Monetary Policy Competition And Cooperation
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Abstract: This paper studies the interactions of monetary policy in the world, by a few assumptions. As we know, monetary policy has a spillover effect. So it carefully discusses the process of policy competition and the structure of the policy cooperation in this paper. As to policy competition, the focus is on competition between the European central bank, American central bank, and the Japanese central bank. As to policy cooperation, the focus is on the same institutions. We conclude that monetary competition between those countries leads to full employment and price stability, and monetary cooperation between those countries can also achieve full employment and price stability.

Key words: monetary policy; competition; cooperation;

In this paper, we consider a world of three monetary regions, let us say Europe, America, and Japan. The exchange rate between the three countries is flexible. There is perfect capital mobility between them. In the short run, nominal wages and prices are rigid. The market for European goods. We assume that the monetary regions in Europe and America are the same size and have the same behavioral functions, and they are both 3 times as large as Japan.

1. Monetary Competition between European, American and Japan

1) The static model. The world consists of three monetary regions, say Europe, America and Japan. The exchange rate among Europe, America and Japan is flexible. There is international trade between Europe, America and Japan. European goods, American goods and Japanese goods are imperfect substitutes for each other. European goods are determined by the demand for European goods. American goods are determined by the demand for American goods. And Japanese goods are determined by the demand for Japanese goods. European money demand equals European supply. American money demand equals American supply. And Japanese money demand equals Japanese supply. There is perfect capital mobility among Europe, America and Japan. Thus the European interest rate, the American interest rate and the Japanese interest rate are equalized. Nominal wages and prices adjust slowly.

As a result, an increase in European money supply raises European output, on the other hand, an increase in European money supply lowers American and Japanese output. Correspondingly, an increase in American money raises American output, lowers European and Japanese output. An increase in Japanese money raises Japanese output, lowers European and American output.

Now have a closer look at the process of adjustment. An increase in European money supply causes a depreciation of the euro, an appreciation of the dollar and yen, and a decline in the world interest rate. The depreciation of euro raises European output, the appreciation of dollar and yen lower American and Japanese output. And the decline in the world interest rate raises European, American and Japanese investment. The net effect is that European output goes up. However, American and Japanese output go down. This model is in the tradition of the Mundell-Fleming model.
The static model can be represented by a system of three equations:

\[ Y_1 = \alpha M_1 - 3\beta M_2 - \beta M_3 \]  \hspace{1cm} (1)

\[ Y_2 = \alpha M_2 - 3\beta M_1 - \beta M_3 \]  \hspace{1cm} (2)

\[ Y_3 = \alpha M_3 - 3\beta M_1 - 3\beta M_2 \]  \hspace{1cm} (3)

Of course this is a reduce form. \( Y_1 \) denotes European output, \( Y_2 \) denotes American output, \( Y_3 \) denotes Japanese output, \( M_1 \) is European money supply, \( M_2 \) is American money supply, \( M_3 \) is Japanese money supply. \( A_1 \) is some other factors bearing on European output, \( A_2 \) is some other factors bearing on American output, \( A_3 \) is some other factors bearing on Japanese output. \( \alpha \) and \( \beta \) denote the monetary policy multiplier. The internal effect of monetary policy is positive \( \alpha > 0 \). By contrast, the external effect of monetary policy is negative \( \beta > 0 \). In absolute values, the internal effect is larger than the external effect \( \alpha > \beta \). The endogenous variables are European output, American output and Japanese output.

2) The dynamic model. At the beginning there is unemployment in Europe, America and Japan. The primary target of the European central bank is price stability in Europe. The secondary target target of the Europe central bank is high employment, in other words, the special target of the European central bank is full employment in Europe. The instrument of the European central bank is European money supply. The European central bank raises European money supply so as to close the output gap in Europe:

\[ M_1 - M_1^{-1} = \frac{Y_1 - Y_1}{\alpha} \]  \hspace{1cm} (4)

Here is a list of the new symbols:

- \( Y_1 \) European output this period
- \( \bar{Y}_1 \) full-employment output in Europe
- \( \bar{Y}_1 - Y_1 \) output gap in Europe this period
- \( M_1^{-1} \) European money supply last period
- \( M_1 \) European money supply this period
- \( M_1 - M_1^{-1} \) increase in European money supply.
The target of American and Japanese central banks are full employment in their own. The instrument of both the central banks are money supply. The American and Japanese banks raise their money supply so as to close the output gap:

$$M_2 - M_2^{−1} = \frac{\bar{Y}_2 - Y_2}{\alpha} \quad (5)$$

$$M_3 - M_3^{−1} = \frac{\bar{Y}_3 - Y_3}{\alpha} \quad (6)$$

Here is a list of the new symbols:

- $Y_2, \ Y_3$: American, Japanese output this period
- $\bar{Y}_2, \ \bar{Y}_3$: full-employment output in America, Japan
- $\bar{Y}_2 - Y_2, \ \bar{Y}_3 - Y_3$: output gap in American, Japan this period
- $M_2^{−1}, \ M_3^{−1}$: American, Japanese money supply last period
- $M_2, \ M_3$: American, Japanese money supply this period
- $M_2 - M_2^{−1}, \ M_3 - M_3^{−1}$: increase in American, Japanese money supply.

In addition there is an output lag. European output next period is determined by European money supply this period as well as by American and Japanese money supply this period:

$$Y_1^{−1} = A_t + \alpha M_1 - 3 \beta M_2 - \beta M_3 \quad (7)$$

Here $Y_1^{−1}$ denotes European output next period, in the same way, American output next period is determined by American supply this period as well as by European and Japanese money supply this period, and so does the Japanese output next period:

$$Y_2^{−1} = A_t + \alpha M_2 - 3 \beta M_1 - \beta M_3 \quad (8)$$

$$Y_3^{−1} = A_t + \alpha M_3 - 3 \beta M_1 - 3 \beta M_2 \quad (9)$$

Here $Y_2^{−1}, \ Y_3^{−1}$ denotes American, Japanese output next period.

On this basis, the dynamic model can be characterized by a system of six equations:

$$M_1 - M_1^{−1} = \frac{\bar{Y}_1 - Y_1}{\alpha} \quad (10)$$

$$M_2 - M_2^{−1} = \frac{\bar{Y}_2 - Y_2}{\alpha} \quad (11)$$
\[ M_3 - M_3^{-1} = \frac{Y_3 - Y_3}{\alpha} \quad (12) \]

\[ Y_1^{-1} = A_1 + \alpha M_1 - 3\beta M_2 - \beta M_3 \quad (13) \]

\[ Y_2^{-1} = A_2 + \alpha M_2 - 3\beta M_1 - \beta M_3 \quad (14) \]

\[ Y_3^{-1} = A_3 + \alpha M_3 - 3\beta M_1 - 3\beta M_2 \quad (15) \]

3) The steady state. In the steady state by definition we have:

\[ M_1 = M_1^{-1} \quad (16) \]

\[ M_2 = M_2^{-1} \quad (17) \]

\[ M_3 = M_3^{-1} \quad (18) \]

Equation (16), (17), (18) mean that European, American, and Japanese money supply does not change any more.

\[ Y_1 = Y_1 \quad (19) \]

\[ Y_2 = Y_2 \quad (20) \]

\[ Y_3 = Y_3 \quad (21) \]

\[ Y_1^{-1} = A_1 + \alpha M_1 - 3\beta M_2 - \beta M_3 \quad (22) \]

\[ Y_2^{-1} = A_2 + \alpha M_2 - 3\beta M_1 - \beta M_3 \quad (23) \]

\[ Y_3^{-1} = A_3 + \alpha M_3 - 3\beta M_1 - 3\beta M_2 \quad (24) \]

The model of the steady state can be compressed to a system of only three equations:

\[ Y_1 = A_1 + \alpha M_1 - 3\beta M_2 - \beta M_3 \quad (25) \]

\[ Y_2 = A_2 + \alpha M_2 - 3\beta M_1 - \beta M_3 \quad (26) \]

\[ Y_3 = A_3 + \alpha M_3 - 3\beta M_1 - 3\beta M_2 \quad (27) \]

Here the endogenous variables are European, American, and Japanese money supply. To simplify notation we introduce:

\[ B_1 = Y_1 - A_1 \quad (28) \]

\[ B_2 = Y_2 - A_2 \quad (29) \]
\[ B_3 = Y_3 - A_3 \]  

(30)

With this, the model of the steady state can be written as follow:

\[ B_1 = \alpha M_1 - 3\beta M_2 - \beta M_3 \]  

(31)

\[ B_2 = \alpha M_2 - 3\beta M_1 - \beta M_3 \]  

(32)

\[ B_3 = \alpha M_3 - 3\beta M_1 - 3\beta M_2 \]  

(33)

The endogenous variables are still \( M_1 \), \( M_2 \), and \( M_3 \).

Next we solve the model for the endogenous variables:

\[ M_1 = \frac{(B_1\alpha + B_2\beta)(\beta^2 + 3\alpha\beta) + (B_1\beta + B_2\alpha)(\alpha^2 - 3\beta^2)}{(\alpha^2 - 2\beta^2 + 3\alpha\beta)(\alpha^2 - 4\beta^2 + 3\alpha\beta)} \]  

(34)

\[ M_2 = \frac{(B_2\beta + B_1\alpha)(\alpha^2 - 3\beta^2) + (B_2\beta + B_1\alpha)(\beta^2 + 3\alpha\beta)}{(\alpha^2 - 2\beta^2 + 3\alpha\beta)(\alpha^2 - 4\beta^2 + 3\alpha\beta)} \]  

(35)

\[ M_3 = \frac{(B_1\beta + B_2\alpha)(\beta^2 + 3\alpha\beta) + (B_1\beta + B_2\alpha)(\alpha^2 - 3\beta^2)}{(\alpha^2 - 2\beta^2 + 3\alpha\beta)(\alpha^2 - 4\beta^2 - 3\alpha\beta)} \]  

(36)

As an alternative, the steady state can be represented in terms of the initial output gap and the total increase in money supply. Taking differences in equations (1), (2), (3), the model of the steady state can be written as follows:

\[ \Delta Y_1 = \alpha \Delta M_1 - 3\beta \Delta M_2 - \beta \Delta M_3 \]  

(37)

\[ \Delta Y_2 = \alpha \Delta M_2 - 3\beta \Delta M_1 - \beta \Delta M_3 \]  

(38)

\[ \Delta Y_3 = \alpha \Delta M_3 - 3\beta \Delta M_1 - 3\beta \Delta M_2 \]  

(39)

Here \( \Delta Y_1 \) is the initial output gap in Europe, \( \Delta Y_2 \) is the initial output gap in America, \( \Delta Y_3 \) is the initial output gap in Japan, \( \Delta M_1 \) is the total increase in European money supply, \( \Delta M_2 \) is the total increase in America money supply, and \( \Delta M_3 \) is the total increase in Japanese money supply. The endogenous variables are \( \Delta M_1 \), \( \Delta M_2 \), and \( \Delta M_3 \). The solution to the system (37), (38), and (39) is:

\[ \Delta M_1 = \frac{(\Delta Y_1\alpha + \Delta Y_2\beta)(\beta^2 + 3\alpha\beta) + (\Delta Y_2\beta + \Delta Y_1\alpha)(\alpha^2 - 3\beta^2)}{(\alpha^2 - 2\beta^2 + 3\alpha\beta)(\alpha^2 - 4\beta^2 + 3\alpha\beta)} \]  

(40)

\[ \Delta M_2 = \frac{(\Delta Y_2\beta + \Delta Y_1\alpha)(\alpha^2 - 3\beta^2) + (\Delta Y_1\beta + \Delta Y_2\alpha)(\beta^2 + 3\alpha\beta)}{(\alpha^2 - 2\beta^2 + 3\alpha\beta)(\alpha^2 - 4\beta^2 + 3\alpha\beta)} \]  

(41)
\[ \Delta M_3 = \frac{(\Delta Y_1 \beta + \Delta Y_2 \alpha) (\beta^2 + 3 \alpha \beta) + (\Delta Y_1 \beta + \Delta Y_2 \alpha) (\alpha^2 - 3 \beta^2)}{(\alpha^2 - 2 \beta^2 + 3 \alpha \beta) (\alpha^2 - 4 \beta^2 - 3 \alpha \beta)} \] (42)

According to equation (40), the total increase in European money supply depends on the initial output gap in Europe, the initial output gap in America, the initial output gap in Japan, the direct multiplier \( \alpha \), and the cross multiplier \( \beta \). The larger the initial output gap in Europe, the larger is the total increase in European money supply. And the total increase in American, Japanese money supply are in the same way.

4) Stability. Eliminate \( Y_1 \) in equation(10) by means of equation(13) and rearrange terms

\[ \bar{Y}_1 = A_1 + \alpha M_1 - 3 \beta M_2^{-1} - \beta M_3^{-1} \] (43)

\[ \bar{Y}_2 = A_2 + \alpha M_2 - 3 \beta M_1^{-1} - \beta M_3^{-1} \] (44)

\[ \bar{Y}_3 = A_3 + \alpha M_3 - 3 \beta M_1^{-1} - 3 \beta M_2^{-1} \] (45)

Here the endogenous variables are European money supply this period \( M_1 \), American money supply this period \( M_2 \), and Japanese money supply this period \( M_3 \). To simplify notation we make use of equations(28), (29), and (30). With this, the dynamic model can be written as follows:

\[ B_1 = \alpha M_1 - 3 \beta M_2^{-1} - \beta M_3^{-1} \] (46)

\[ B_2 = \alpha M_2 - 3 \beta M_1^{-1} - \beta M_3^{-1} \] (47)

\[ B_3 = \alpha M_3 - 3 \beta M_1^{-1} - 3 \beta M_2^{-1} \] (48)

The endogenous variables are still \( M_1 \), \( M_2 \) and \( M_3 \).

Now substitute equation(47), (48) into (46) and solve for:

\[ \alpha M_1 = (B_1 \beta + B_2 \alpha) \frac{\alpha - \beta}{\alpha^2 - 3 \alpha \beta} + \frac{B_3 \beta}{\alpha + \beta} + \frac{\beta^2 M_3^{-1}}{\alpha} \] (49)

Then differentiate equation(49) for \( M_1^{-2} \):

\[ \frac{dM_1}{dM_1^{-2}} = \frac{\beta^2}{3 \alpha^2} \text{, finally, the stability condition is } \frac{\beta^2}{\alpha^2} < 0, \text{ or } \alpha > \beta \, . \]

5) Conclusion. That means, the steady is stable if and only if the internal effect of monetary
policy is larger than the external effect of monetary policy. As a result, there is a stable steady state of monetary competition. In other words, the process of monetary competition leads to full employment in Europe, America and Japan. And what is more, it leads to price stability in Europe, America and Japan. However, it does mean that this will lead to full employment and price stability in European member countries.

2. Monetary Cooperation between European, American and Japan

1. As a point of mentioned before, take the output model, it can be represented by a system of three equations:

\[ Y_1 = A_1 + \alpha M_1 - 3\beta M_2 - \beta M_3 \]  
\[ Y_2 = A_2 + \alpha M_2 - 3\beta M_1 - \beta M_3 \]  
\[ Y_3 = A_3 + \alpha M_3 - 3\beta M_1 - 3\beta M_2 \]  

At the beginning there is unemployment in Europe, America, and Japan. The primary target of the European central bank is price stability in Europe. The secondary target target of the European central bank is high employment, in other words, the special target of the European central bank is full employment in Europe. The instrument of the European central bank is European money supply. The European central bank raises European money supply so as to close the output gap in Europe. The instruments of monetary cooperation are European money supply, American money supply and Japanese money supply. So there are three targets and three instruments.

2) The price model. On this basis, the policy model can be characterized by a system of three equations:

\[ \bar{Y}_1 = A_1 + \alpha M_1 - 3\beta M_2 - \beta M_3 \]  
\[ \bar{Y}_2 = A_2 + \alpha M_2 - 3\beta M_1 - \beta M_3 \]  
\[ \bar{Y}_3 = A_3 + \alpha M_3 - 3\beta M_1 - 3\beta M_2 \]  

To simplify notation, we introduce

\[ B_1 = \alpha M_1 - 3\beta M_2 - \beta M_3 \]  
\[ B_2 = \alpha M_2 - 3\beta M_1 - \beta M_3 \] 
\[ B_3 = \alpha M_3 - 3\beta M_1 - 3\beta M_2 \]

Then we solve the model for the endogenous variables:

\[ M_1 = \frac{(B_1 + \alpha B_3)(\beta^2 + 3\alpha \beta) + (B_2 + B_2 \alpha)(\alpha^2 - 3\beta^2)}{(\alpha^2 - 2\beta^2 + 3\alpha \beta)(\alpha^2 - 4\beta^2 + 3\alpha \beta)} \]
\[ M_2 = \frac{(B_1 + B_2 \alpha)(\alpha^2 - 3\beta^2) + (B_2 + B_2 \alpha)(\beta^2 + 3\alpha \beta)}{(\alpha^2 - 2\beta^2 + 3\alpha \beta)(\alpha^2 - 4\beta^2 + 3\alpha \beta)} \]
\[ M_3 = \frac{(B_1 + B_2 \alpha)(\beta^2 + 3\alpha \beta) + (B_2 + B_3 \alpha)(\alpha^2 - 3\beta^2)}{(\alpha^2 - 2\beta^2 + 3\alpha \beta)(\alpha^2 - 4\beta^2 - 3\alpha \beta)} \]
3). Another version of the policy model. As an alternative, the policy model can be stated in terms of the initial output gap and required increase in money supply. Taking differences in equations (1), (2), and (3), the policy model can be written as follows:

\[ \Delta Y = \alpha \Delta M_1 - 3 \beta \Delta M_2 - \beta \Delta M_3 \]  
\[ \Delta Y = \alpha \Delta M_2 - 3 \beta \Delta M_1 - \beta \Delta M_3 \]  
\[ \Delta Y = \alpha \Delta M_3 - 3 \beta \Delta M_1 - 3 \beta \Delta M_2 \]

The endogenous variables are \( \Delta M_1 \), \( \Delta M_2 \), and \( \Delta M_3 \). The solution to the system (10), (11), and (12) is:

\[ \Delta M_1 = \frac{(\Delta Y \alpha + \Delta Y \beta)(\beta^2 + 3\alpha \beta) + (\Delta Y \beta + \Delta Y \alpha)(\alpha^2 - 3 \beta^2)}{\alpha^2 - 2 \beta^2 + 3 \alpha \beta}(\alpha^2 - 4 \beta^2 + 3 \alpha \beta) \]  
\[ \Delta M_2 = \frac{(\Delta Y \beta + \Delta Y \alpha)(\alpha^2 - 3 \beta^2) + (\Delta Y \beta + \Delta Y \alpha)(\beta^2 + 3 \alpha \beta)}{\alpha^2 - 2 \beta^2 + 3 \alpha \beta}(\alpha^2 - 4 \beta^2 + 3 \alpha \beta) \]  
\[ \Delta M_3 = \frac{(\Delta Y \beta + \Delta Y \alpha)(\beta^2 + 3 \alpha \beta) + (\Delta Y \beta + \Delta Y \alpha)(\alpha^2 - 3 \beta^2)}{\alpha^2 - 2 \beta^2 + 3 \alpha \beta}(\alpha^2 - 4 \beta^2 - 3 \alpha \beta) \]

4). Conclusion. According to equations (13), (14), (15), the total increase in European money supply depends on the initial output gap in Europe, the initial output gap in America, the initial output gap in Japan, the direct multiplier \( \alpha \), and the cross multiplier \( \beta \). The larger the initial output gap in Europe, the larger is the total increase in European money supply. And the total increase in America, Japanese money supply are in the same way.

This is a solution if and only if \( \alpha \neq \beta \). Due to the assumption \( \alpha > 2 \beta \), this condition is met. As a result, monetary cooperation can achieve full employment in Europe, America, and Japan. And what is more, it leads to price stability in Europe, America, and Japan. However, it does mean that this will lead to full employment and price stability in European member countries. It is worth to point out here that the solution to monetary cooperation is identical to the steady state of monetary competition.

References: