau^{1-e}, \omega, \alpha, f_m) \geq 0 \text{ and } \varphi_2 \geq \varphi_1\}$$

$$X_{\varphi_m} = \{(\varphi_1, \varphi_2) \in X_{m^{\&}a}: \varphi_m = \varphi_1^{(1-\omega)/\omega}\}$$

Next, we define the distribution of productivity of post-M&A firms as $h(.)$. Taking the symmetry of Figure 6 into consideration, we need to determine twice the line integral over the set $X_{\varphi_m}$ of the joint distribution $\mu(\varphi_1) \mu(\varphi_2)$. To do so, we first determine $\varphi_2$ as a function of $\varphi_1$ given $\varphi_m$ such that:

$$\varphi_2(\varphi_1|\varphi_m) = \varphi_m^{1/\omega} \varphi_1^{-(1-\omega)/\omega}.$$ 

Using $\varphi_2(\varphi_1|\varphi_m)$ in the joint distribution of productivity, the value of the line integral over the set $X_{\varphi_m}$ is

$$h(\varphi_m) = 2 \int_{X_{\varphi_m}} \mu(\varphi_1) \mu(\varphi_2(\varphi_1|\varphi_m)) d\varphi_1$$

Finally, to describe the productivity distribution conditional on having merged, we note that integrating $h(\varphi_m)$ over all values leads to the economy-wide probability for a firm to engage in a viable merger $\bar{p}$ such that

$$\int_{\varphi_\mu}^{\infty} h(\varphi) d\varphi = \bar{p}.$$ 

The distribution of productivity of successfully merged firms is therefore $h(\varphi)/\bar{p}$.

5.3 M&As and Viability Cut-off

We can now characterize the equilibrium based on Melitz and Redding (2014) and Feenstra (2016), in which there is a competitive fringe of firms that can enter the market by paying a sunk
entry cost \( f_{en} \). Potential entrants face uncertainty about their productivity. Once the sunk entry cost is paid, a firm draws its productivity \( \varphi \) from the distribution \( g(\varphi) \) and determines whether to exit the sector or produce. This decision yields the viability cut-off productivity \( \varphi_* \) at which the potential entrant makes zero profits; this is the free entry condition. One can think of the equilibrium below as the stationary outcome of a dynamic model if the aggregate conditions remain stable over time, as in Melitz (2003). In that case, there is an exogenous probability \( \delta \) per period of time (independent of the productivity level \( \varphi \)) that a firm is hit by a negative shock and forced to exit the market. Firm profit in the conditions below is then replaced by firm value, which is the discounted value of the expected future profits based on probability \( \delta \). Appendix B provides the characterization of the autarky and trade equilibria using this framework, which we consider as a reference case without M&As. Please note that in this setting an increase in the firm viability cut-off implies an increase in welfare (Feenstra, 2016, p. 163).

In an equilibrium in which M&As occur, the profit functions for domestic firms and exporting firms are: \( \pi_{\varphi,d} = B\varphi^{(\epsilon-1)} - f \) and \( \pi_{\varphi,x} = \tau^{1-\epsilon} B\varphi^{(\epsilon-1)} - f_x \). In these expressions, \( B \) is the relevant constant if M&As are possible, defined by \( B = \frac{L^{\epsilon-\epsilon}}{P^{1-\epsilon} (\epsilon-1)^{1-\epsilon}} \), where \( P \) is the M&A price index. The zero profit conditions for domestic and exporting firms are: \( \varphix^{\epsilon-1} = f / B \) and \( \varphi\tau^{\epsilon-1} = \tau^{\epsilon-1} f_x / B \). This implies that \( \varphi_* = \eta \varphi_* \), (where \( \eta > 1 \) is a parametric constant – see Appendix B for the derivation).

When the possibility of M&As is introduced, the free entry condition depends on the expected profits of three types of firms. First, the fraction \( 1 - \beta \) of firms who are not given the opportunity for a possible M&A have domestic and exporting profits as analysed in the trade equilibrium (Appendix B). Second, there is a fraction \( \beta \left(1 - p(\varphi)\right) \) of firms who are given the opportunity to merge, but decide not to do so (see section 5.2). They have (positive) domestic and exporting profits as analysed in the standard trade equilibrium (Appendix B). Third, there is a fraction \( \beta \bar{p} \) of firms who are given the opportunity to merge and decide to do so (see section 5.1). Their distribution is given by \( h(\varphi) \), see section 5.2, and by Proposition 1, they have profits \( 4B\varphi_m^{\epsilon-1} - 2(1+\alpha)f - f_m \). The free entry condition is thus:

\[
(20) \quad (1 - \beta) \int_{\varphi_*}^{\infty} g(\varphi) (B\varphi^{\epsilon-1} - f) d\varphi + (1 - \beta) \int_{\tau^{1-\epsilon} B\varphi^{\epsilon-1} - f_x}^{\infty} g(\varphi) d\varphi + \\
+ \beta \int_{\varphi_*}^{\infty} g(\varphi) (1 - p(\varphi)) (B\varphi^{\epsilon-1} - f) d\varphi + \beta \int_{\tau^{1-\epsilon} B\varphi^{\epsilon-1} - f_x}^{\infty} g(\varphi) (1 - p(\varphi)) d\varphi + \\
+ \beta \int_{\varphi_*}^{\infty} h(\varphi) (4B\varphi_m^{\epsilon-1} - 2(1+\alpha)f - f_m) d\varphi = f_{en}
\]

\[\text{Note that our model can be given a dynamic interpretation as explained in Melitz and Redding (2014) if firms are confronted with a death shock of probability } \delta, \text{ which is independent of productivity, as in Melitz (2003).}\]
As in the case without M&As, we can write domestic and export profits as functions of the viability and exporting productivity cut-off values, like in sections 3.1 and Appendix B. The main difference with respect to the case without M&As is the last term before the equality sign in equation (20), related to the profitability of merged firms. We know that the condition for a successful merger is \( \pi_m - (\pi_h + \pi_l) \geq 0 \) and that each merged firm corresponds to two pre-merger firms with productivity \( \varphi_h \) and \( \varphi_l \). This implies that the term describing profits of post-merger firms must be bigger than or equal to the matching components of the pre-merger profits, which has density \( p(\varphi) \) as analysed in section 5.1:

\[
\int_{\varphi_*}^{\infty} h(\varphi)(4B\varphi_m^{e-1} - 2(1 + \alpha)f - f_m)d\varphi 
\geq 
\int_{\varphi_*}^{\infty} g(\varphi)p(\varphi)(B\varphi^{e-1} - f) d\varphi + \int_{\eta\varphi_*}^{\infty} g(\varphi)p(\varphi)(\tau^{1-\varepsilon}B\varphi^{e-1} - f_x)d\varphi
\]

Define the function \( H(\varphi_*) \geq 0 \) to be the difference between the left-hand-side of the inequality in equation (21) and the right-hand-side of this inequality. It is the difference in aggregate post-merger and pre-merger profits for all firms engaged in M&As. Now proceed as follows. First, substitute the function \( H(\varphi_*) \) in equation (20) by eliminating the last term before the equality sign. Second, combine the terms with the densities \( g(\varphi)(1 - p(\varphi)) \) in the second line of equation (20) with the terms with densities \( g(\varphi)p(\varphi) \) of the function \( H(\varphi_*) \) (see equation 21) to get the simple densities \( g(\varphi) \) for both domestic and exporting profits. Third, note that the combinations of the second step still have the probability \( \beta \) in front of the integral signs (indicating the probability of being given the opportunity to merge), which can now be combined with the \( 1 - \beta \) terms in the first line of equation (20) to get 1 for both domestic and exporting profits. Fourth, use the \( f(.) \) function defined in Appendix B to simplify the resulting free entry condition, where \( J(\varphi_*) \equiv \int_{\varphi_*}^{\infty} g(\varphi) \left( \varphi \varphi_{au}^{e-1} - 1 \right) d\varphi \).

We have now re-written the free-entry condition (20) to:

\[
(20') \quad f_{en} = J(\varphi_*)f + J(\eta\varphi_*)f_x + H(\varphi_*)
\]

The free entry condition with M&As now shows that the viability cut-off \( \varphi_* \) is determined by \( J(\varphi_*)f + J(\eta\varphi_*)f_x = f_{en} - H(\varphi_*) \leq f_{en} \). This implies \( \varphi_* > \varphi_{*tr} \) if \( H(\varphi_*) > 0 \), since \( f(.) \) is monotonically declining and \( \varphi_{*tr} \) is determined by \( J(\varphi_*)f + J(\eta\varphi_*)f_x = f_{en} \). If we assume that M&As are economically relevant, by which we mean that the economy-wide probability of a merger is positive (\( \beta \bar{p} > 0 \)), then \( H(\varphi_*) > 0 \), since \( \pi_m = (\pi_h + \pi_l) \) is only possible on a set of measure zero.

**Proposition 4 (impact of mergers on the productivity distribution)**

If mergers are economically relevant (that is: \( \beta \bar{p} > 0 \)), then the viability cut-off is higher than under a regime of free trade only, which in turn is higher than under autarky: \( \varphi_* > \varphi_{*tr} > \varphi_{au} \).
As a result, following Appendix B, permitting mergers and acquisitions in autarky raises average productivity and welfare.

What are the consequences of imperfect knowledge transfers for equilibrium outcomes? We argued that relaxing the perfect transfer assumption leads to different predictions in terms of merger patterns, but does it also change the welfare conclusions? We get to the impact of the degree of transfers \( \omega \) on welfare in three steps. First, note that (other things equal) a merger leaves the total number of varieties in the market unchanged. Second, for a given successful merger, the post-M&A price is linear in \( \phi_{m}^{\varepsilon-1} \), which is increasing in \( \omega \) as \( \phi_{h}^{\varepsilon-1} > \phi_{l}^{\varepsilon-1} \). Third, a rise in knowledge transfers increases the probability of a merger for all \( \varphi \) [except for the exiters].

To see this, realize that every firm has one draw for a partner, and merges with probability \( \beta \) if the merge is profitable. The probability for a firm of productivity \( \varphi \) of drawing a partner with whom the merger is successful is \( p(\varphi) \), see section 4.4. The merger value function is strictly increasing in the level of knowledge transfers since its derivative with respect to knowledge transfers is: \( \frac{\partial \theta}{\partial \omega} = 4B(\varepsilon - 1)\phi_{m}^{\varepsilon-1}(\ln(\varphi_{h}) - \ln(\varphi_{l})) > 0 \), irrespective of the case of takeover (merger of domestic-domestic; domestic-exporter and exporter-exporter, see sections 3.3 and 4.3). Fourth, the cut-off productivity increases in \( \omega \). From step three, \( h(\varphi) \) increases in \( \omega \) for all \( \varphi \); and the merger value (excess merger profits) \( \theta \) increase in \( \omega \) for all \( \varphi \). Hence, the term \( H(\varphi_{*}) \) increases in \( \omega \), so the cut-off productivity rises in \( \omega \).

**Proposition 5 (knowledge transfers and welfare)**

Better knowledge transfers (higher \( \omega \)) raises M&A viability and the firm viability cut-off \( \varphi_{*} \), and thus raises welfare.

**6 Conclusions**

The bulk of foreign direct investments (FDI) takes the form of cross-border mergers and acquisitions (M&As) as opposed to greenfield investments. Both models of greenfield FDI and cross-border M&As usually assume that productivity can be transferred perfectly, such that in the case of cross-border M&A, the post-M&A firm operates at productivity levels of the most productive pre-merger partner – typically the acquirer. The stylized facts that we presented on cross-border M&As suggest, however, that this is not actually true: post-M&A firms generally operate at lower productivity levels than one might expect if productivity would transfer perfectly between the partners. Furthermore, all sorts of M&As occur in reality; between low productive partners, between high productive partners, and between combinations of high and low productivity.

Against this background, we develop a model of matching between heterogeneous firms in cross-border M&As based on the Melitz (2003) trade model of monopolistic competition, with the

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16 As all firms in the Melitz setup have the same proportional markup, the monopolistic price is not distortive. This might differ, if there was a group of firms operating on perfectly competitive markets in an asymmetric setting (Demidova, 2008)
addition of productivity transfers between the M&A partners. We allow for imperfect knowledge transfers, and hence imperfect productivity transfers. Our model includes perfect productivity transfers as a special case; this special case yields a familiar prediction most models of FDI and cross-border M&As: highly productive firms take over the least productive firms. However, relaxing the assumption of perfect productivity transfers changes the underpinning for the observed patterns in cross-border M&As substantially. Once some of the characteristics the target (or the low-productive partner) surface in the post-merger firm, two-sided heterogeneity becomes important: the productivity levels of both partners matter for the profitability of an M&A decision.

Our model shows that viability of a cross-border M&A between a pair of firms crucially hinges on two key model parameters, the extent of knowledge transfer in the wake of the M&A on the one hand and the combination of pre-merger productivities between the two firms on the other hand. A combination of very unproductive firms will generally never merge, irrespective of the extent of the knowledge transfer. For a combination of sufficiently productive firms, an M&A is viable if the productivity difference within the combination is not too large. The range of productivity differences for viable M&As is larger, when knowledge transfers between the merging firms are larger.

The main implications from our model with imperfect knowledge transfers are threefold. First, a cross-border M&A only occurs if the productivity difference between potential partners is not too large. Matching with a weak partner dilutes firm productivity, making a strategy of exporting for the individual firm more profitable than a merger. Consequently, there is weak positive assortative matching: the M&A is not viable if the productivities of the two partners are too far apart. The productivity range of potential matches for a given firm rises with the degree to which productivity levels can be transferred. This implication of the model is backed up by the stylized facts on cross-border M&As that we brought to the fore in the first part of our paper, and in particular the fact that these M&As are more likely to occur between firm when productivity levels are not too far apart.

A second novel implication to which our model gives rise is that the M&As are not restricted to the most productive firms. M&As are generally more profitable for highly productive firms, as opposed to models with one-sided heterogeneity only, but less productive firms can engage in M&As, provided that they are matched to similar firms. Hence, M&As occur across the board of productivities, and merging firms are typically similar in terms of productivity. These implications are in line with the stylized facts that we reported for cross-border M&As and also with the results of Braguinsky et al. (2015) and Rhodes-Kropf and Robinson (2008). At the same time with imperfect knowledge transfers in the wake of the M&A, the post-merger productivity is also determined or influenced by the low-productive partner which is also in line with the facts on cross-border M&As as illustrated in the empirical part of our paper.
Finally, and by way of third implication, our model shows that eliminating barriers to M&As always increases the average productivity of firms in the economy. Consequently, liberalization in the sense of permitting foreign direct investments is welfare-improving even if the post-merger productivity is in between the highest and lowest pre-merger productivities. However, the welfare-benefits significantly depend on how productivities are transferred in the wake of cross-border M&As. With lower levels of productivity transfer between the partners, the welfare is lower than when the highest productivity level is perfectly duplicated across the firms involved in the cross-border M&A.
Appendix A

Table A1 Firm type characteristics

<table>
<thead>
<tr>
<th>population</th>
<th>observations</th>
<th>mean log sales/employee</th>
<th>s.d.</th>
<th>10\textsuperscript{th} percentile</th>
<th>90\textsuperscript{th} percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>All firms</td>
<td>414,333</td>
<td>5.06</td>
<td>1.64</td>
<td>2.99</td>
<td>6.82</td>
</tr>
<tr>
<td>Targets</td>
<td>25,731</td>
<td>4.64</td>
<td>1.75</td>
<td>2.36</td>
<td>6.57</td>
</tr>
<tr>
<td>Acquirers</td>
<td>37,581</td>
<td>5.41</td>
<td>1.44</td>
<td>3.89</td>
<td>6.95</td>
</tr>
</tbody>
</table>

Table A2 Distribution of M&As over productivity quintiles, different samples

\textit{a. Within sectors (per cent of M&As)}

<table>
<thead>
<tr>
<th>High-productivity quintile</th>
<th>Low-productivity quintile</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>I</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>7.4</td>
<td>5.4</td>
</tr>
<tr>
<td>III</td>
<td>5.9</td>
<td>9.3</td>
</tr>
<tr>
<td>IV</td>
<td>5.3</td>
<td>10.6</td>
</tr>
<tr>
<td>V</td>
<td>4.7</td>
<td>5.8</td>
</tr>
<tr>
<td>Total</td>
<td>27.4</td>
<td>31.1</td>
</tr>
</tbody>
</table>

\textit{b. All M&As, at least 5 firms in sector-country group (per cent of M&As)}

<table>
<thead>
<tr>
<th>High-productivity quintile</th>
<th>Low-productivity quintile</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>I</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>7.9</td>
<td>5.1</td>
</tr>
<tr>
<td>III</td>
<td>7.5</td>
<td>8.2</td>
</tr>
<tr>
<td>IV</td>
<td>6.1</td>
<td>7.7</td>
</tr>
<tr>
<td>V</td>
<td>5.6</td>
<td>7.0</td>
</tr>
<tr>
<td>Total</td>
<td>31.1</td>
<td>28.0</td>
</tr>
</tbody>
</table>

\textit{c. Within sectors, at least 5 firms in sector-country group (per cent of M&As)}

<table>
<thead>
<tr>
<th>High-productivity quintile</th>
<th>Low-productivity quintile</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>I</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>7.9</td>
<td>5.4</td>
</tr>
<tr>
<td>III</td>
<td>6.3</td>
<td>8.8</td>
</tr>
<tr>
<td>IV</td>
<td>5.6</td>
<td>8.4</td>
</tr>
<tr>
<td>V</td>
<td>5.0</td>
<td>6.1</td>
</tr>
<tr>
<td>Total</td>
<td>29</td>
<td>28.8</td>
</tr>
</tbody>
</table>
Table A2 continued

d. All M&As, at least 50 firms in sector-country group (per cent of M&As)

<table>
<thead>
<tr>
<th>High-productivity quintile</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>4.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.7</td>
</tr>
<tr>
<td>II</td>
<td>7.9</td>
<td>5.3</td>
<td></td>
<td></td>
<td></td>
<td>13.1</td>
</tr>
<tr>
<td>III</td>
<td>7.5</td>
<td>9.1</td>
<td>3.7</td>
<td></td>
<td></td>
<td>20.3</td>
</tr>
<tr>
<td>IV</td>
<td>5.3</td>
<td>6.7</td>
<td>8.6</td>
<td>5.0</td>
<td></td>
<td>25.6</td>
</tr>
<tr>
<td>V</td>
<td>6.3</td>
<td>7.2</td>
<td>8.6</td>
<td>9.3</td>
<td>5.0</td>
<td>36.3</td>
</tr>
<tr>
<td>Total</td>
<td>31.6</td>
<td>28.3</td>
<td>20.9</td>
<td>14.3</td>
<td>5.0</td>
<td>100</td>
</tr>
</tbody>
</table>

e. Within sectors, at least 50 firms in sector-country group (per cent of M&As)

<table>
<thead>
<tr>
<th>High-productivity quintile</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>5.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.5</td>
</tr>
<tr>
<td>II</td>
<td>7.5</td>
<td>5.9</td>
<td></td>
<td></td>
<td></td>
<td>13.3</td>
</tr>
<tr>
<td>III</td>
<td>7.5</td>
<td>8.4</td>
<td>4.2</td>
<td></td>
<td></td>
<td>20.1</td>
</tr>
<tr>
<td>IV</td>
<td>5.1</td>
<td>7.1</td>
<td>9.0</td>
<td>5.5</td>
<td></td>
<td>26.7</td>
</tr>
<tr>
<td>V</td>
<td>6.0</td>
<td>5.7</td>
<td>9.5</td>
<td>8.8</td>
<td>4.4</td>
<td>34.4</td>
</tr>
<tr>
<td>Total</td>
<td>31.6</td>
<td>27.1</td>
<td>22.7</td>
<td>14.3</td>
<td>4.4</td>
<td>100</td>
</tr>
</tbody>
</table>

Table A3 Post-M&A sales per employee explained by pre-merger productivity

<table>
<thead>
<tr>
<th>Indexation</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M&amp;As all or within sector</td>
<td>global</td>
<td>global</td>
<td>global</td>
<td>global</td>
<td>sectoral</td>
<td>sectoral</td>
</tr>
<tr>
<td>High-productive partner</td>
<td>0.68*** (0.018)</td>
<td>0.12*** (0.035)</td>
<td>0.65*** (0.035)</td>
<td>0.23*** (0.066)</td>
<td>0.58*** (0.047)</td>
<td>0.23*** (0.066)</td>
</tr>
<tr>
<td>Low-productive partner</td>
<td>0.30*** (0.022)</td>
<td>0.32*** (0.040)</td>
<td>0.42*** (0.053)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Combined pre-M&amp;A</td>
<td>0.86*** (0.038)</td>
<td>0.75*** (0.071)</td>
<td>0.75*** (0.071)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>3,065</td>
<td>3,065</td>
<td>832</td>
<td>832</td>
<td>612</td>
<td>832</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.968</td>
<td>0.975</td>
<td>0.976</td>
<td>0.979</td>
<td>0.980</td>
<td>0.979</td>
</tr>
</tbody>
</table>

OLS estimates with robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1; dependent variable: post M&A log sales/employee; independent variable 1-year pre-M/A log sales/employee of participating firms.
Appendix B

Autarky Equilibrium
In autarky \( \pi_{\varphi,d} = B_{au} \varphi^{\varepsilon-1} - f \), where \( B_{au} \) is the relevant constant in autarky, which is equal to (Feenstra, p. 158): \( B_{au} = \frac{L_{e}^{\varepsilon}}{P_{au}^{1-\varepsilon}(\varepsilon-1)^{1-\varepsilon}} \), where \( L \) is the size of the labour force and \( P_{au} \) is the autarky price index. The zero-profit condition implies \( \varphi_{*au}^{\varepsilon-1} = f / B_{au} \). Firms pay a fixed entry cost \( f_{en} \) to enter the market. The free entry condition is thus provided by:

\[(B1) \quad \int_{\varphi_{*au}}^{\infty} g(\varphi) (B_{au} \varphi^{\varepsilon-1} - f) \, d\varphi = f_{en} \]

Making use of the fact that \( \varphi_{*au}^{\varepsilon-1} = f / B_{au} \), we can write (domestic) profits as:

\[B_{au} \varphi^{\varepsilon-1} - f = \left( \frac{\varphi}{\varphi_{*au}} \right)^{\varepsilon-1} B_{au} \varphi_{*au}^{\varepsilon-1} - f = \left( \left( \frac{\varphi}{\varphi_{*au}} \right)^{\varepsilon-1} - 1 \right) f\]

The free entry condition can thus be written as:

\[(B1') \quad f_{en} = \int_{\varphi_{*au}}^{\infty} g(\varphi) \left( \left( \frac{\varphi}{\varphi_{*au}} \right)^{\varepsilon-1} - 1 \right) f \, d\varphi = J(\varphi_{*au}) f\]

Where the function \( J(\varphi_{*au}) \equiv \int_{\varphi_{*au}}^{\infty} g(\varphi) \left( \left( \frac{\varphi}{\varphi_{*au}} \right)^{\varepsilon-1} - 1 \right) d\varphi \) is positive and monotonically declining in the cut-off value \( \varphi_{*au} \), which implies there is a unique value \( \varphi_{*au} \) satisfying the free entry and zero profit conditions.

Determining the Trade Equilibrium
With free trade \( \pi_{\varphi,d} = B_{tr} \varphi^{\varepsilon-1} - f \) and \( \pi_{\varphi,x} = \tau^{1-\varepsilon} B_{tr} \varphi^{\varepsilon-1} - f_{x} \), where \( B_{tr} \) is the relevant constant under trade, which is equal to \( B_{tr} = \frac{L_{e}^{\varepsilon}}{P_{tr}^{1-\varepsilon}(\varepsilon-1)^{1-\varepsilon}} \), where \( P_{tr} \) is the trade price index.

There are now two zero profit conditions, one for viability and one for engaging in export activities, which provide the cut-offs: \( \varphi_{*tr}^{\varepsilon-1} = f / B_{tr} \) and \( \varphi_{*x,tr}^{\varepsilon-1} = \tau^{1-\varepsilon} f_{x} / B_{tr} \). Note that \( \varphi_{*x,tr} = \tau(\frac{f_{x}/f}{1/(\varepsilon-1)}) \varphi_{*tr} \equiv \eta \varphi_{*tr} \), with \( \eta > 1 \) because we assumed \( f_{x} > \tau^{1-\varepsilon} f \). The free entry condition is now provided by:

\[(B2) \quad \int_{\varphi_{*tr}}^{\infty} g(\varphi) (B_{tr} \varphi^{\varepsilon-1} - f) \, d\varphi + \int_{\eta \varphi_{*tr}}^{\infty} g(\varphi) (\tau^{1-\varepsilon} B_{tr} \varphi^{\varepsilon-1} - f_{x}) \, d\varphi = f_{en}\]

Making use of the fact that \( \varphi_{*x,tr}^{\varepsilon-1} = \tau^{1-\varepsilon} f_{x} / B_{tr} \), we can write exporting profits as:

\[\tau^{1-\varepsilon} B_{tr} \varphi^{\varepsilon-1} - f_{x} = \left( \frac{\varphi}{\varphi_{*x,tr}} \right)^{\varepsilon-1} \tau^{1-\varepsilon} B_{tr} \varphi_{*x,tr}^{\varepsilon-1} - f_{x} = \left( \left( \frac{\varphi}{\varphi_{*x,tr}} \right)^{\varepsilon-1} - 1 \right) f_{x}\]

We can thus use the \( J(\cdot) \) function again to simplify the free entry condition:

\[(B2') \quad f_{en} = J(\varphi_{*tr}) f + J(\eta \varphi_{*tr}) f_{x}\]

34
We already know that $\varphi_{x,tr} > \varphi_{tr}$. The free entry condition with trade now shows that $\varphi_{tr}$ is determined from $J(\varphi_{tr}) f = f_{en} - J(\eta \varphi_{tr}) f_{k} < f_{en}$, which thus implies $\varphi_{tr} > \varphi_{au}$ since $J(.)$ is monotonically declining and $\varphi_{au}$ is determined by $J(\varphi_{au}) f = f_{en}$.

References


Journal of Economics, 98, 185-199.